New nonperturbative approach to dynamic critical phenomena

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[arXiv:1307.1700]

Alexander von Humboldt
Stiftung / Foundation

Thank you, Kodama-san!

- Undergrad Classical Mechanics lecturer
- Professor of the same group for many years

Happy birthday and many many more years to come!

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Outline

- **Motivation:** dynamic critical phenomena + QCD Critical Point
- Real-time Nonperturbative RG framework
- Analysis of the case of a relaxational order parameter coupled to a conserved density -> **Model C**
- Conclusions and outlook

- ***Dynamic*** critical properties are much richer and hard to predict from the microscopic theory:

Static Universality Classes

fully determined by symmetries and dimensionality

x

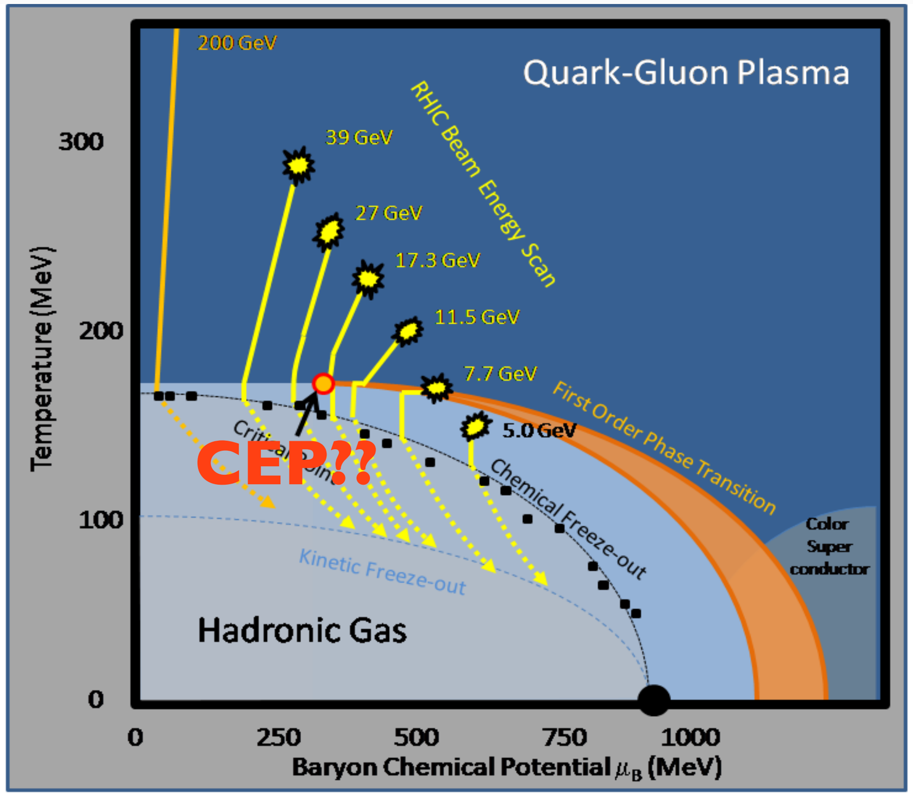
Dynamic Universality Classes

require the knowledge of the relevant long wavelength dofs: order parameter(s), conserved densities + their couplings

- Wide range of applications: **universality.**

➡ In particular: dynamical critical phenomena could be important in the CEP search in HICs

BES @ RHIC



The chiral CEP

- <u>General features:</u>

Second order phase transition

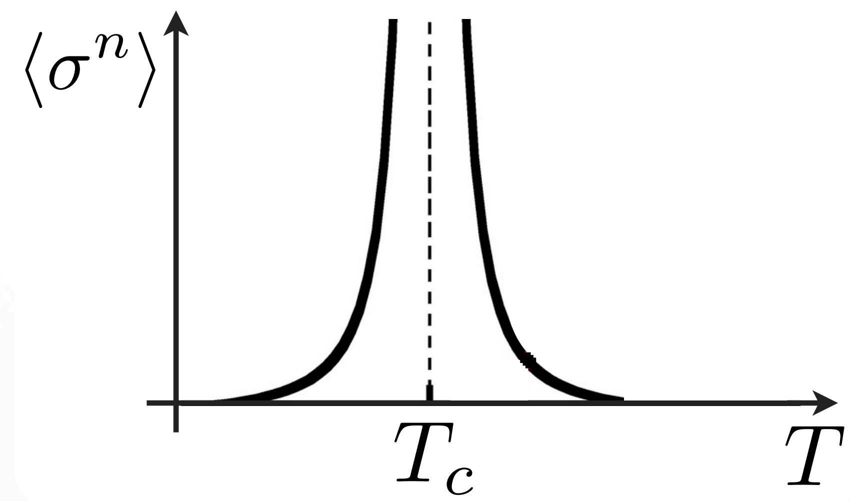
⇒ Diverging correlation length

⇒ Conformal invariance at criticality

⇒ large fluctuations at all scales

Correlations of the chiral condensate:

