Thermalization of the quark-gluon plasma and Bose-Einstein condensation in unusual circumstances

RANP
Rio de Janeiro
September 25, 2013

European Research Council

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THERMALIZATION

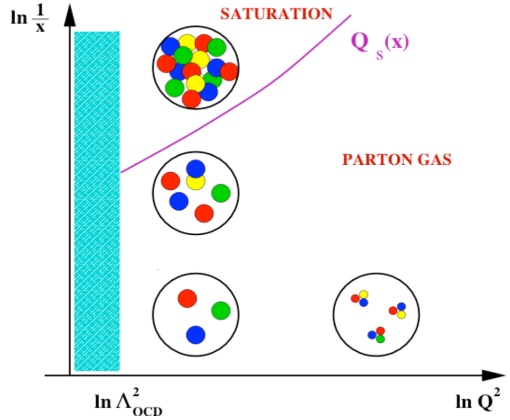
- What is the origin of the apparent strongly coupled character of the quark-gluon plasma ? (The QCD coupling constant is not (cannot be) infinite.....)
- How do we go from the initial nuclear wave-functions to the locally equilibrated fluid «seen» in experiments ?
- What are the initial d.o.f.'s : partons ? color fields (CGC)? mixture of both ?
- Initial fields are typically unstable (e.g. if anisotric momentum distributions of particles). Instabilities provide 'fast' isotropization of momentum distributions
- CGC picture suggests that the *initial gluon density is too large to be accommodated by an equilibrium distribution*.

(for a summary see arXiv: 1203.2042)

The over-populated quark-gluon plasma

CGC initial conditions

$$Q_s^2 \approx \alpha_s \frac{xG(x,Q^2)}{\pi R^2}$$

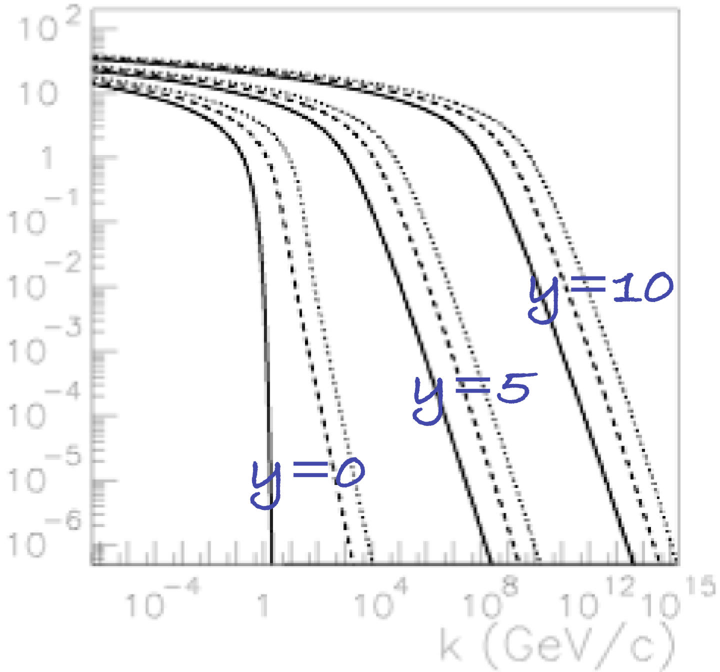


At saturation, occupation numbers are large

$$\frac{xG(x,Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$

Most partons taking part in collision have $k_T \sim Q_s$

$$f_A(k_\perp \ll Q_s) \approx \frac{1}{\alpha N_c} \ln \frac{Q_s^2}{k_\perp^2}$$



$$Q_s^2(x,A) \simeq Q_0^2 A^{1/3} \left(\frac{x_0}{x}\right)^\lambda$$

$$\lambda = 0.2 \div 0.3$$

$$f_A(k_\perp \gg Q_s) \approx \frac{1}{\alpha N_c} \frac{Q_s^2}{k_\perp^2}$$

Thermodynamical considerations

Initial conditions $(t_0 \sim 1/Q_s)$

$$\epsilon_0 = \epsilon(\tau = Q_s^{-1}) \sim \frac{Q_s^4}{\alpha_s} \qquad n_0 = n(\tau = Q_s^{-1}) \sim \frac{Q_s^3}{\alpha_s} \qquad \epsilon_0 / n_0 \sim Q_s$$

overpopulation parameter $\quad n_0\, \epsilon_0^{-3/4} \sim 1/\alpha_s^{1/4}$

In equilibrated quark-gluon plasma

$$\epsilon_{\rm eq} \sim T^4 \qquad n_{\rm eq} \sim T^3 \qquad n_{\rm eq}\, \epsilon_{\rm eq}^{-3/4} \sim 1$$

mismatch by a large factor (at weak coupling) $\quad \alpha_s^{-1/4}$

Will the system accommodate the particle excess by forming a Bose-Einstein condensate ?

(JPB, F. Gelis, J. Liao, L. McLerran, R. Venugopalan, 2012)

How does an over-populated system evolve towards equilibrium ?

An interesting problem in itself

Simplifying assumptions

- spatially uniform, non expanding systems
- isotropic in momentum space
- elastic scattering only

Question : is onset of BEC reached in a finite time ?

