From Jet Quenching to Turbulence

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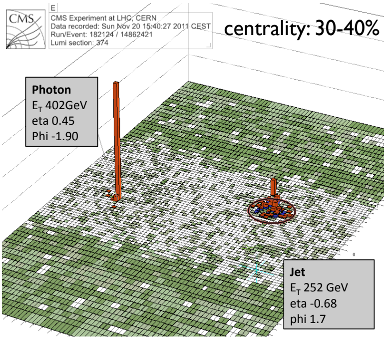

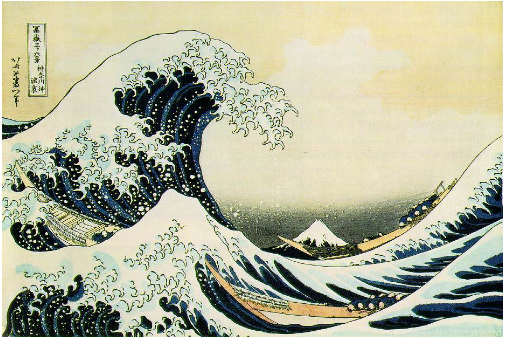

Takeshi and I ... a 10 years old story

Takeshi and I ... the spirit of RANP



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In the honour of Takeshi, I would have loved to give a talk on hydro ...

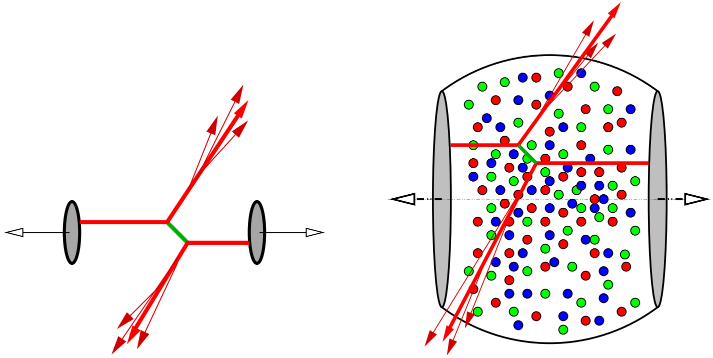
Takeshi and I ... a 10 years old story

In the honour of Takeshi, I would have loved to give a talk on hydro ...

but I can't honestly do that, so I will just speak about turbulence

Jet quenching

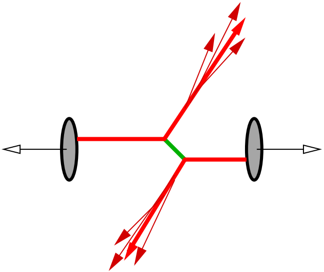
- How the measure the 'quark–gluon plasma' created at the intermediate stages of a heavy ion collision (at RHIC or the LHC) ?

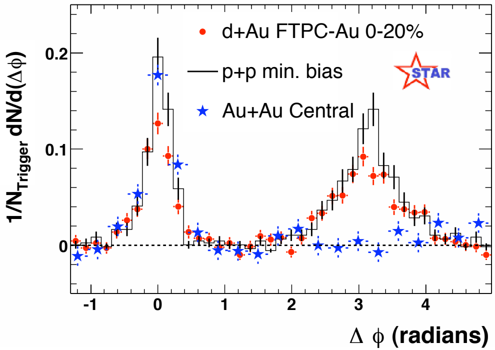


- Shut a hard parton and measure its interactions
- Hard partons are typically created in pairs which propagate back–to–back in the transverse plane
- In–medium interactions may alter this azimuthal correlation