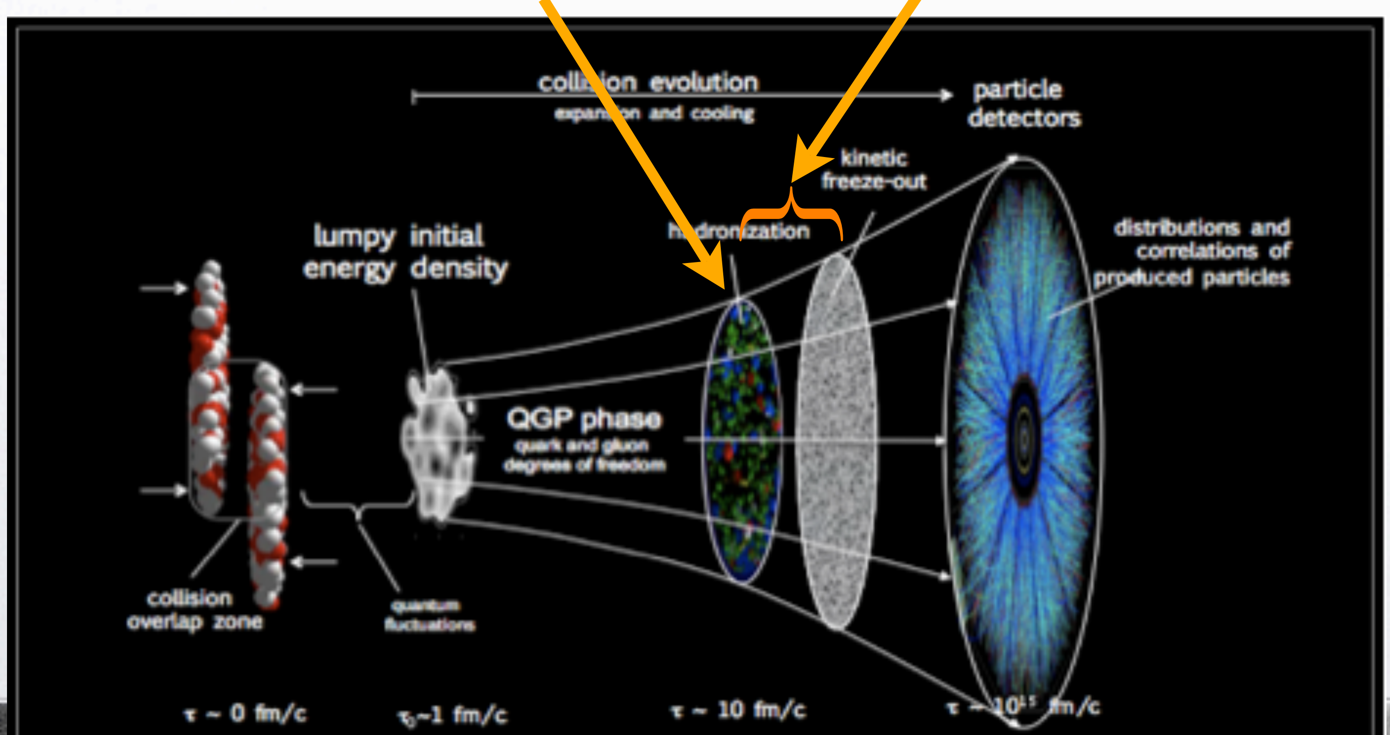
$$\langle \sigma^n \rangle \sim \xi^{p_n} \to \infty$$



- <u>In HICs:</u>

Chiral Ph. Trans.

Hadronic medium



Elliptic Flow: A Study of Space-Momentum Correlations In Relativistic Nuclear Collisions.
P. Sorensen, Quark Gluon Plasma, Vol. 4, World Scientific, arXiv:0905.0174 [nucl-ex]