Similar studies (cold atoms and cosmology):

- R. Lacaze et al. Physica D 152-153 (2001) 779
- D.V. Semikoz, I.I. Tkachev, PRA 55(1997) 489

[work in collaboration with Jinfeng Liao and Larry McLerran]

Boltzmann equation with 2->2 scattering

Gluon distribution function

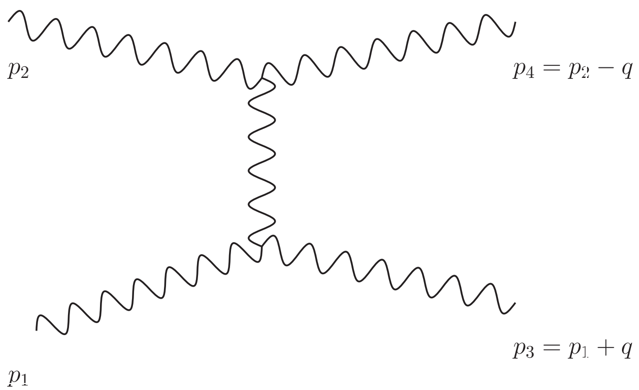
$$f(\boldsymbol{x}, \boldsymbol{p}) = \frac{(2\pi)^3}{2(N_c^2 - 1)} \frac{dN}{d^3\boldsymbol{x} d^3\boldsymbol{p}}$$

Boltzmann equation

$$\mathcal{D}_t f_1 = \frac{1}{2} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \frac{1}{2E_1} \mid M_{12\to 34} \mid^2$$
$$\times (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4)\{f_3 f_4 (1+f_1)(1+f_2) - f_1 f_2 (1+f_3)(1+f_4)\}$$

$$\mathcal{D}_t \equiv \partial_t + \vec{v}_1 \cdot \vec{\bigtriangledown}$$

$M_{12\to 34}$ =



Small angle approximation

$$\mathcal{D}_t f = -\boldsymbol{\nabla} \cdot \mathcal{J} = -\frac{\partial \mathcal{J}_i}{\partial p_i}$$

Simplified kinetic equation

$$\mathcal{D}_\tau f(\boldsymbol{p}) = \boldsymbol{\nabla} \cdot \left[I_a \boldsymbol{\nabla} f(\boldsymbol{p}) + \frac{\boldsymbol{p}}{p} I_b \, f(\boldsymbol{p})[1 + f(\boldsymbol{p})] \right]$$

$$I_a = \int \frac{d^3p}{(2\pi)^3} f(\boldsymbol{p})(1 + f(\boldsymbol{p}))$$ [~diffusion constant]

$$I_b = \int \frac{d^3p}{(2\pi)^3} \frac{2f(\boldsymbol{p})}{p}$$ [~screening mass]

'Universal' equation

$$\tau = 36\pi\alpha^2 \mathcal{L} t$$

$$\mathcal{L} = \int \frac{dq}{q}$$ [Coulomb logarithm]

Used in 'linear' approximation, (1+f)--> 1 , by

A.H. Mueller, PLB475 (2000) 220

J. Bjoraker, R. Venugopalan PRC 63, 024609

Isotropic solutions

Look for isotropic solutions

$$\partial_\tau f(\tau,p) = -\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 \mathcal{J}(\tau,p)\right) \qquad \mathcal{J} = -I_a\,\partial_p f - I_b\, f(1+f)$$

with initial condition

$$f(p) = f_0\,\theta(Q_s - p) \qquad \epsilon_0 = f_0\,\frac{Q_s^4}{8\pi^2} \qquad n_0 = f_0\,\frac{Q_s^3}{6\pi^2}$$

Onset of BEC

$$n_0\,\epsilon_0^{-3/4} = f_0^{1/4}\,\frac{2^{5/4}}{3\,\pi^{1/2}}. \qquad n\,\epsilon^{-3/4}|_{SB} = \frac{30^{3/4}\,\zeta(3)}{\pi^{7/2}} \approx 0.28.$$

$$f_0^c \approx 0.154$$

Two types of solutions

$$f_0 < f_0^c \quad \text{or} \quad f_0 > f_0^c$$

Two types of solution

$f_0 < f_0^c$ (under-occupied) $$f_{eq}(p) = \frac{1}{\mathrm{e}^{(p-\mu)/T} - 1}$$

$f_0 > f_0^c$ (over-occupied) $$f_{eq}(p) = \frac{1}{\mathrm{e}^{p/T} - 1} + n_c\, \delta^{(3)}(\boldsymbol{p})$$

