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HYDRODYNAMIC FLUCTUATION IN RELATIVISTIC HEAVY ION COLLISIONS

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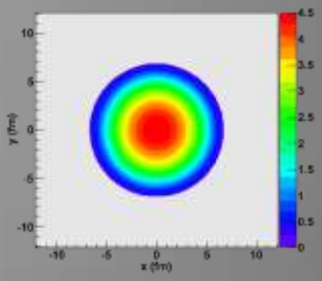
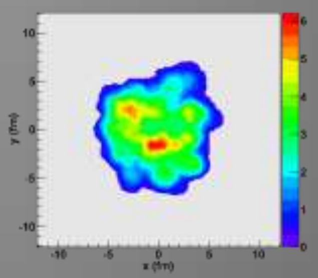
Outline

1. Introduction
2. Relativistic fluctuating hydrodynamics
3. Summary and outlook

Interpretation of data using hydro

	coarse graining size	initial profile
• Most of the people did not believe hydro description of the QGP (~ 1995)		
• Hydro at work to describe elliptic flow (~ 2001)	$d \lesssim 5$ fm	
• E-by-e hydro at work to describe higher harmonics (~ 2010)	$d \lesssim 1$ fm	
• Even in p+p and/or p+A?	$d \lesssim 1$ fm?	?

Thermal Fluctuation

- Conventional hydro describes space-time evolution of (coarse-gained) thermodynamic quantities.
- Some of microscopic information must be lost through coarse-graining process.
- Does the lost information play an important role in dynamics on an e-by-e basis?
  → Thermal (Hydrodynamic) fluctuation!

J.Kapusta, B.Muller, M.Stephanov, PRC85(2012)054906.
J.Kapusta, J.M.Torres-Rincon, PRC86(2012)054911.
P.Kovtun, G.D.Moore, P.Romatschke, PRD84(2011)025006.
J.Peralta-Ramos, E.Calzetta, JHEP1202(2012)085.

Green-Kubo Formula

$$\eta = \lim_{\omega\to 0}\lim_{q\to 0}\frac{1}{2\omega}\int dtdx\, e^{i(\omega t-qx)}$$

$$\times\left\langle\left[T_{xy}(t,x),T_{xy}(0,0)\right]\right\rangle$$

Slow dynamics → How slow?

Macroscopic time scale ~ $1/\omega$ ← $t_{\text{"macro"}}$

Microscopic time scale ~ τ

cf.) Long tail problem (liquid in 2D, glassy system, super-cooling, etc.)

Relaxation and Causality

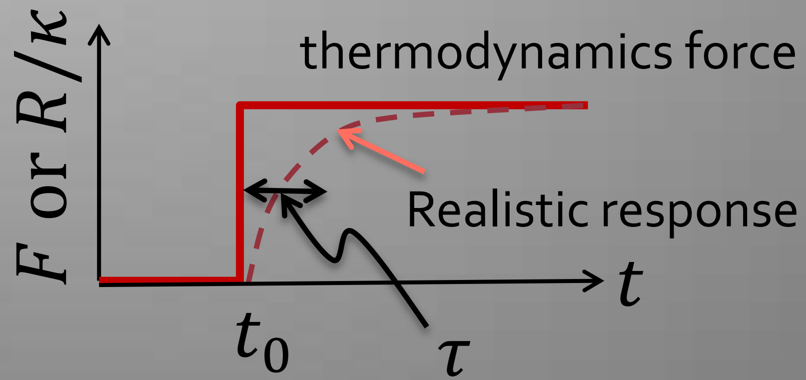
Constitutive equations at Navier-Stokes level

$$\pi^{\mu\nu} = 2\eta\partial^{<\mu}u^{\nu>},$$
$$\Pi = -\varsigma\partial_\mu u^\mu,$$
$$\ldots$$

Instantaneous response violates causality

→ Critical issue in relativistic theory

→ Relaxation plays an essential role



Causal Hydrodynamics

Linear response to thermodynamic force

$$\Pi(t) = \int dt' G_R(t,t')F(t')$$

Retarded Green function (as an example)

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

Differential form

$$\dot{\Pi}(t) = -\frac{\Pi(t) - \kappa F(t)}{\tau}, \qquad v_{\mathrm{signal}} = \sqrt{\frac{\kappa}{\tau}} < c$$

➡ Maxwell-Cattaneo Eq. (simplified Israel-Stewart Eq.)

