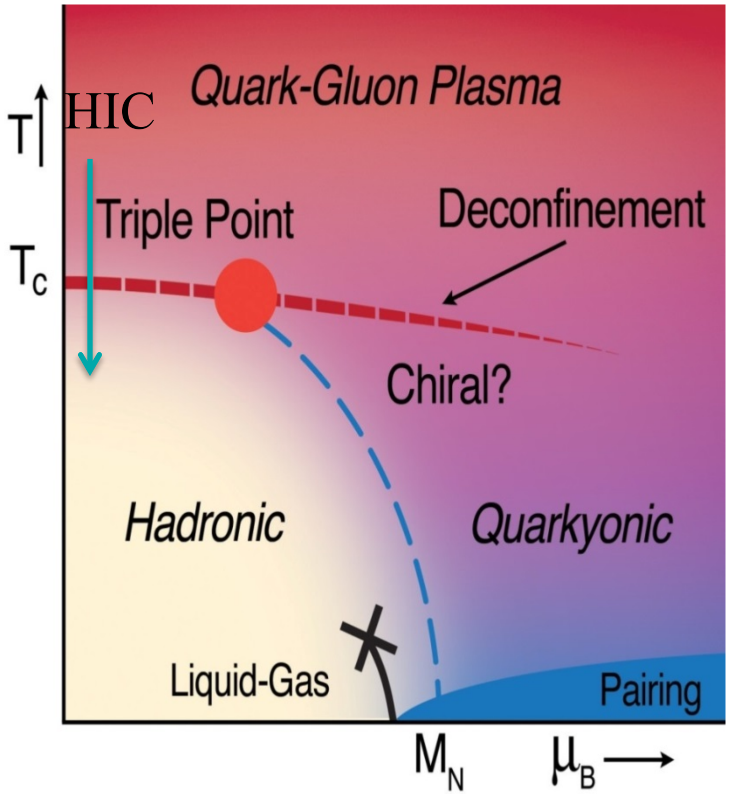
Polyakov loop and charge fluctuations as probes of QCD phase transition

Krzysztof Redlich University of Wroclaw & EMMI/GSI



- Fluctuations of the Polyakov loop and deconfinement in a pure SU(N) gauge theory and in QCD
- Fluctuations of conserved charges as a probe for the chiral phase transition and deconfinement
- Probability distribution and O(4) criticality

  ⇨ expectations and STAR data

In collaboration with: P. Braun-Munzinger, B. Friman, O. Kaczmarek, Pok. M. Lo, F. Karsch, K. Morita, C. Sasaki & V. Skokov

Susceptibilities of the net charge and order parameters

– The generalized susceptibilities probing fluctuations of the net charge
number in a system and its critical properties

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

generalized susceptibilities

$$\chi_q^{(i+j+k)} = \frac{\partial^{(i+j+k)} p / T^{\wedge 4}}{\partial T^i \partial \mu_x^j \partial m^i} :$$

Order parameter

$$< O_h >= \frac{1}{V} \frac{\partial \ln Z}{\partial h}$$

particle number density

$$\frac{n_q}{T^3} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q / T}$$

quark number susceptibility

$$\chi_q^{(2)} = \frac{\partial \, n_q / T^3}{\partial \mu_q / T}$$

4th order cumulant

$$\chi_q^{(4)} = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q / T)^4}$$

$$\chi_q^1 = \frac{1}{VT^3} < N >, \quad \chi_q^2 = \frac{1}{VT^3} (< N^2 > - < N >^2) \quad \chi_q^4 = \frac{1}{VT^3} (< (\delta N)^4 > -3 < (\delta N)^3 >)$$

expressed by $\boxed{N = N_q - N_{\bar{q}}}$ **and central moment** $\boxed{\delta N = N - < N >}$

Polyakov loop on the lattice needs renormalization

- Introduce Polyakov loop:

$$\boxed{L \Rightarrow c_N L} \qquad \boxed{c_N = e^{2\pi i k/N} \in Z(N)}$$

$$L_{\vec{x}}^{\text{bare}} = \frac{1}{N_c} Tr \prod_{\tau=1}^{N_\tau} U_{(\vec{x},\tau),4}$$