Equilibrium parameters are determined from number and energy densities

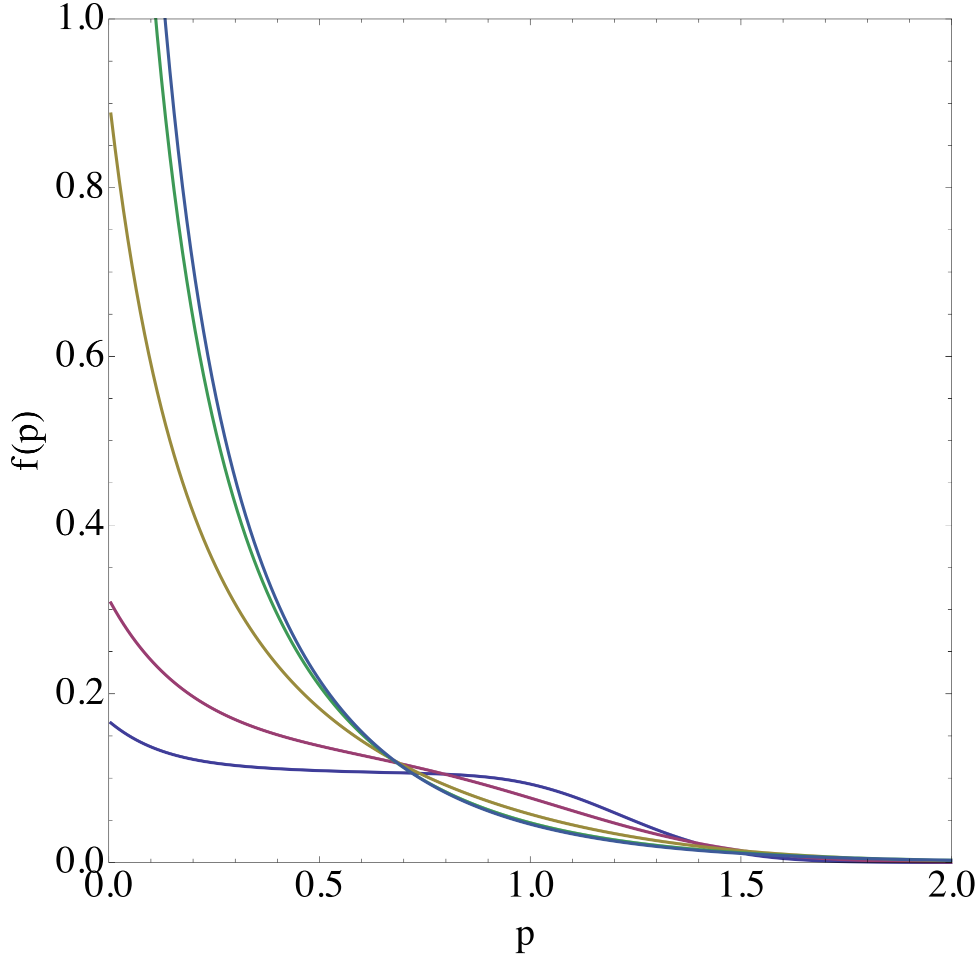
NB i) In equilibrium, the entropy is maximum and the current vanishes

$$\frac{d\mathcal{S}}{d\tau} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\partial f}{\partial \tau} \ln \frac{1+f}{f} = -\int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \mathcal{J}(p) \frac{1}{f(1+f)} \frac{\partial f}{\partial p}$$

ii) In equilibrium

$$I_b[f_{eq}] = -\int \frac{d^3p}{(2\pi)^3} \frac{\partial f_{eq}}{\partial p} = \frac{1}{T} I_a[f_{eq}]$$

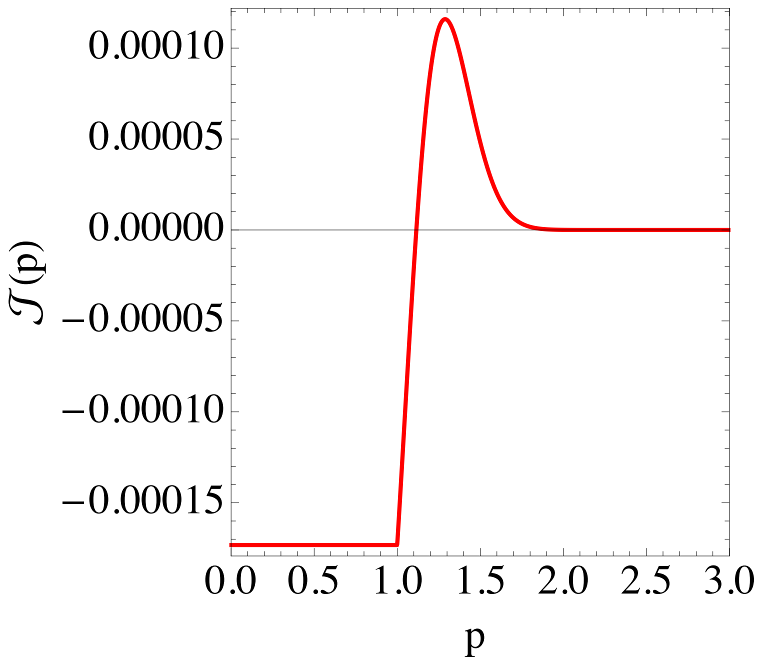
Generic pattern of evolution towards equilibrium (under-populated case)



The rapidly growing population of low momentum modes

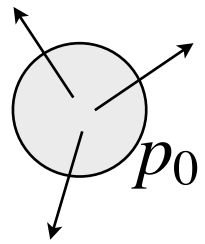
$$\partial_\tau f(\tau,p) = -\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2\mathcal{J}(\tau,p)\right)$$