Jet quenching

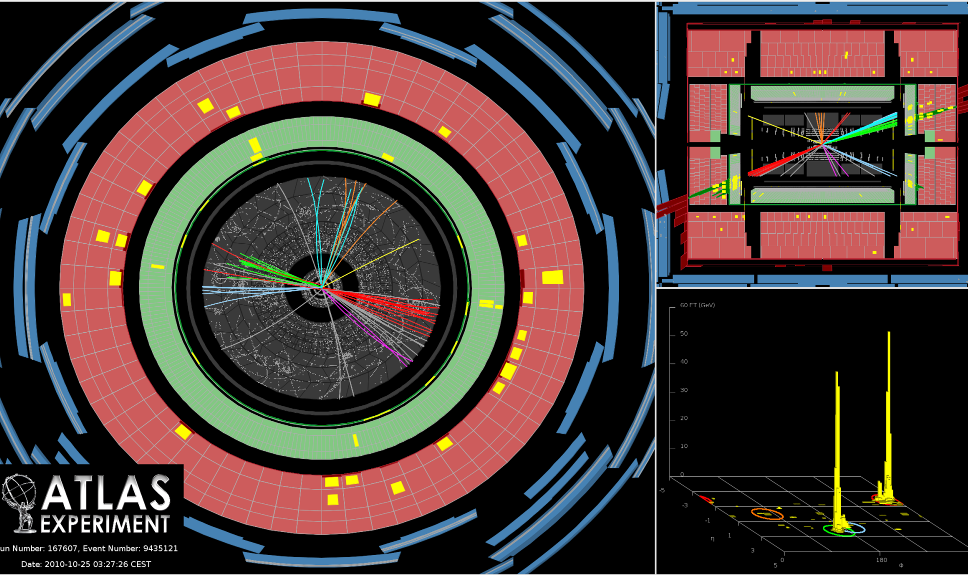
- How the measure the 'quark–gluon plasma' created at the intermediate stages of a heavy ion collision (at RHIC or the LHC) ?





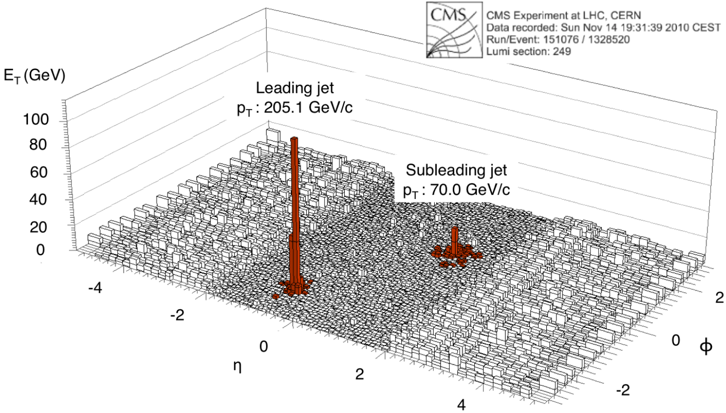
- Au+Au collisions at RHIC: the away peak is washed out
- 'Jet quenching' ... although jets were not really measured at RHIC
  - energy loss & transverse momentum broadening for the leading particle

Jet production at the LHC



- The LHC gives us access to real jets

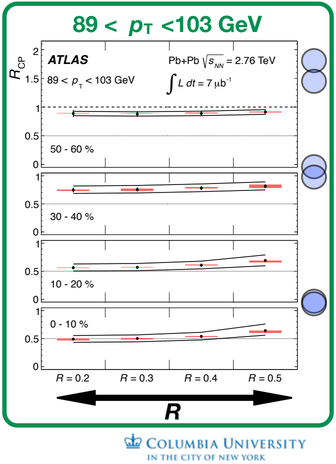
Di–jet asymmetry

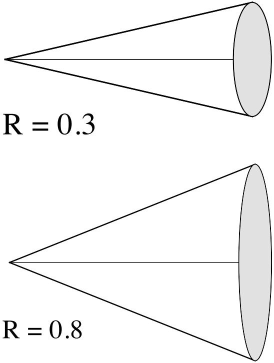


- Central Pb+Pb: the secondary jet is barely visible
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 4$ GeV) hadrons propagating at large angles

Energy transport at large angles

- The 'missing energy' is not gradually recovered when progresively increasing the jet opening : it is lost fully at large angles

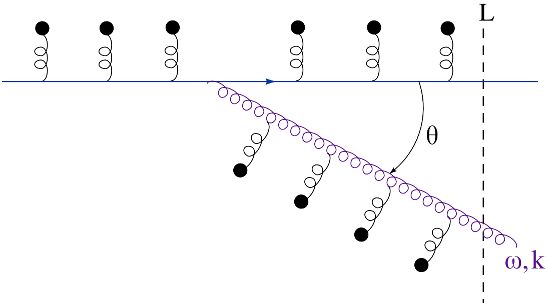




- What is the mechanism for energy transport at large angles ?

Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)

- Additional gluon radiation triggered by interactions in the medium



- Many related studies and applications to phenomenology ...

  Wiedemann (2000); Guylassy, Levai, Vitev (2000);

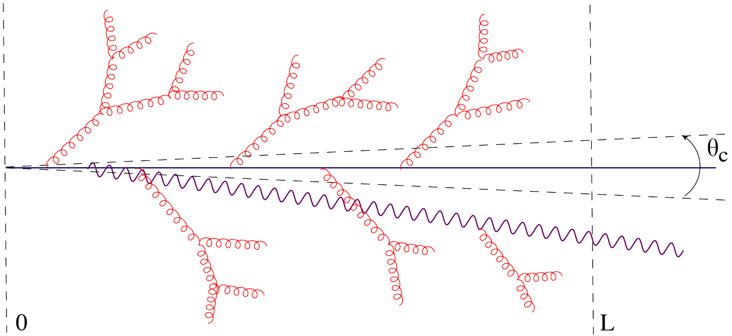
  Arnold, Moore, Yaffe (2002); Wang and Wang (2002) ...

- ... which have mostly focused on a single emission
  (the total energy loss by the leading particle)

pQCD : the BDMPSZ mechanism

Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)

- Additional gluon radiation triggered by interactions in the medium



- Recently generalized to multiple emissions & jet evolution

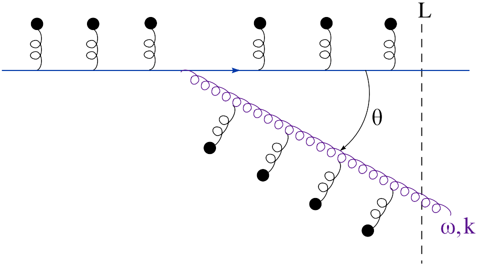
  Blaizot, Dominguez, E.I., Mehtar-Tani (2012–13)

Medium–induced gluon emissions

- Gluon emission is linked to transverse momentum broadening

$$\Delta k_\perp^2 \simeq \hat{q}\,\Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$



  - destroys the coherence between the gluon and its parent parton
  - increases the emission angle

- Gluon emissions can occur anywhere inside the medium (with size L)
- ... but they are not instantaneous : formation time $\tau_f = 1/\Delta E$

$$\tau_f \simeq \frac{\omega}{k_\perp^2} \simeq \frac{\omega}{\hat{q}\,\tau_f} \Longrightarrow \tau_f(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

Formation time (τ_f) & angle (θ_f)

$$\tau_f(\omega) \simeq \sqrt{\frac{\omega}{\hat{q}}} \qquad \& \qquad \theta_f(\omega) \sim \frac{k_\perp}{\omega} \simeq \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

- Maximal ω for this mechanism : $\tau_f \simeq L \Rightarrow \omega_c = \hat{q}L^2$
- The BDMPSZ spectrum: bremsstrahlung $\times$ the available phase space

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- LPM effect : the emission rate decreases with increasing ω
  - coherence: many soft collisions contribute to a single, hard, emission
- The total energy loss by the leading particle

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega\, \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$$

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- Relatively hard emissions with $\omega \sim \omega_c$:
  - large formation times: $\tau_f \sim L$
  - rare events : probability of $\mathcal{O}(\alpha_s)$
  - control the energy loss by the leading particle
  - small emission angles $\Rightarrow$ the energy remains inside the jet
- Arguably, not so important for the di–jet asymmetry

Formation time (τ_f) & angle (θ_f)