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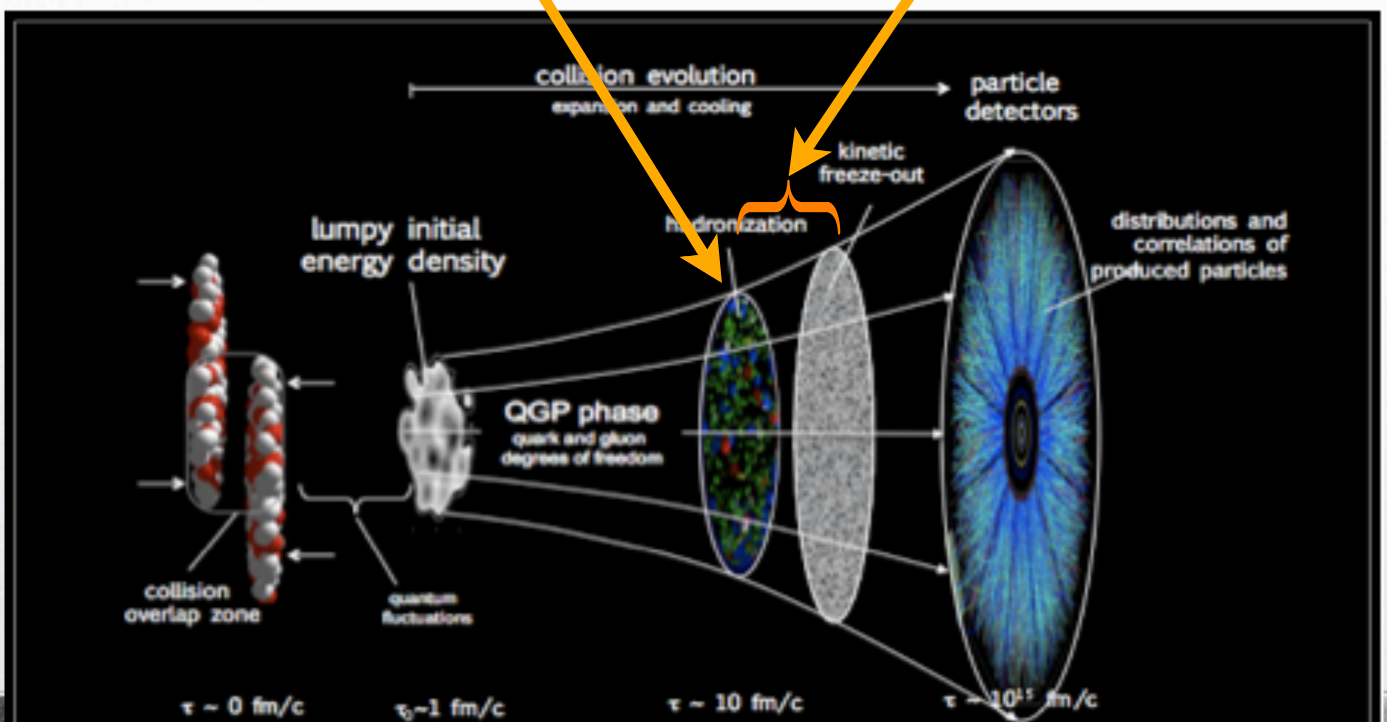

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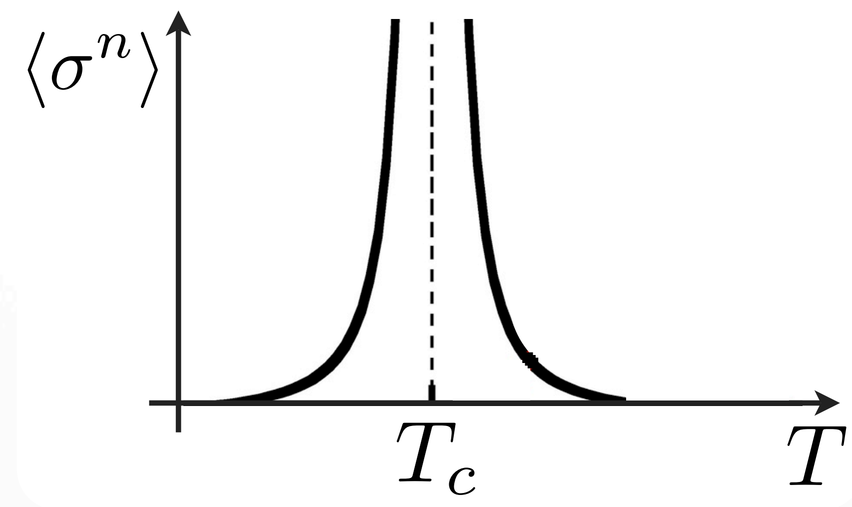
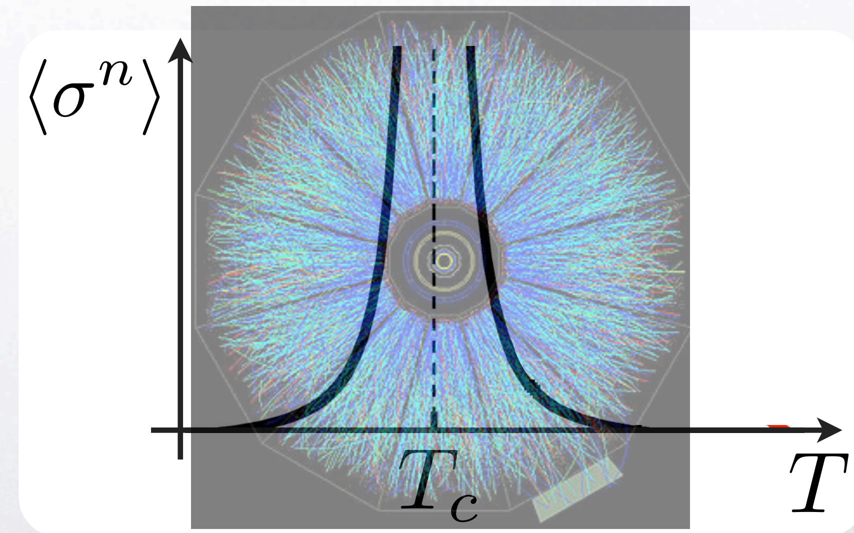
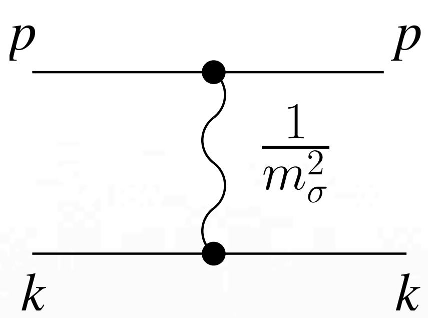