$$\mathcal{J} = -I_a\,\partial_p f - I_b\, f(1+f)$$

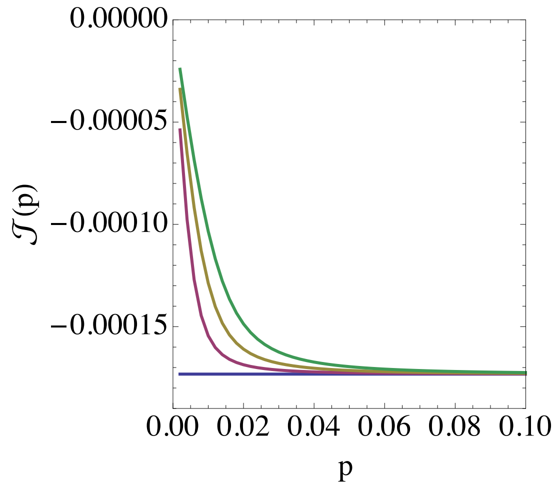


initial current

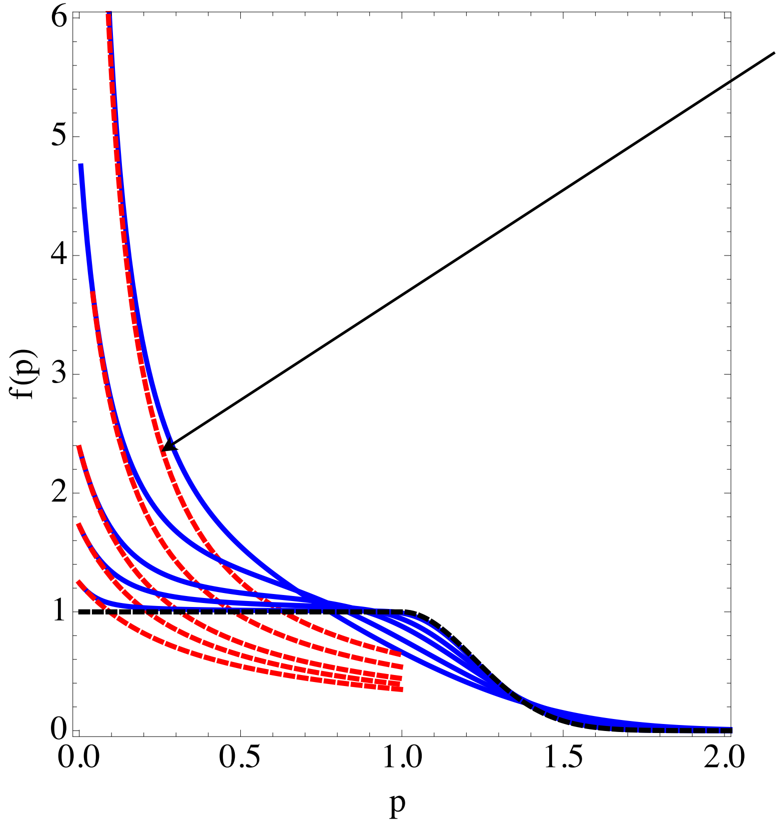
$$\partial_\tau \int_0^{p_0} dp\, p^2 f(p) = -p_0^2\,\mathcal{J}(p_0)$$



$$\frac{\partial f(0)}{\partial \tau} = -\frac{3}{p_0}\mathcal{J}(p_0)$$

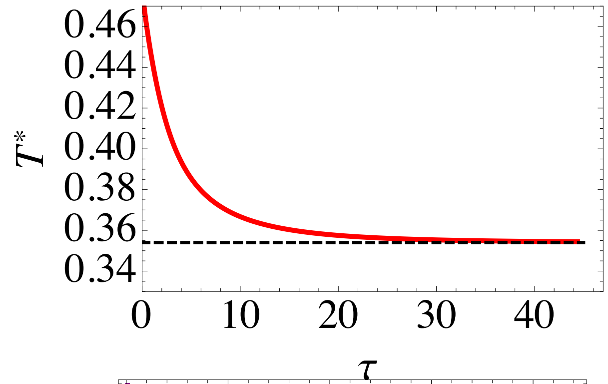


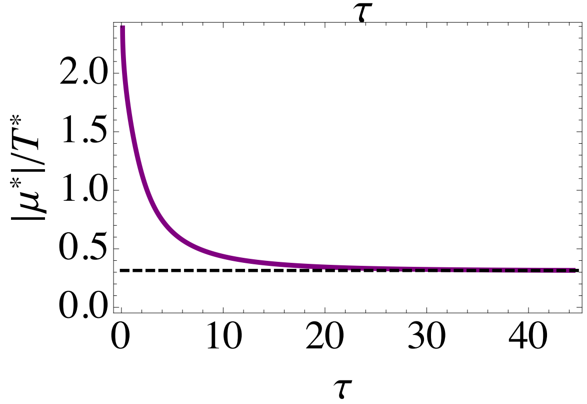
Small momentum tail well reproduced by a 'classical' distribution



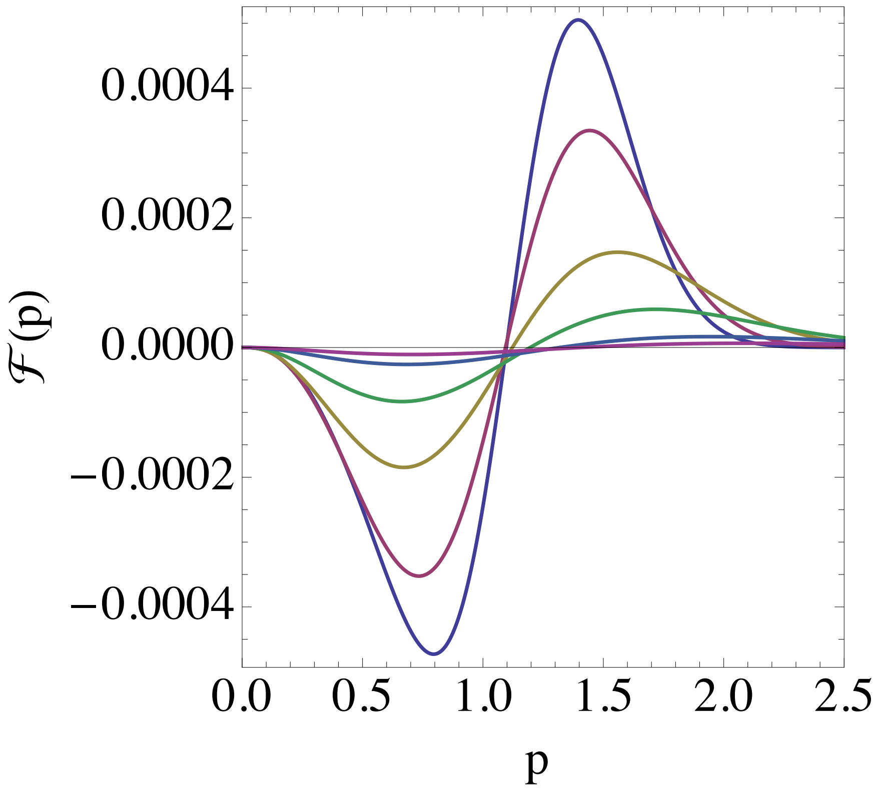
$$f = \frac{T^*}{p - \mu^*} \qquad T^* \equiv \frac{I_a}{I_b}$$