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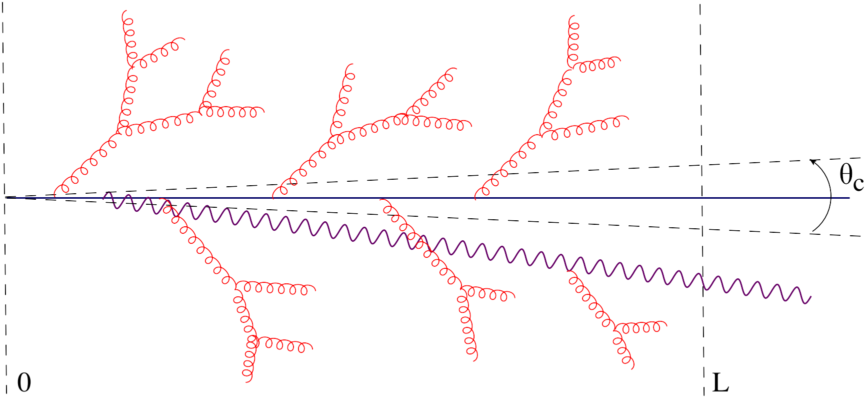
$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

- Relatively soft emissions with $\omega \ll \omega_c$:
    - small formation times : $\tau_f \ll L$
    - quasi–deterministic : probability of $\mathcal{O}(1)$ for $\omega \lesssim \alpha_s^2 \omega_c$
    - a relatively smaller contribution to the energy loss : $\Delta E_{\rm soft} \sim \alpha_s^2 \omega_c$
    - ... but this can be lost at arbitrarily large angles
- Potentially relevant for the di–jet asymmetry

A typical gluon cascade

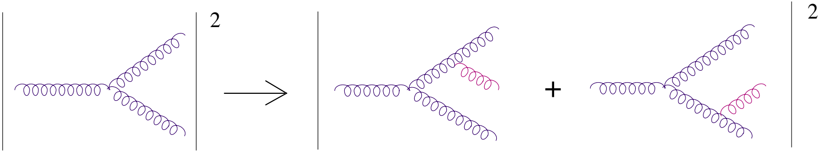
- One needs to understand multiple medium–induced branchings



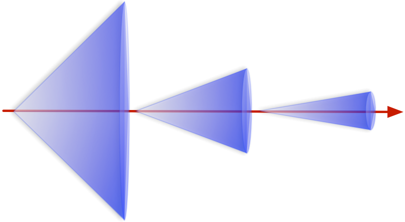
- A 'rain' of soft gluons plus (sometimes) a harder one $(\omega \sim \hat{q}L^2)$
- From now on, we shall focus on the 'rain' !

Multiple emissions : vacuum

- Successive medium–induced branchings are independent
- Non–trivial ! Not true for jet evolution in the vacuum !



- The daughter gluons keep their color coherence until the next emission

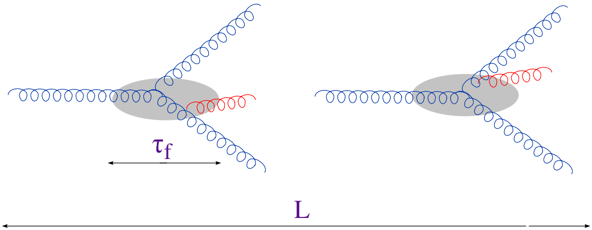


- Destructive interference effects leading to angular ordering

Multiple emissions : medium

- In medium, color coherence is rapidly lost via rescattering

  Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)



- The interference effects are suppressed by a factor $\tau_f/L \ll 1$

  Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)

Multiple emissions : medium

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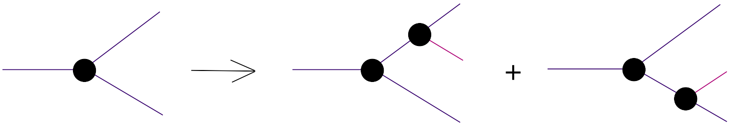

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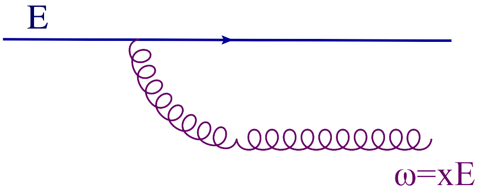
- Medium–induced jet evolution $\approx$ a classical branching process
  - a Markovian process in $D = 3 + 1$: ω, $k_\perp$, time t (or medium size L)
  - the $g \to gg$ splitting vertex (the 'blob') : the BDMPSZ spectrum
  - the propagator (the 'line') : transverse momentum broadening
- Well suited for Monte Carlo simulations

Quasi–democratic branchings

- The branchings of the soft gluons are quasi–democratic

  ▷ the daughter gluons carry comparable energy fractions: $x \sim 1/2$

- Non–trivial ! Not true for bremsstrahlung in the vacuum !