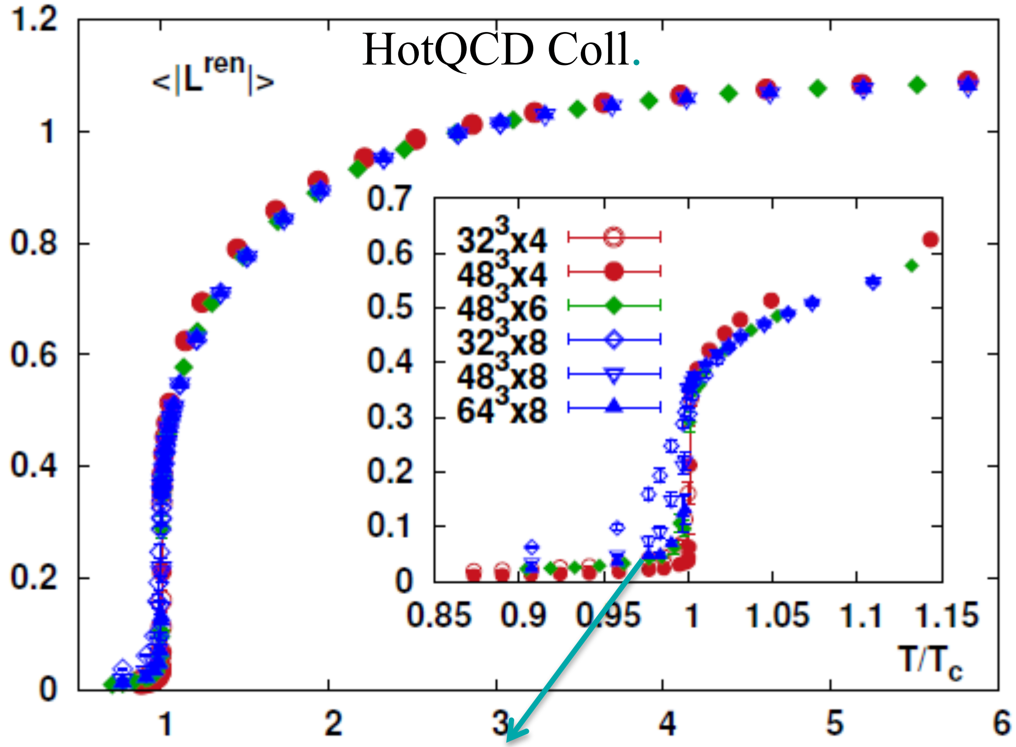
$$\langle | L^{\text{ren}} | \rangle = e^{-\beta F_q^{\text{ren}}} \rightarrow \begin{cases} \neq 0 \Leftrightarrow \text{deconfined } T > T_c \\ 0 \Leftrightarrow \text{confined } T < T_c \end{cases}$$

$$L^{\text{bare}} = \frac{1}{N_\sigma^3} \sum_{\vec{x}} L_{\vec{x}}^{\text{bare}}$$

- Renormalized ultraviolet divergence

$$L^{\text{ren}} = \left(Z(g^2)\right)^{N_\tau} L^{\text{bare}}$$

- Usually one takes $\langle |L^{\text{ren}}| \rangle$ as an order parameter



$1/\sqrt{V}$

To probe deconfinement : consider fluctuations

- Fluctuations of modulus of the Polyakov loop

$$T^3\chi_A = \frac{N_\sigma^3}{N_\tau^3}\left(\langle|L^{\rm ren}|^2\rangle - \langle|L^{\rm ren}|\rangle^2\right)$$