Relativistic Fluctuating Hydrodynamics (RFH)

Generalized Langevin Eq.

dissipative current | thermodynamic force | hydrodynamic fluctuation

$$\Pi(x) = \int d^4x' G_R(x,x') F(x') + \delta\Pi(x)$$

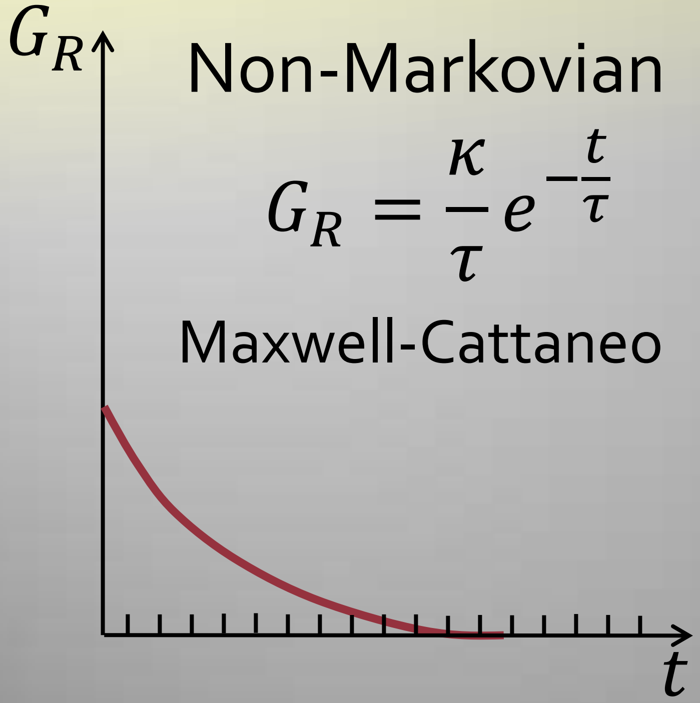
Fluctuation-Dissipation Relation (FDR)

$$\langle \delta\Pi(x)\delta\Pi(x')\rangle = TG^*(x,x')$$

G^* : Symmetrized correlation function

For non-relativistic case, see Landau-Lifshitz, Fluid Mechanics

Coarse-Graining in Time



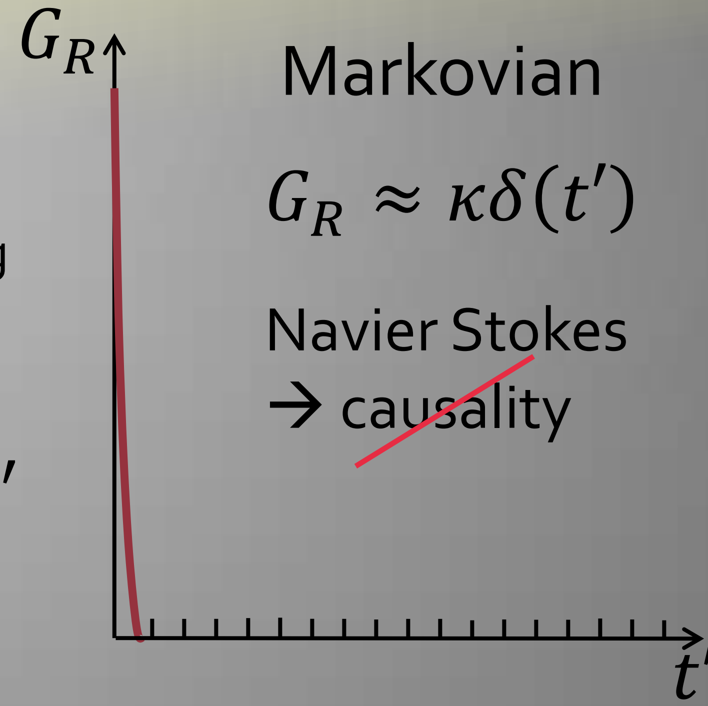
Non-Markovian

$$G_R = \frac{\kappa}{\tau} e^{-\frac{t}{\tau}}$$

Maxwell-Cattaneo

coarse graining ???