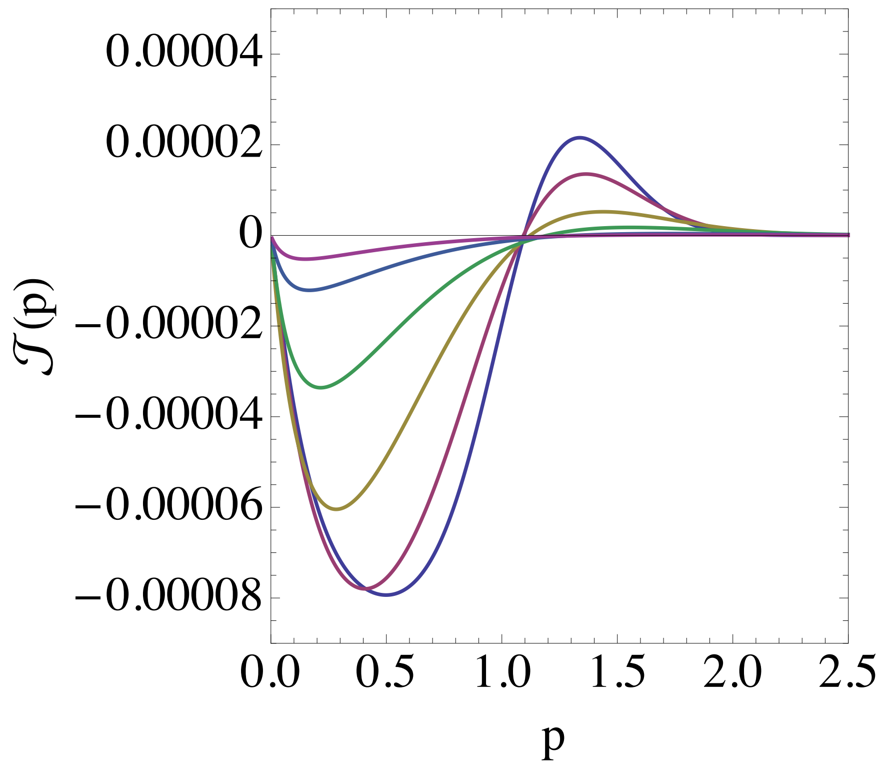
$$(\mu^* < 0)$$





Evolution towards equilibrium (under-populated case)