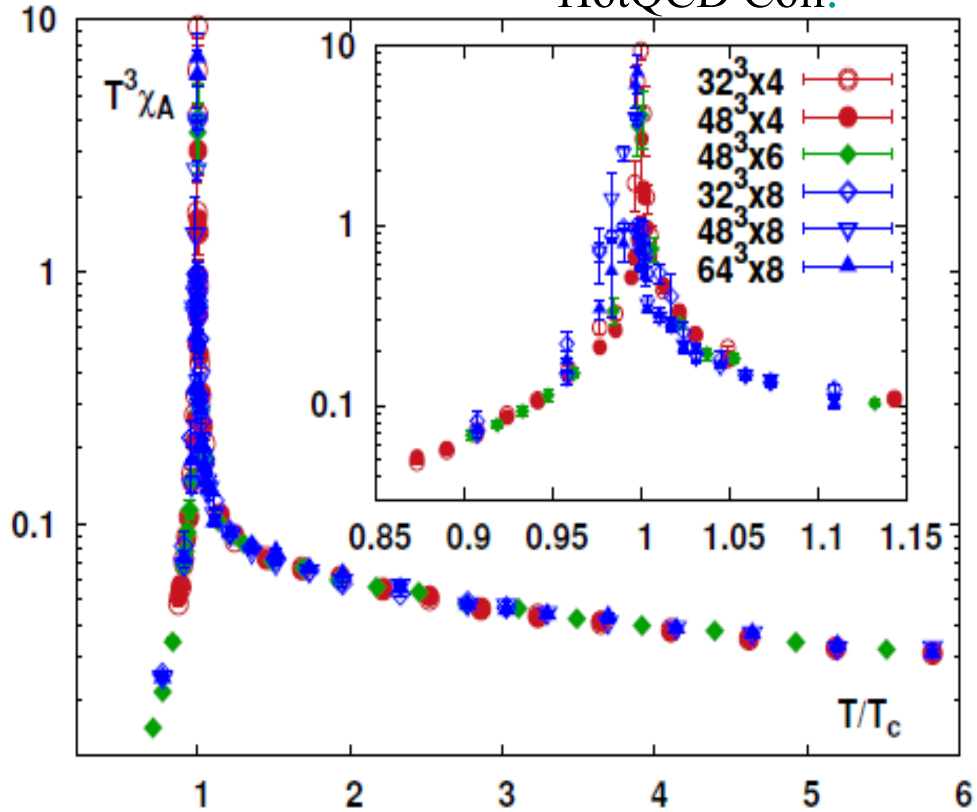
However, the Polyakov loop

$$L = L_R + iL_I$$

Thus, one can consider fluctuations of the real χ_R and the imaginary part χ_I of the Polyakov loop.

SU(3) pure gauge: LGT data

HotQCD Coll.