CEP search in HICs

- **A signature of the chiral CEP:** critical correlations of the chiral condensate will be transmitted to particles coupled to the sigma field, e.g. pions ($G\sigma\pi\pi$) and nucleons ($g_N \sigma \bar{N} N$): **[Stephanov et al]**



$$\langle \delta n_p \delta n_k \rangle_\sigma = \frac{1}{T} \frac{f_p(1+f_p)}{\omega_p} \frac{f_k(1+f_k)}{\omega_k} \frac{G^2}{m_\sigma^2} \sim \xi^2 \to \infty$$

Skewness: $$\omega_3(N)_\sigma = \frac{2\lambda_3}{T} \frac{G^3}{m_\sigma^6} \left(\int_p \frac{v_p^2}{\omega_p} \right)^3 \left(\int_p \bar{n}_p \right)^{-1} \sim \xi^{9/2}$$

Kurtosis: $$\omega_4(N)_\sigma = \frac{6}{T} \left[2\frac{\lambda_3^2}{m_\sigma^2} - \lambda_4 \right] \frac{G^4}{m_\sigma^8} \left(\int_p \frac{v_p^2}{\omega_p} \right)^4 \left(\int_p \bar{n}_p \right)^{-1} \sim \xi^7$$

- **However, the growth of the correlation length is limited in HICs:**

[LFP, Fraga, Kodama, JPG (2011)]

➡ *finite size effects* → Sizable for QCD transitions in HIC; FSS signature

→ No sign of scaling in data ***[Fraga, LFP, Sorensen, PRC (2011)]***

➡ ***finite lifetime:*** how much does the correlation length grow?

$$\xi \sim t^z$$

z: universal dynamic critical exponent

The Functional Renormalization Group

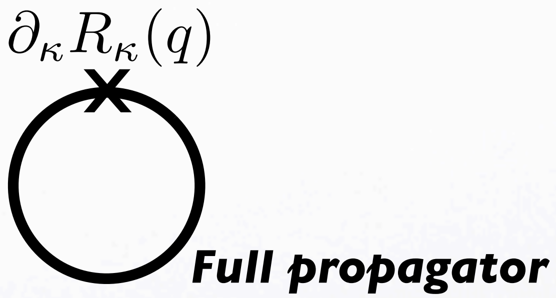
➡ ***'Microscope' with varying resolution ~1/k:***

- Construction of a set of effective actions $\Gamma_\kappa[\varphi]$ which interpolate between the classical action $S[\varphi]$ and the full effective action $\Gamma[\varphi]$.

- The trajectory parameterized by the scale κ in the effective-action space is determined by an exact renormalization group equation, which actually encodes an infinite hierarchy of coupled exact RG equations involving the n−point functions.

e.g. Scalar case:

$$\partial_\kappa \Gamma_\kappa[\varphi] = \frac{1}{2} \mathrm{Tr} \int_q \partial_\kappa R_\kappa(q) \left[\Gamma_\kappa^{(2)}[q;\varphi] + R_\kappa(q)\right]^{-1}$$



- At finite κ, the interpolating theory presents a suppression of IR modes, being totally finite. The renormalization in this context is implicitly contained in the initial conditions of the flow.

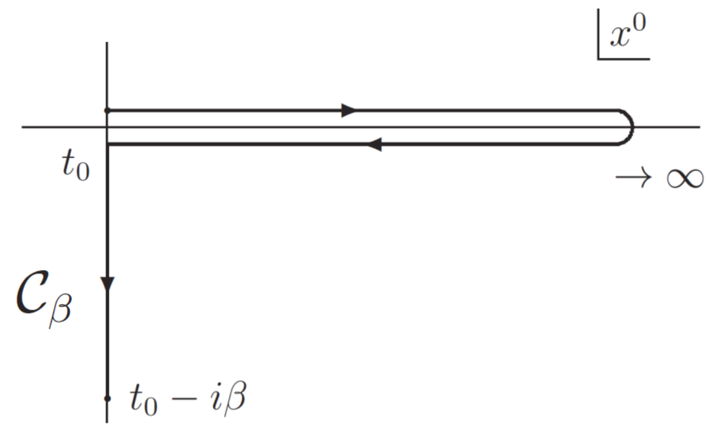
FRG is a practical tool only if a *sensible* truncation is implemented!

(= gives a description of the ingredients you are interested in and at least some control of the approximation)

Real-time FRG

- To describe dynamic properties (even close to thermal equilibrium) a real-time technique is needed.

Closed time path



➡ Forward and backward correlations distinguished

$F = \langle \{\phi, \phi\} \rangle$: statistical correlator

$\rho = \langle [\phi, \phi] \rangle$: spectral function

➡ C_β : initial density matrix *in equilibrium*

Fluctuation-Dissipation Relation: $iF^{(eq)} = \left(\frac{1}{2} + n_{BE}\right)\rho^{(eq)}$

- FRG on the CTP:
  - matrix structure due to doubling of degrees of freedom
  - built at the propagator level (instead of effective action) fully in terms of commutators (using FDR).