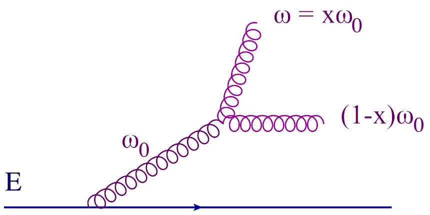
$$\mathrm{d}\mathcal{N} \sim \alpha_s \frac{\mathrm{d}\omega}{\omega} \sim \alpha_s \frac{\mathrm{d}x}{x}$$

- probability of $\mathcal{O}(1)$ when $\alpha_s \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$
- argument independent of the original energy E

  ▷ all that matters is the splitting fraction x

Quasi–democratic branchings

- The branchings of the soft gluons are quasi–democratic
  - ▷ the daughter gluons carry comparable energy fractions: $x \sim 1/2$
- In–medium radiation: a consequence of the LPM effect

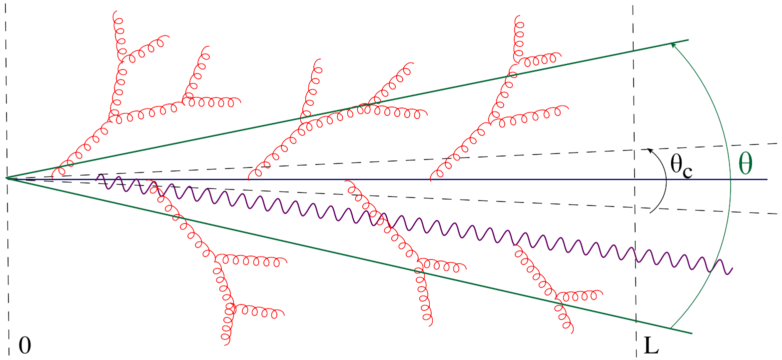


$$\mathrm{d}\mathcal{N} \sim \alpha_s \frac{\mathrm{d}\omega}{\omega}\sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha_s \frac{\mathrm{d}x}{x}\sqrt{\frac{\omega_c}{x\omega_0}}$$

- the rate also depends upon the parent gluon energy ω_0
- probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha_s^2\omega_c$ for any value of x
- the phase space favors generic values of x: 'quasi–democratic'

Wave turbulence



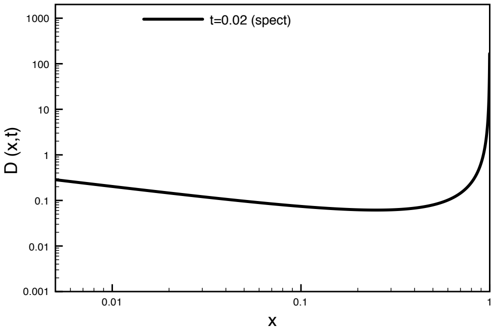
- This quasi–democratic cascade develops wave turbulence
- Via successive branchings, the energy flows from large x toward $x = 0$, without accumulating in any bin $x > 0$
- A very efficient mechanism for angular spreading

  J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

The gluon spectrum

- The time evolution of the gluon spectrum $D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x}$
- At small times: single branching $\Rightarrow$ BDMPSZ spectrum :

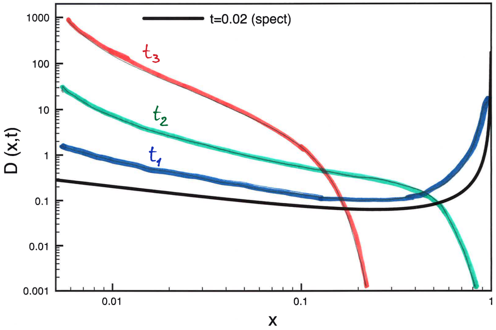
$$D^{(1)}(x,L) \simeq \alpha_s \frac{L}{\tau_f(\omega)} = \frac{t}{\sqrt{x}} \qquad (t = L \text{ in appropriate units})$$