Fluctuations of the real and imaginary part of the renormalized Polyakov loop

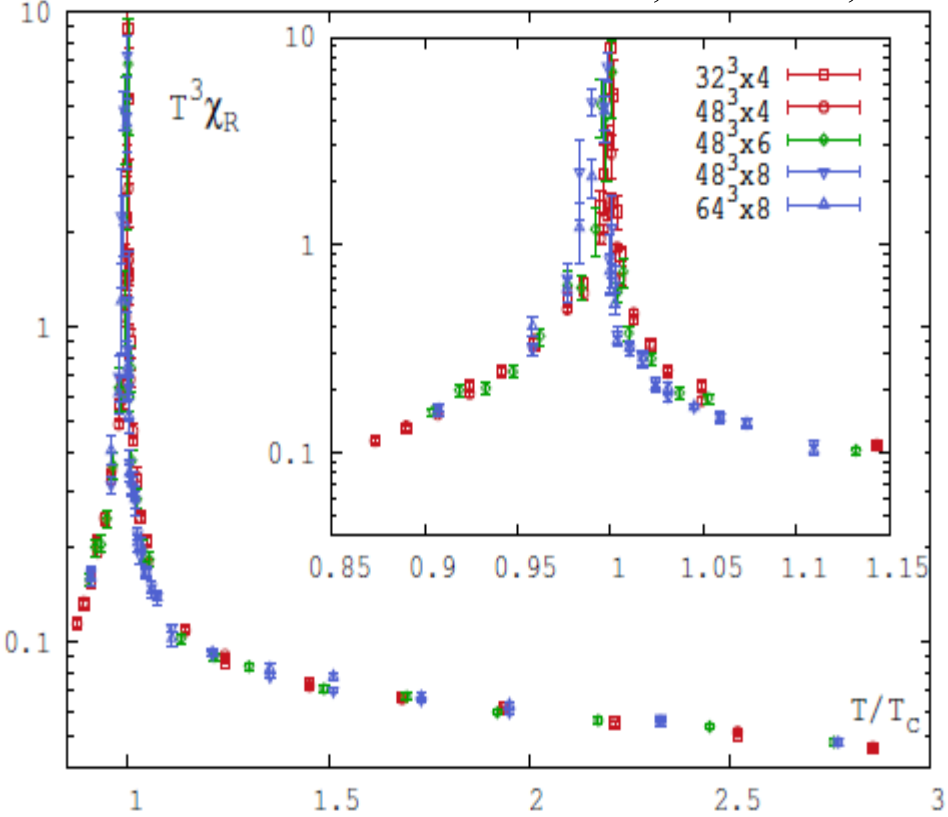
- Real part fluctuations

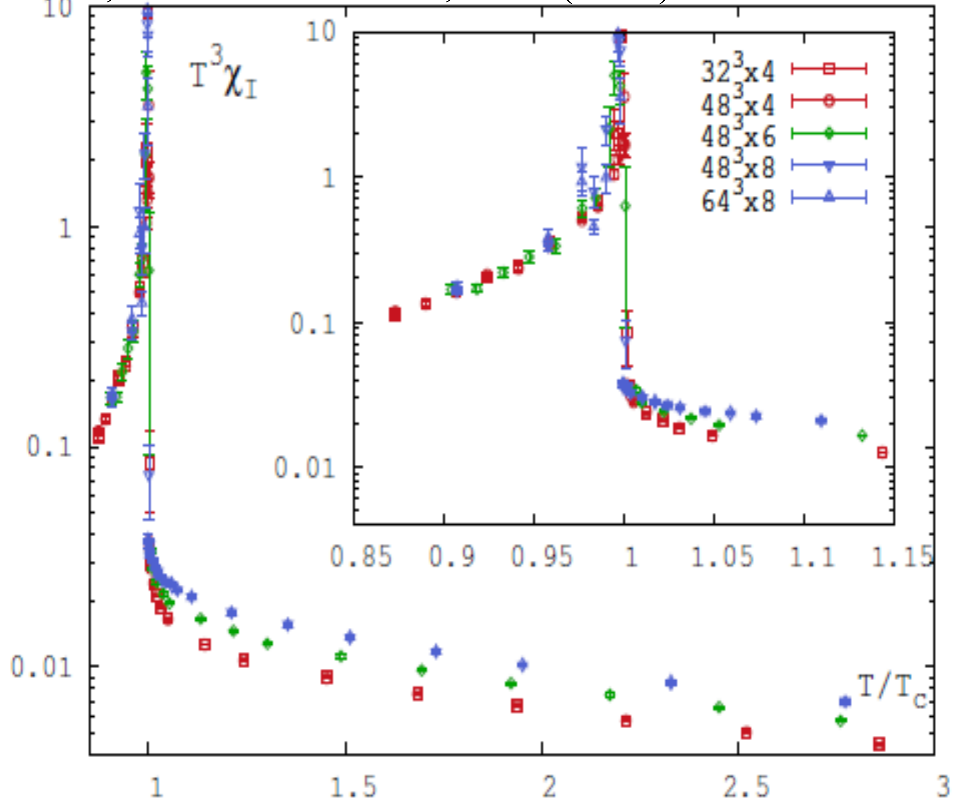
$$T^3\chi_R = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_R^{\rm ren})^2\rangle - \langle L_R^{\rm ren}\rangle^2\right]$$

- Imaginary part fluctuations

$$T^3\chi_I = \frac{N_\sigma^3}{N_\tau^3}\left[\langle (L_I^{\rm ren})^2\rangle - \langle L_I^{\rm ren}\rangle^2\right]$$