Model C: coupling to conserved density

- Relaxational dynamics of an *N-component order parameter* *coupled to a conserved density*:

$$\frac{\partial}{\partial t}\varphi_a(x,t) = -\Omega\frac{\delta\mathcal{H}[\varphi,\varepsilon]}{\delta\varphi_a(x,t)} + \eta_a(x,t)$$

$$\frac{\partial}{\partial t}\varepsilon(x,t) = \Omega_\varepsilon\nabla^2\frac{\delta\mathcal{H}[\varphi,\varepsilon]}{\delta\varepsilon(x,t)} + \zeta(x,t)$$

Noise

Ω: relaxation rate

Ω_ε: diffusion rate

$$\mathcal{H} = \int d^dx\left\{\frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}\bar{m}^2\varphi^2 + 3\frac{\bar{\lambda}}{4!}\left(\varphi^2\right)^2 + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\bar{\gamma}\varepsilon\varphi^2\right\}$$

- Equivalent to the variational principle of an MSR action:

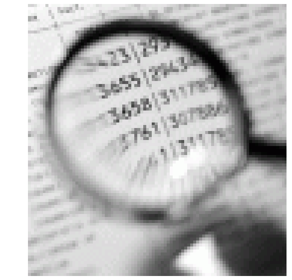
$$S = \int_{[t_0,\infty)} d^dx\,dt\left\{\tilde{\varphi}_a\left(\Omega^{-1}\frac{\partial}{\partial t}\varphi_a + \frac{\delta\mathcal{H}}{\delta\varphi_a}\right) - \Omega^{-1}\tilde{\varphi}^2 + \tilde{\varepsilon}\left(\Omega_\varepsilon^{-1}\frac{\partial}{\partial t}\varepsilon - \nabla^2\frac{\delta\mathcal{H}}{\delta\varepsilon}\right) + \Omega_\varepsilon^{-1}\tilde{\varepsilon}\nabla^2\tilde{\varepsilon}\right\}$$

  - quadratic in ε: auxiliary field that encodes complicated momentum-dependent interactions in a microscopic theory for the field ϕ.
  - may in principle be obtained from a microscopic theory on the CTP

Ansatz for the FRG flowing action

$k > \Lambda$ UV

‘Microscopic’ physics: relativistic (QCD), unitary (cold atoms), etc



- Ansatz: (non-relativistic) MSR action with k-dependent couplings.

$\rho_k,\ \lambda_k,\ Z_k,$ (statics) $Z_{\varepsilon,k},\ \gamma_k,\ \Omega_k$ (dynamics)

$k \sim T$

- In the deep IR limit: $Z_k \sim k^{-\eta}$ $\quad Z_{\varepsilon,k} \sim k^{-\eta_\varepsilon}$ $\quad \Omega_k \sim k^{\eta_\Omega}$

$$z = 2 - \eta + \eta_\Omega \qquad z_\varepsilon = 2 - \eta_\varepsilon \qquad [\xi \sim t^z]$$

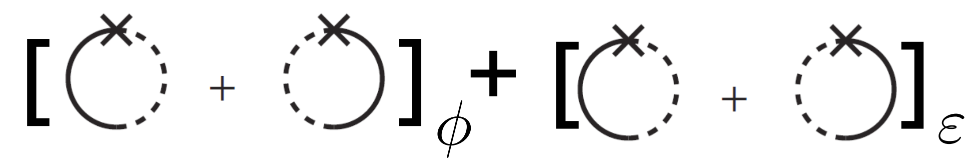
non-relativistic regime

- Litim regulator in spatial momentum; frequencies are not regulated.

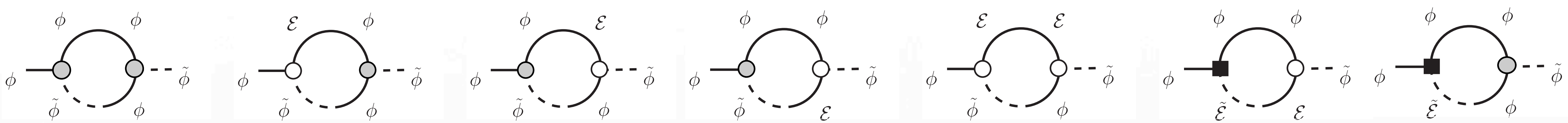
$k \to 0$ IR

Flow equations

$$\partial_\kappa \Gamma_\kappa[\varphi] = \frac{1}{2} \mathrm{Tr} \int_q \partial_\kappa R_\kappa(q) \left[\Gamma_\kappa^{(2)}[q;\varphi] + R_\kappa(q) \right]^{-1}$$



- The 2-point correlation functions receive contribution from >50 diagrams, but the nontrivial (and new) ones are the cuts of the following one-loop diagrams:



- Statics is unchanged!