- For generic times, this is described by a rate equation

A fake DGLAP–like scenario

- Via successive branchings, gluons fall at smaller and smaller values of x
- At any t, the energy remains in the spectrum: $\int_0^1 \mathrm{d}x\, D(x,t) \,=\, 1$



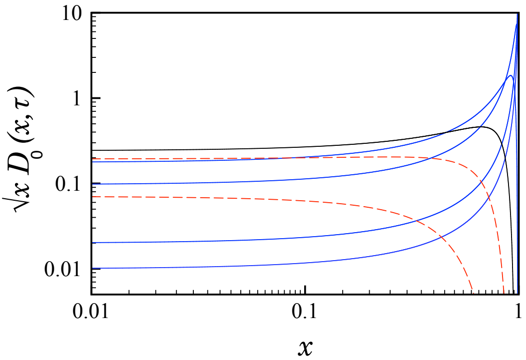
- The spectrum becomes steeper and steeper at small x, yet the total energy stored in the bins with $x \ll 1$ remains small

  ▷ very little energy can be lost in this way at large angles

The scaling spectrum

- The actual scenario (exact solution to the rate equation)

$$D(x \ll 1, t) \simeq \frac{t}{\sqrt{x}}\, \mathrm{e}^{-\pi t^2} \quad \& \quad \int_0^1 \mathrm{d}x\, D(x,t) = \mathrm{e}^{-\pi t^2}$$



- The 'scaling' spectrum $1/\sqrt{x}$ is a fixed point : 'Kolmogorov–Zakharov'
- The energy flows out from the spectrum

Where does the energy go ?

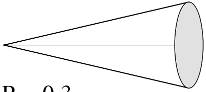
- Via successive branchings, the energy flows down to $x = 0$
  - formally, it accumulates into a 'condensate' at $x = 0$
  - physically, it goes below $x_{\rm th} = T/E \ll 1$, meaning it thermalizes
- The energy carried away by this flow ends up at arbitrarily large angles

$$E_{\rm flow}(t) \equiv 1 - \int_0^1 {\rm d}x\, D(x,t) = 1 - {\rm e}^{-\pi t^2}$$

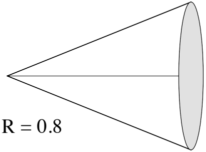
- In practice, $t = \alpha_s \sqrt{2\omega_c/E} \sim 0.3$ is not that large ($E = 100$ GeV)
  - $1 - {\rm e}^{-\pi t^2} \sim 0.25 \Rightarrow$ about 25% of the energy is lost at large angles
  - ... irrespective of the jet angle & the thermalization mechanism ($x_{\rm th}$)
- A universal mechanism (as usual with turbulence):
  - a property of the gluon cascade, not of the in–medium dissipation

Energy flow at large angles

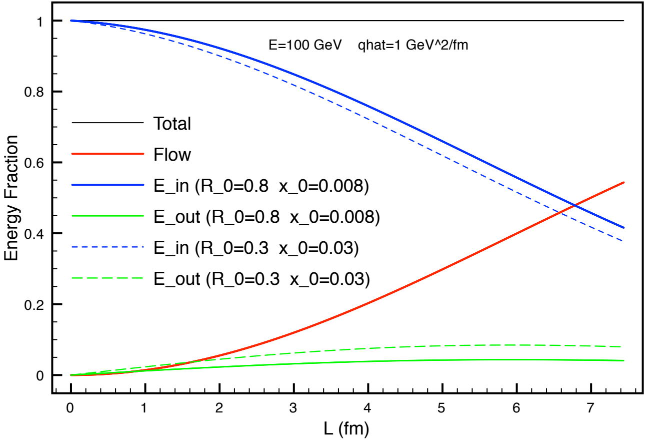
- The energy inside the jet is only weakly increasing with the jet angular opening R, within a wide range of values for R ☺