$t \rightarrow t'$



Markovian

$$G_R \approx \kappa\delta(t')$$

Navier Stokes

→ ~~causality~~

Existence of upper bound in coarse-graining time (or lower bound of frequency) in relativistic theory???

K.Murase and TH, arXiv:1304.3243[nucl-th]

Colored Noise in Relativistic System

$$G_R(t,t') = \frac{\kappa}{\tau}\exp\left(-\frac{t-t'}{\tau}\right)\theta(t-t')$$

G^*: Extention to t<t'

Correlation in Fourier space

$$\langle \delta\Pi^*_{\omega,\boldsymbol{k}}\delta\Pi_{\omega',\boldsymbol{k}'}\rangle = 2\kappa\frac{(2\pi)^4\delta(\omega-\omega')\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')}{1+\omega^2\tau^2}$$

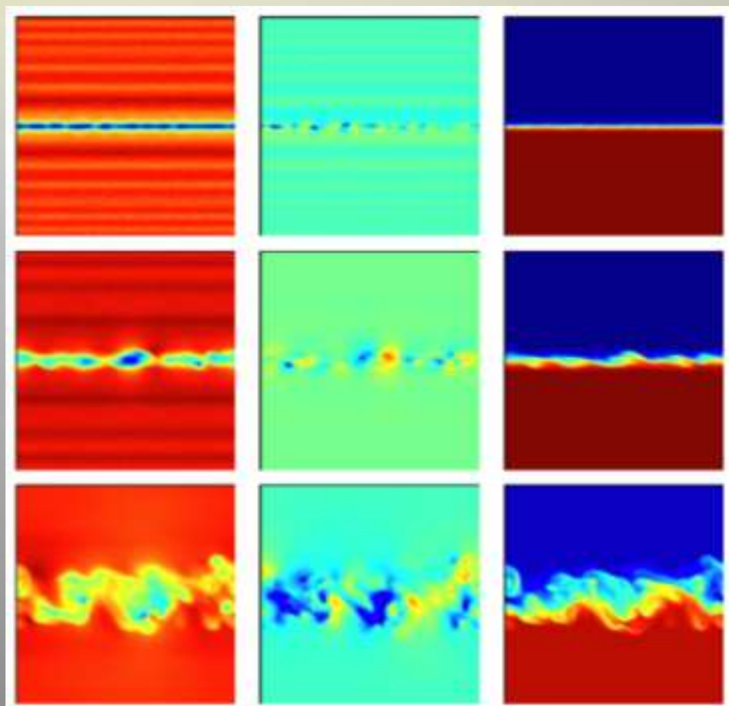
→ Colored noise!

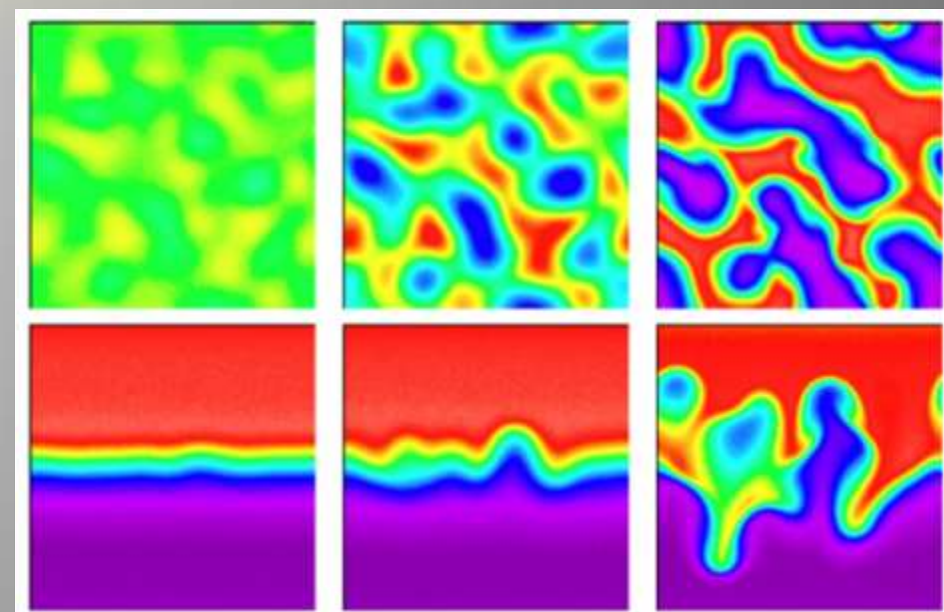
→ (Indirect) consequence of causality

→ Note: white noise in differential form

Need Fluctuation?