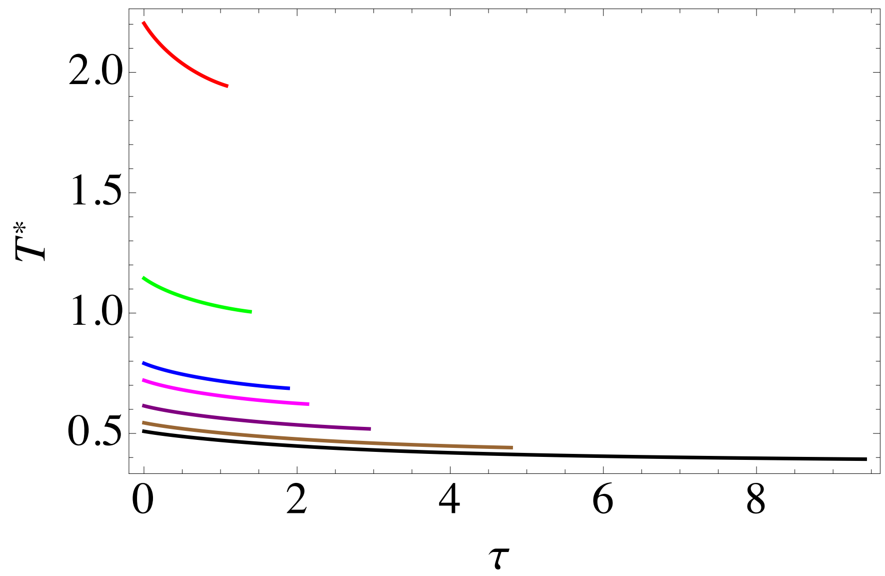


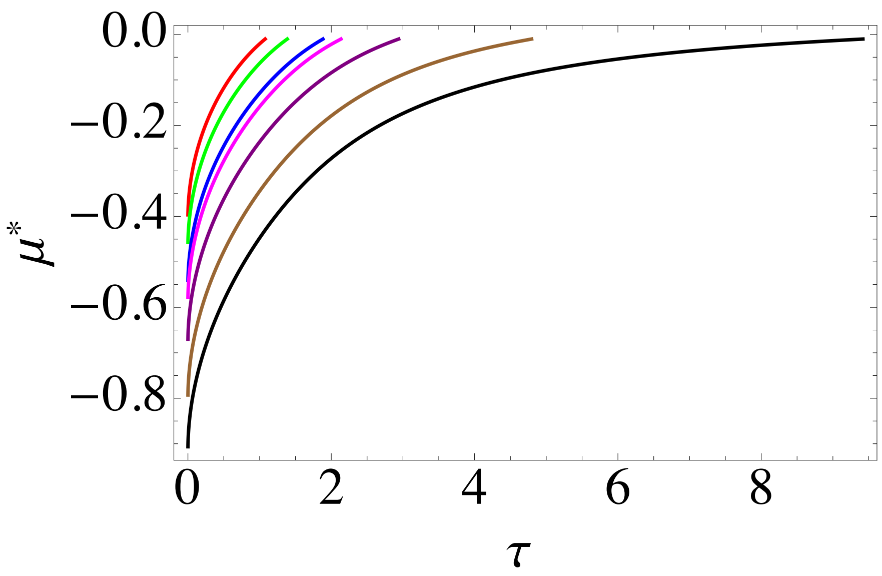


$$\mathcal{F}(p) \equiv 4\pi p^2 \mathcal{J}(p)$$

Onset of Bose-Einstein condensation (over-populated case)

The effective chemical potential vanishes in a finite time

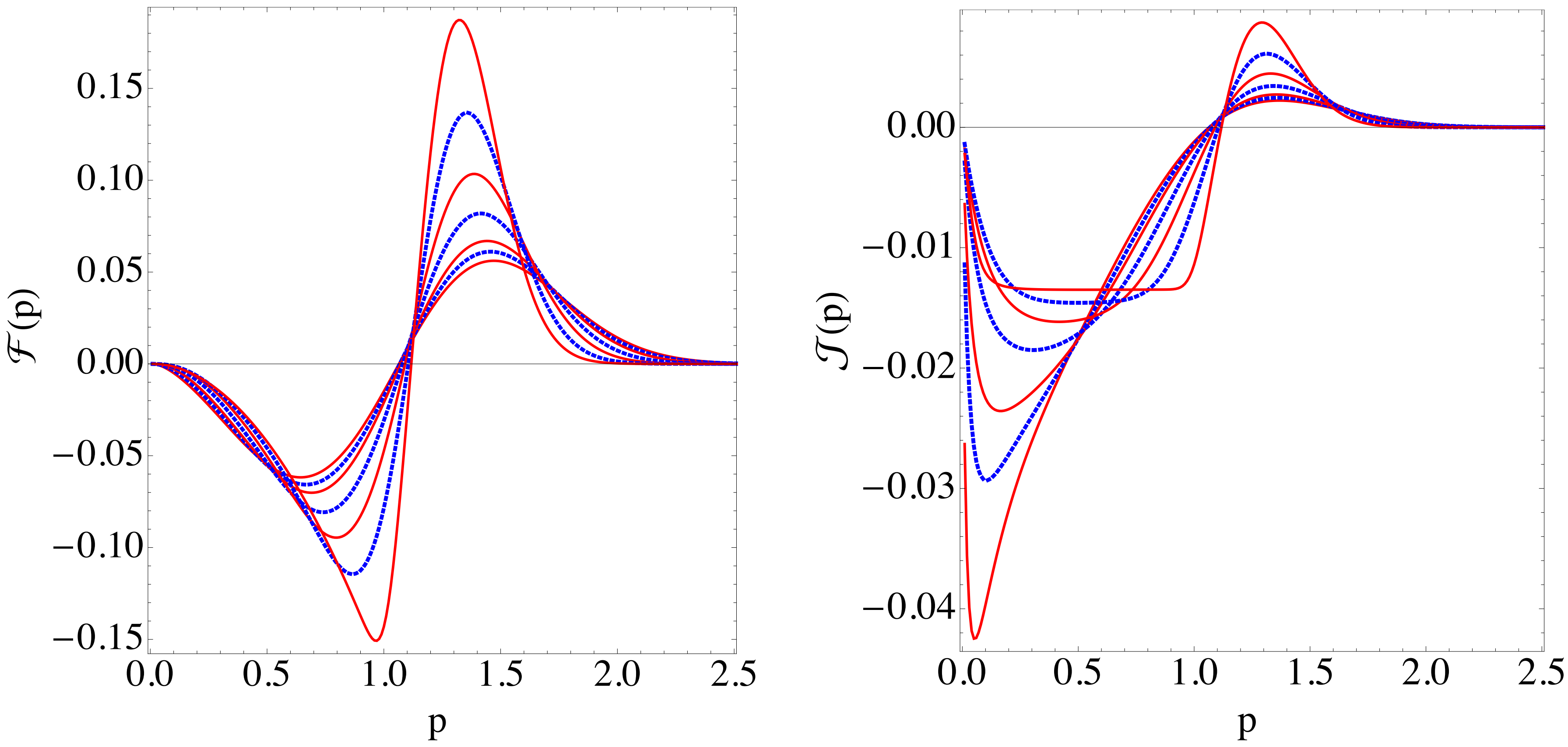




$$f_0 = 0.2,\ 0.3,\ 0.5,\ 0.8,\ 1,\ 2,\ 5$$

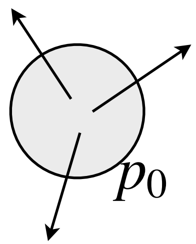
Onset of Bose-Einstein condensation (over-populated case)