R = 0.3



R = 0.8



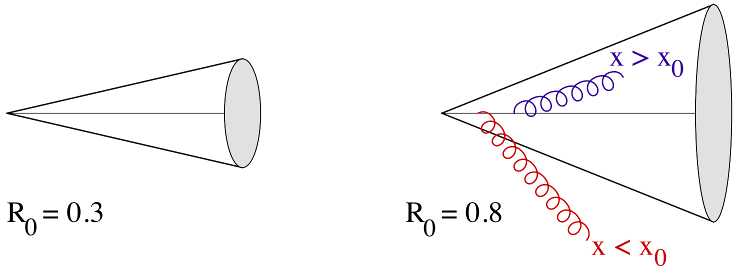
- $E_{\rm flow}$: independent of R, $x_{\rm th}$, and the original energy E

$$E_{\rm flow} \simeq v\,\alpha_s^2\,\hat{q}L^2 \qquad (\sim 20\,\text{GeV for } L = 5\text{ fm})$$

Joyeux Anniversaire, Takeshi !

Energy flow at large angles

- The energy inside the jet is only weakly increasing with the jet angular opening R_0, within a wide range of values for R_0 ☺

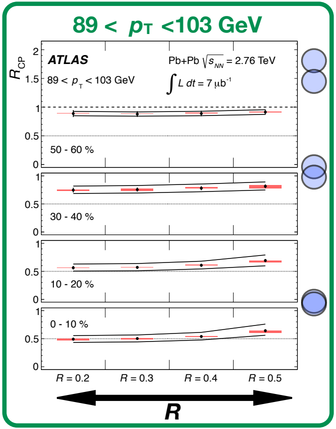


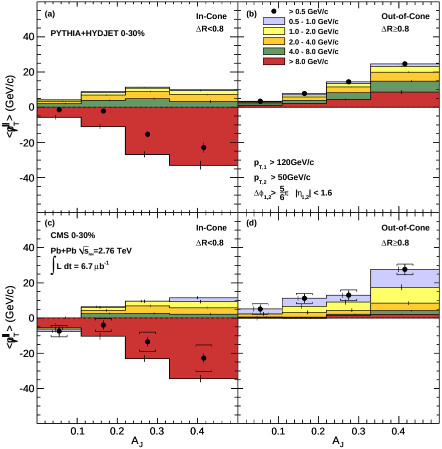
- The energy inside the jet E_{in} : the energy in the spectrum at $x > x_0$
- The energy outside the jet : $E_{\text{out}}(x_{\text{th}} < x < x_0) + E_{\text{flow}} \simeq E_{\text{flow}}$
- E_{flow} : independent of R_0, x_{th}, and the original energy E

$$E_{\text{flow}} \simeq v\,\alpha_s^2\,\hat{q}L^2 \qquad (\sim 20\,\text{GeV for } L = 5 \text{ fm})$$

Energy flow at large angles

- The 'missing energy' is lost fully at large angles
  - ▷ it cannot be gradually recovered when increasing the jet opening

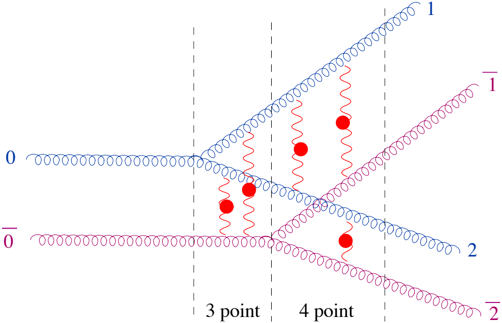




- ... in qualitative and even semi-quantitative agreement with the data

A few words on the formalism

- Quantum emission: amplitude × the complex conjugate amplitude



- 'Medium' = randomly distributed scattering centers (Gaussian)
  - Coulomb scattering with Debye screening
  - multiple scattering in eikonal approximation (one Wilson line per gluon)
  - $1 \rightarrow 2$ gluon branching $\Rightarrow$ 3–p and 4–p functions of the Wilson lines

A few words on the formalism

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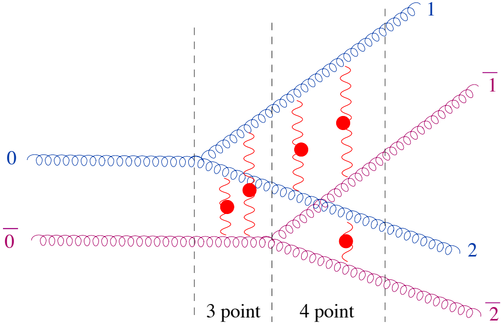


- Contains and extends the original BDMPSZ/AMI formalisms already at the level of a single medium–induced emission ...