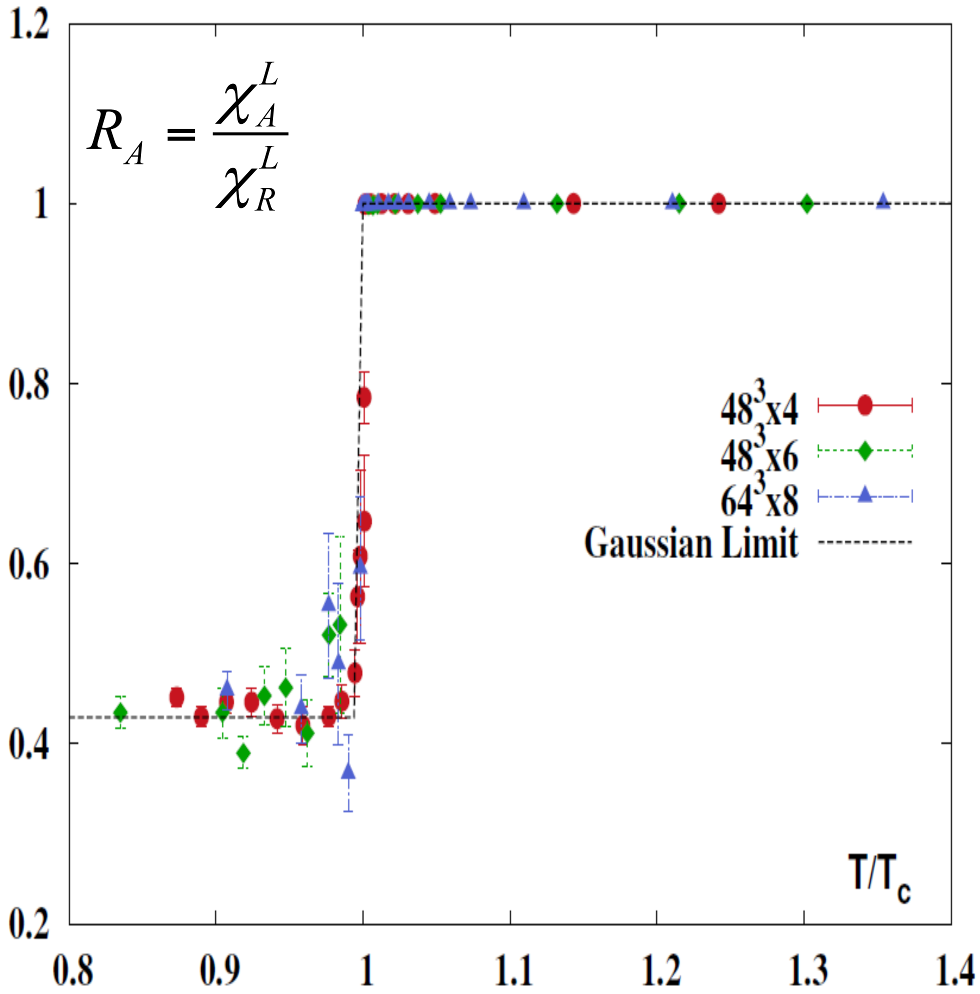
Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)





Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the deconfined phase $R_A \approx 1$

Indeed, in the real sector of Z(3)

$$L_R \approx L_0 + \delta L_R \quad \text{with} \quad L_0 = < L_R >$$

$$L_I \approx L_0^I + \delta L_I \quad \text{with} \quad L_0^I = 0, \quad \text{thus}$$

$$\chi_R^L = V < (\delta L_R)^2 >, \quad \chi_I^L = V < (\delta L_I)^2 >$$

Expand the modulus,

$$|L| = \sqrt{L_R^{\ 2} + L_I^{\ 2}} \approx L_0 \left(1 + \frac{\delta L_R}{L_0} + \frac{(\delta L_I)^2}{2 L_0^{\ 2}}\right)$$