The current exhibits a singular behavior at small momentum



Small momentum analysis

$$\partial_\tau \int_0^{p_0} dp\, p^2 f(p) = (\mu^*)^2 \frac{\partial T^*}{\partial \tau}\, h_1(y) + T^* |\mu^*| \frac{\partial \mu^*}{\partial \tau}\, h_2(y)$$



$$f = \frac{T^*}{p - \mu^*}$$

$$h_1(y) \equiv \log\left(1+y\right) + \frac{1}{2} y(y-2)$$

$$h_2(y) \equiv 2\log\left(1+y\right) - \frac{y(2+y)}{1+y}$$

$$-\mathcal{J}(\tau, p_0) = \frac{\partial T^*}{\partial \tau} \frac{h_1(y)}{y^2} + \frac{T^*}{|\mu^*|} \frac{\partial \mu^*}{\partial \tau} \frac{h_2(y)}{y^2} \qquad y \equiv \frac{p_0}{|\mu^*|}$$

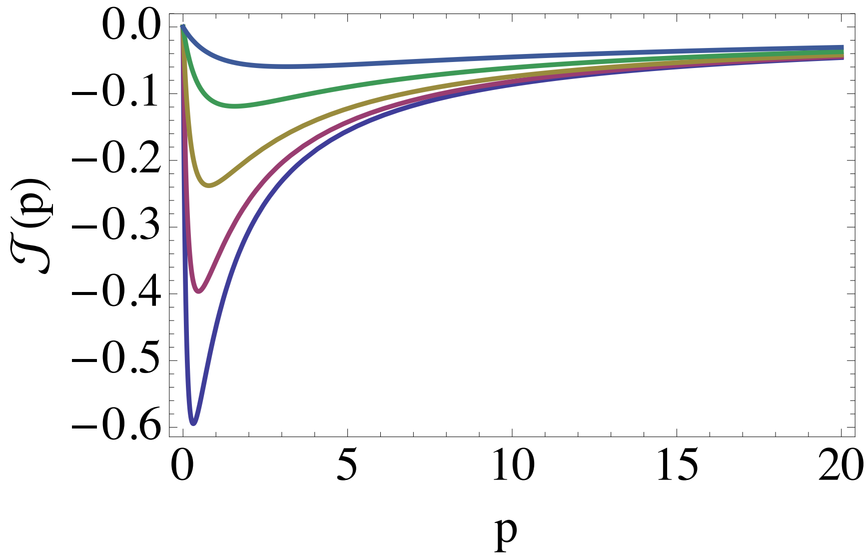
$$p_0 \ll |\mu^*| \qquad -\mathcal{J}(\tau, p_0) = \left(\frac{1}{|\mu^*|} \frac{\partial T^*}{\partial \tau} - \frac{T^*}{|\mu^*|^2} \frac{\partial |\mu^*|}{\partial \tau} \right) p_0 = \frac{\dot{f}(0)}{3}\, p_0$$

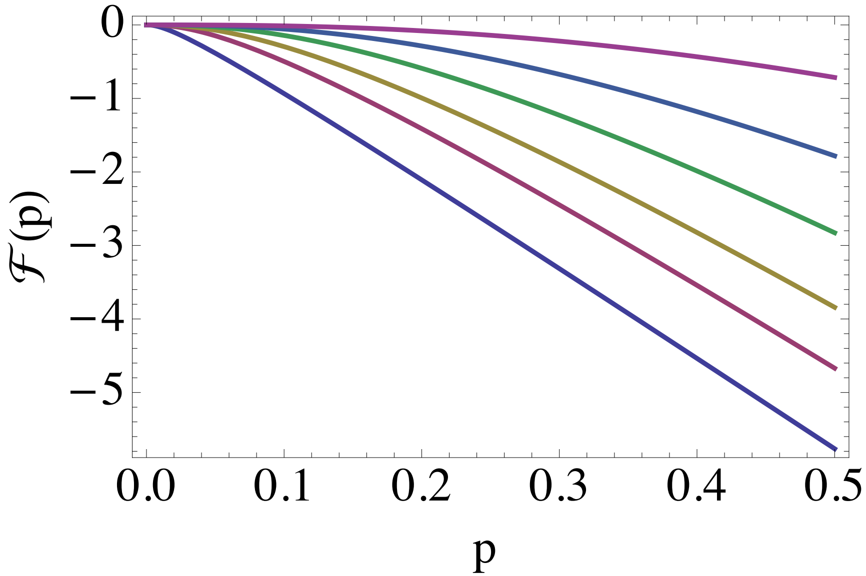
$$|\mu^*| \ll p_0 \ll T^* \qquad \mathcal{J}(\tau, p_0) \approx \frac{1}{p_0} \frac{\partial (|\mu^*| T^*)}{\partial \tau}$$