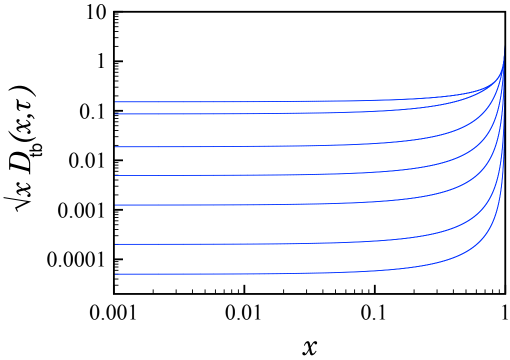
  ▷ transverse momentum dependence for the emission vertex, correct inclusion of single scattering, color (de)coherence after emission ...

- Permits the treatment of interference & multiple branchings

The usual turbulence set-up

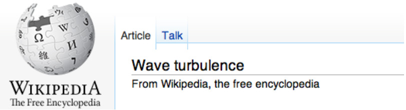
- Steady source at $x = 1$ and sink at $x = x_{\rm th}$ (here $x_{\rm th} = 0$)

$$D_{\rm tb}(x,t) = \frac{1}{2\pi\sqrt{x(1-x)}}\left(1 - {\rm e}^{-\pi\frac{t^2}{1-x}}\right)$$



- The spectrum approaches a steady shape when $\pi t^2 \gtrsim 1$

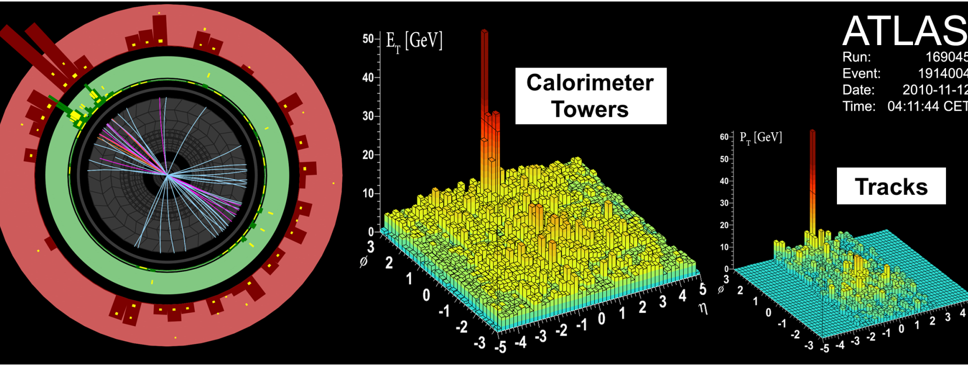
With due respect to Wikipedia ...



- *Wave turbulence is a set of waves deviated far from thermal equilibrium. Such state is accompanied by dissipation. It is either decaying turbulence or requires external source of energy to sustain it. Examples are waves on a fluid surface excited by winds or ships, and waves in plasma excited by electromagnetic waves etc. The wave system can be described by kinetic equations and their stationary solutions called Kolmogorov-Zakharov (KZ) energy spectra. They have the form $1/k^{\nu}$, with k the wavenumber and ν a positive constant depending on the specific wave system. The form of KZ-spectra does not depend on the initial magnitude of the total energy or on the details of initial energy distribution over the modes. Only the fact the energy is conserved at some inertial interval is important.*

- *V. Zakharov, V. L´vov, and G. Falkovich, Kolmogorov Spectra of Turbulence, Wave Turbulence (Springer-Verlag, 1992)*

Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV
- Additional energy imbalance as compared to p+p : 20 to 30 GeV