- <u>Fixed point solution</u>: e.g. kinetic parameter

$$\kappa = \frac{1}{1 + \frac{Z_\epsilon}{\Omega Z}}$$

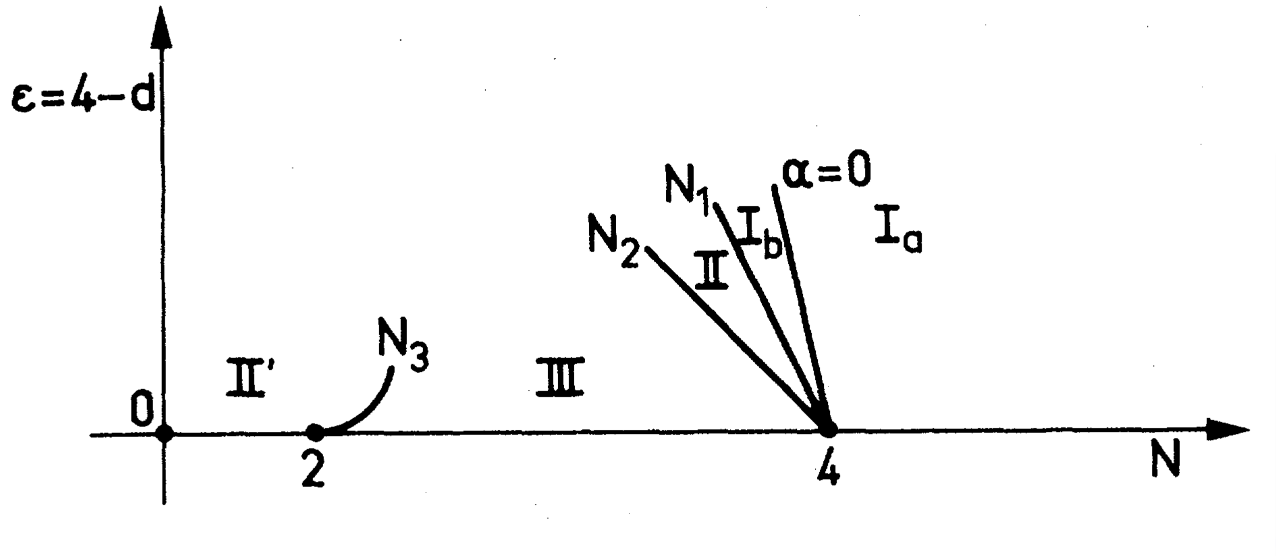
Relaxation rate of the OP	$\overset{\kappa=1}{\gg}$ $\overset{\kappa=0}{\ll}$	Diffusion rate of conserved density

$$\dot{\kappa} = \kappa(1-\kappa)[\eta_\Omega(\kappa) - \eta + \eta_\varepsilon]$$

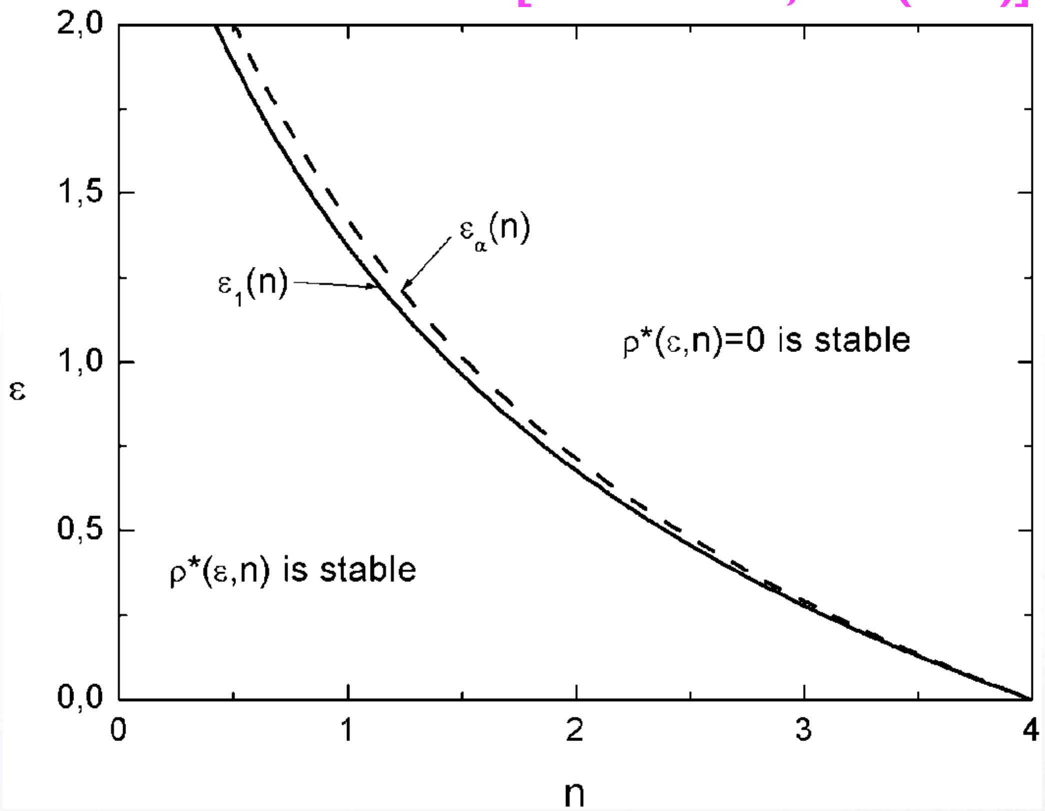
Previous results: the Model C phase diagram