Ex.) Seeds for instabilities





Rayleigh-Taylor instability

Kelvin-Helmhortz instability

Non-linearity, instability, dynamic critical phenomena,...

Summary and Outlook

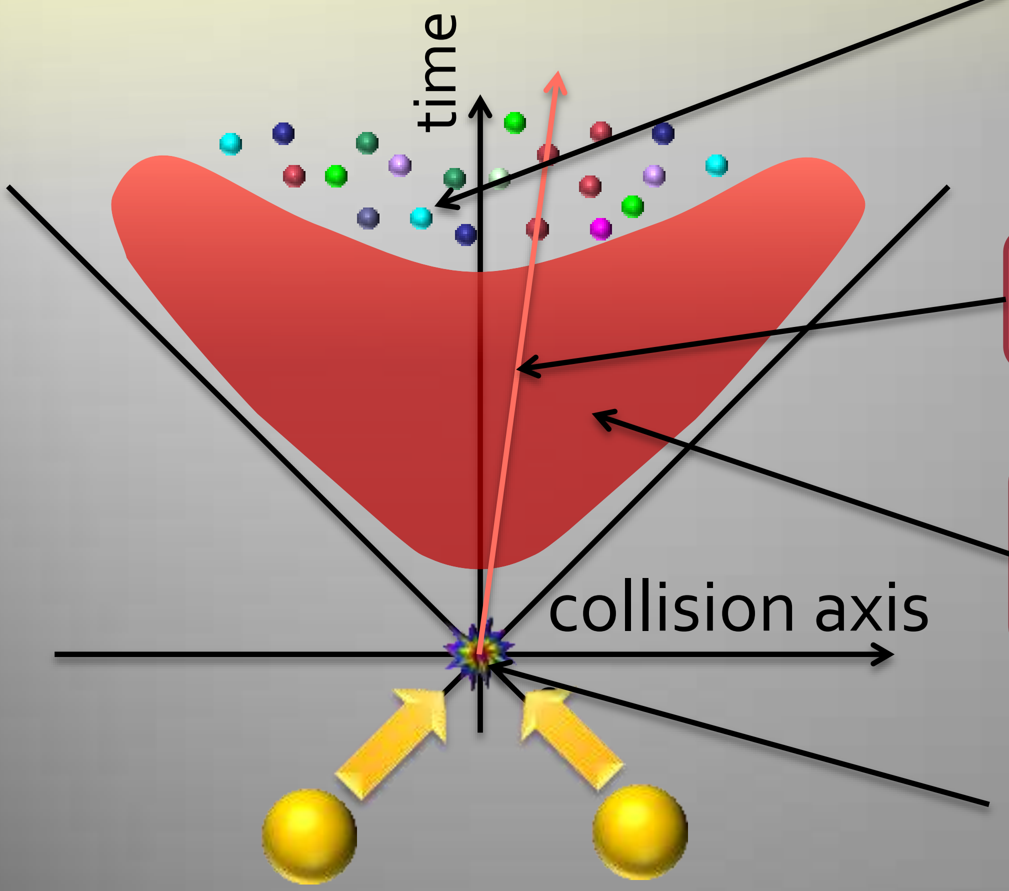
- Implement of hydrodynamic fluctuation into causal hydrodynamics
- Colored noise as a consequence of causality
- Numerical implementation and its consequences in observables
- Development of a more sophisticated dynamical model towards precision heavy-ion physics

Happy 70th Birthday, Kodama-san

Greetings and a present from
Shin Muroya,
Chiho Nonaka and
Tetsufumi Hirano
(Alumni of Waseda Univ.)

Japanese spirits,
"Waseda Spirits"

Fluctuation Appears Everywhere



Finite number of hadrons

Propagation of jets

Hydrodynamic fluctuation

Initial fluctuation and particle production