Scaling analysis

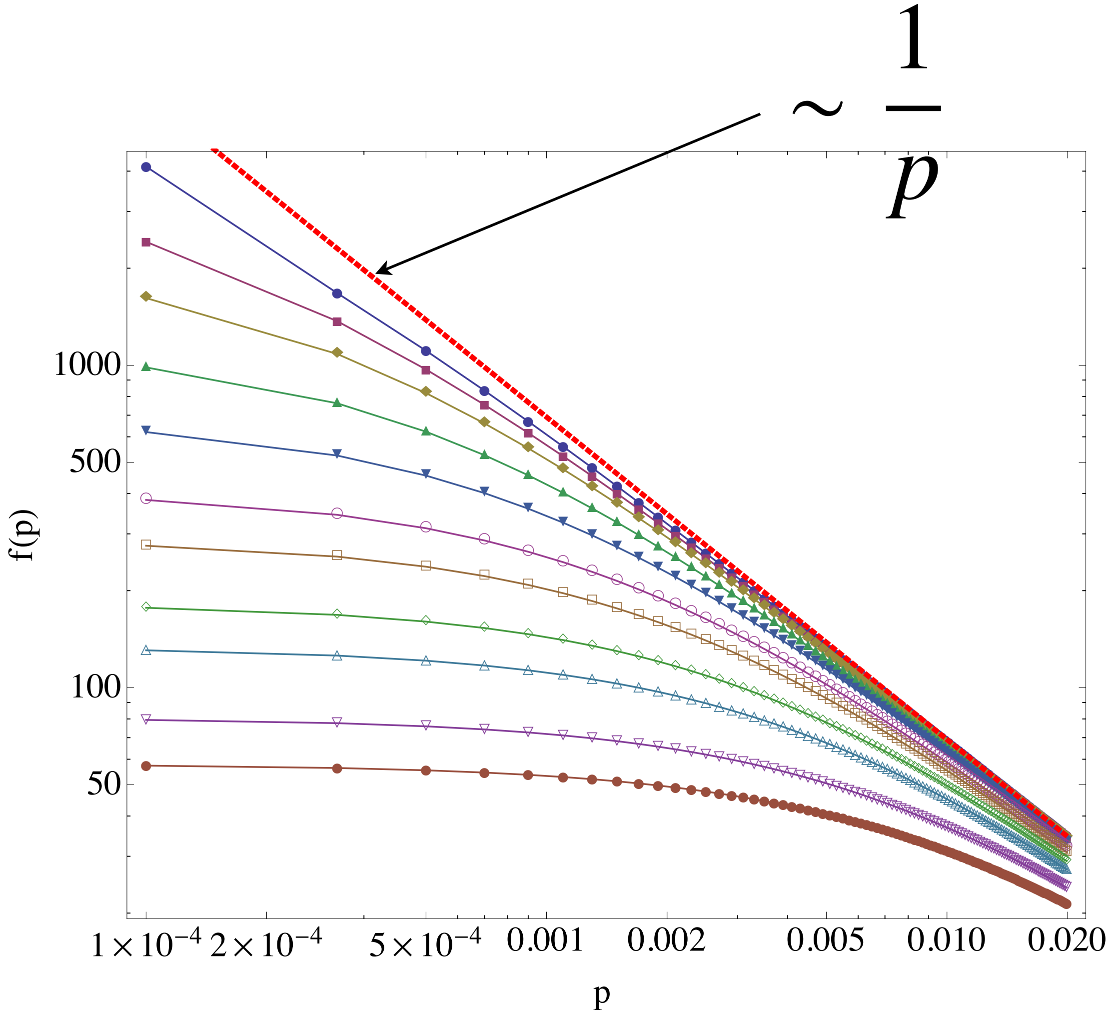
$$|\mu^*| = C(\tau_c - \tau)^\eta \qquad \eta \simeq 1$$

$$\mathcal{J}(\tau, p) \simeq \frac{\eta T^*}{\tau_c - \tau} \frac{h_2(y)}{y^2} \qquad y = \frac{p_0}{C(\tau_c - \tau)^\eta}$$

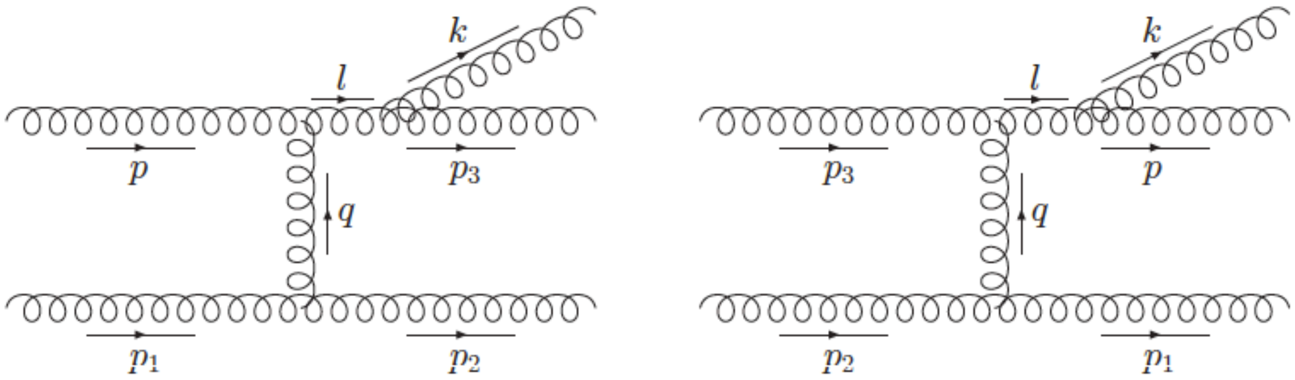




The distribution at small momentum near the onset



Inelastic processes



In principle, of higher order in the gauge coupling

However soft emissions are enhanced

In principle, prevent the formation of BEC (particle number is no longer conserved)

But explicit calculations indicate that they may shorten the time to reach the onset of BEC....

[Xu-Guang Huang, Jinfeng Liao, arXiv: 1303.7214]

Summary

- initial states of colliding heavy nuclei at high energy are characterized by **'over-populated'** gluonic state. Because of the large occupation, the system remains **'strongly interacting'** in spite of the small coupling constant

- the (dynamical) **growth of (very) soft modes** seems to a be a robust feature. It may lead to the formation of a (transient) **Bose condensate**.

- the phenomenon is well established in simulations of scalar field theory (and in other context, e.g. inflationary cosmology)

- simulations of gauge theories are inconclusive

- the nature of the condensate, if it exists, is unclear