- Status of the Model C dynamics phase diagram:

[Brézin & De Dominicis, PRB (1975)]
Hohenberg, Halperin, Ma '74 -- '78



[Volk & Moser, PRL (2003)]



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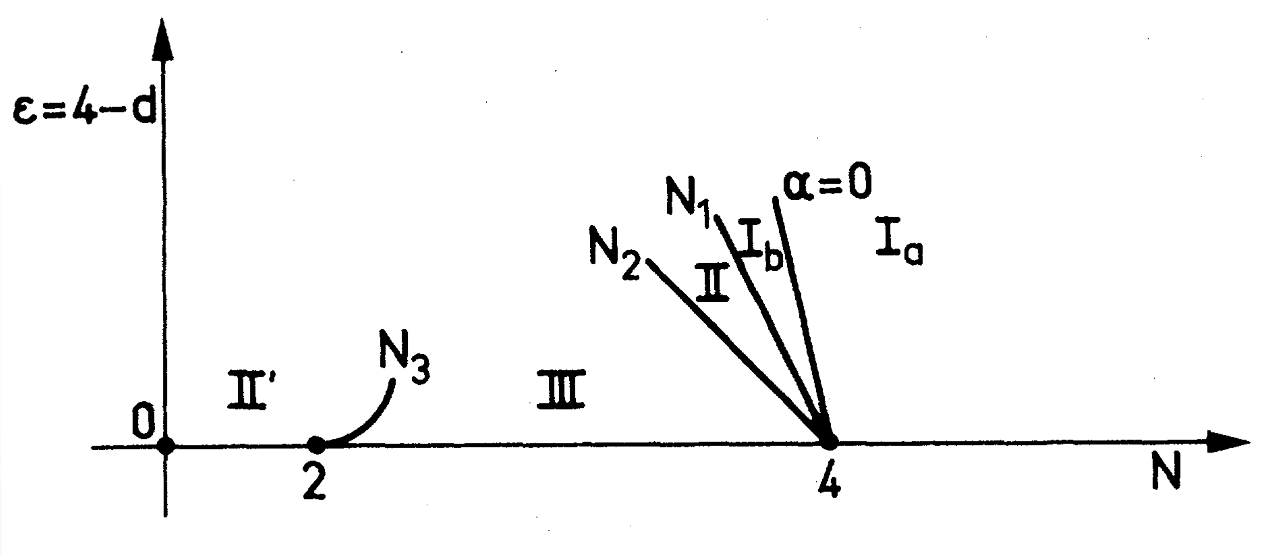

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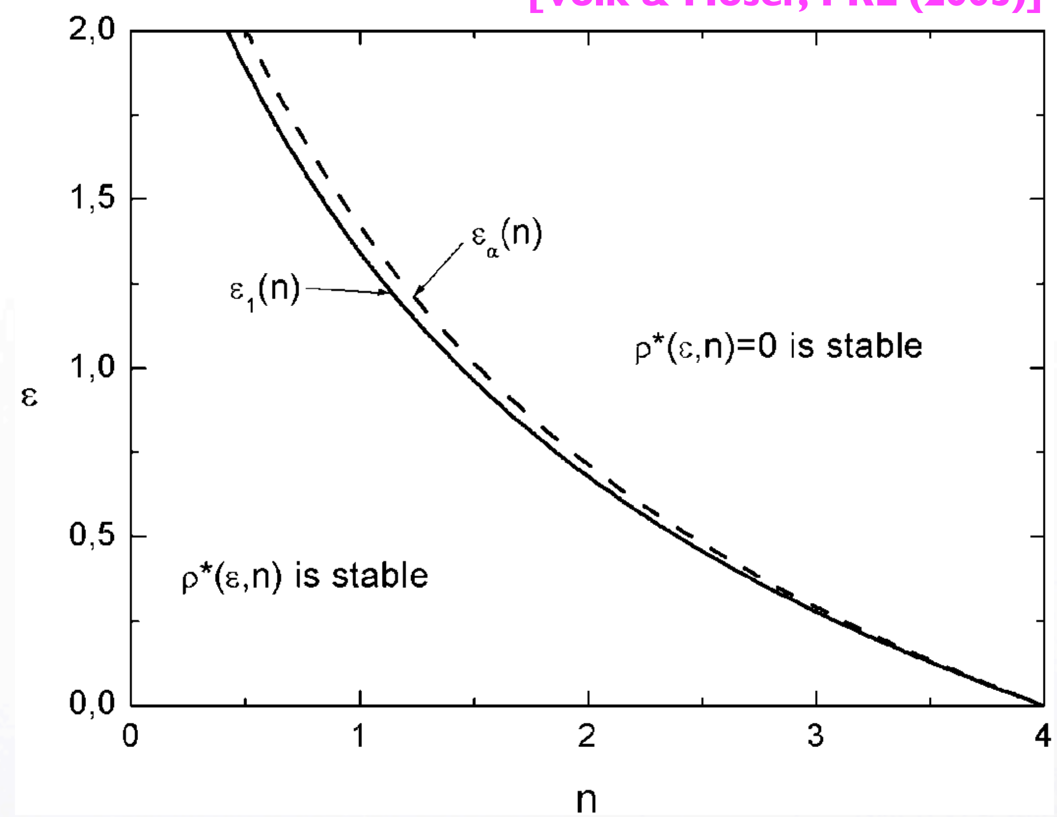
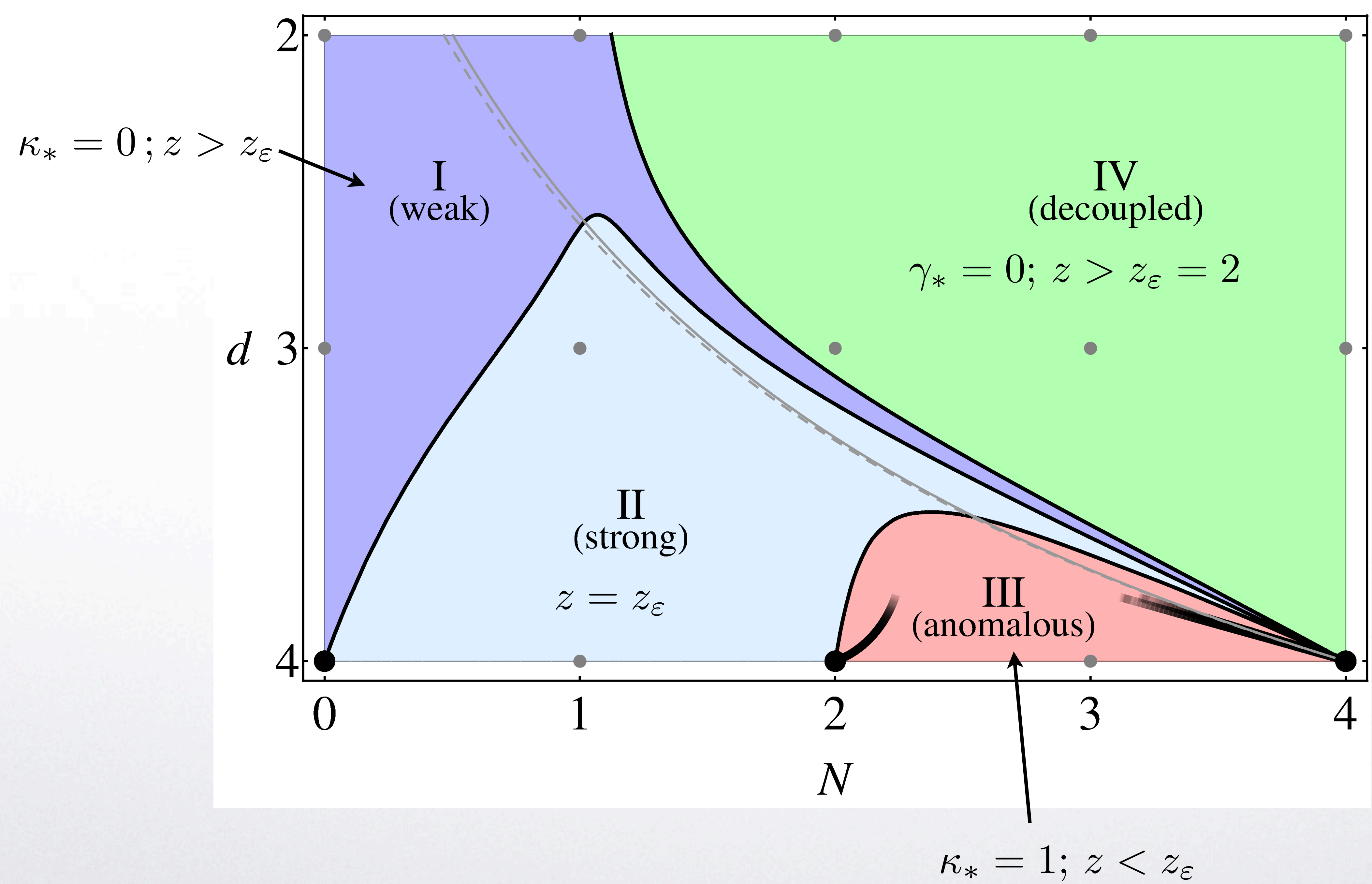


→ **One-loop ε-expansion predicted 5 possible regions**

→ **30 years later: 2-loop results claim breakdown of ε-expansion for 2<N<4...**

Model C phase diagram: nonperturbative results



Conclusions and Outlook

• Dynamic critical phenomena are present in a wide variety of physical systems in Nature and experiments. In particular, they may be important in the QCD CEP search in HIC's.

• Yet, their classification is much more complicated than that of static critical phenomena: requires the knowledge of relevant IR degrees of freedom.

• Real-time FRG is a powerful nonperturbative tool which is especially suitable to describe universal phenomena in the vicinity of 2nd order phase transitions. [Canet et al, PRL (2004)]

• We have showed results for the full nonperturbative Model C phase diagram and established the existence of an anomalous scaling region and a significant change for low N's.

• The framework alows for the investigation of the transition between micro and IR physics.

• It can also treat couplings between different order parameters, as should be the case of QCD.

→

Thank you for your attention!

One should always be cautious with hydro...

Boat trip (ISMD 2006)