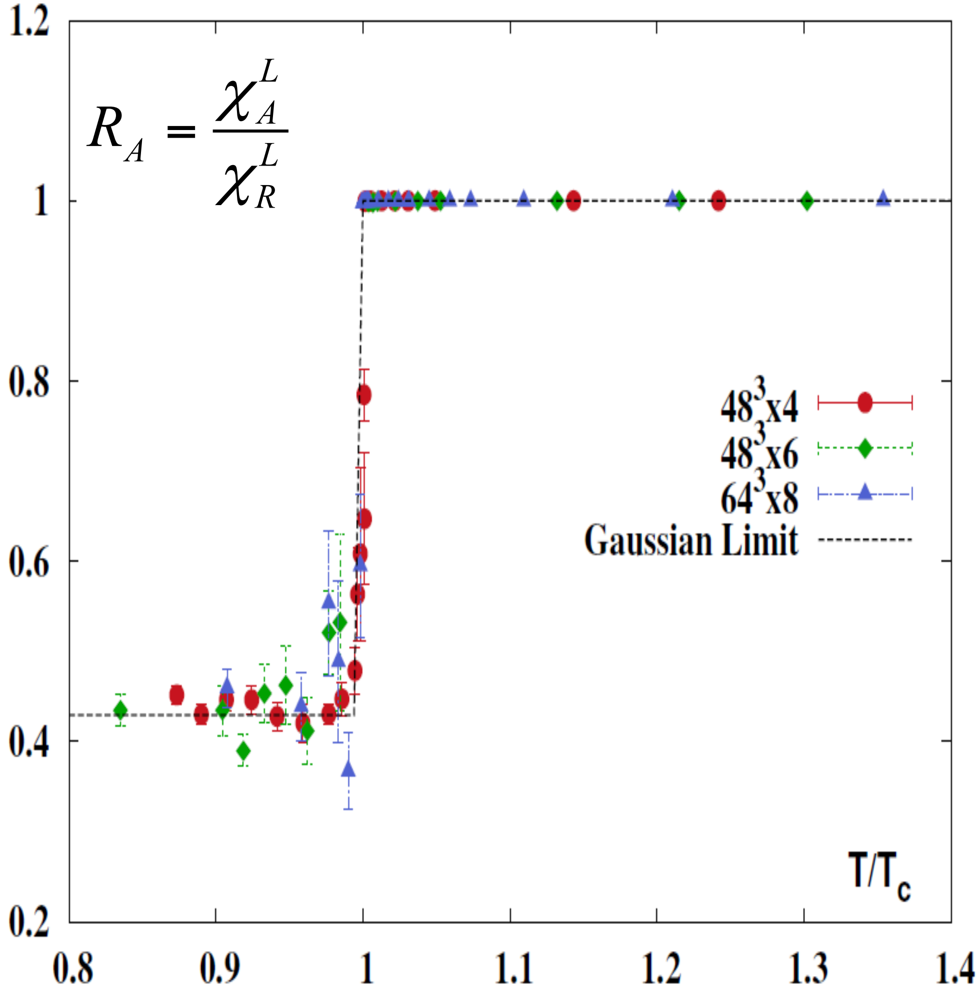
get in the leading order

$$\boxed{<|L|^2> - <|L|>^2 \approx \, < (\delta L_R)^2 >}$$

thus $\chi_A \approx \chi_R$

Ratios of the Polyakov loop fluctuations as an excellent probe for deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R. , PRD (2013)



- In the confined phase $R_A \approx 0.43$

Indeed, in the Z(3) symmetric phase, the probability distribution is Gaussian to the first approximation,

with the partition function

$$Z = \int dL_R dL_I e^{VT^3[\alpha(T)(L_R{}^2 + L_I{}^2)]}$$

Thus $\chi_R = \frac{1}{2\alpha T^3}, \quad \chi_I = \frac{1}{2\alpha T^3}$ and

$\chi_A = \frac{1}{2\alpha T^3}(2 - \frac{\pi}{2})$, consequently

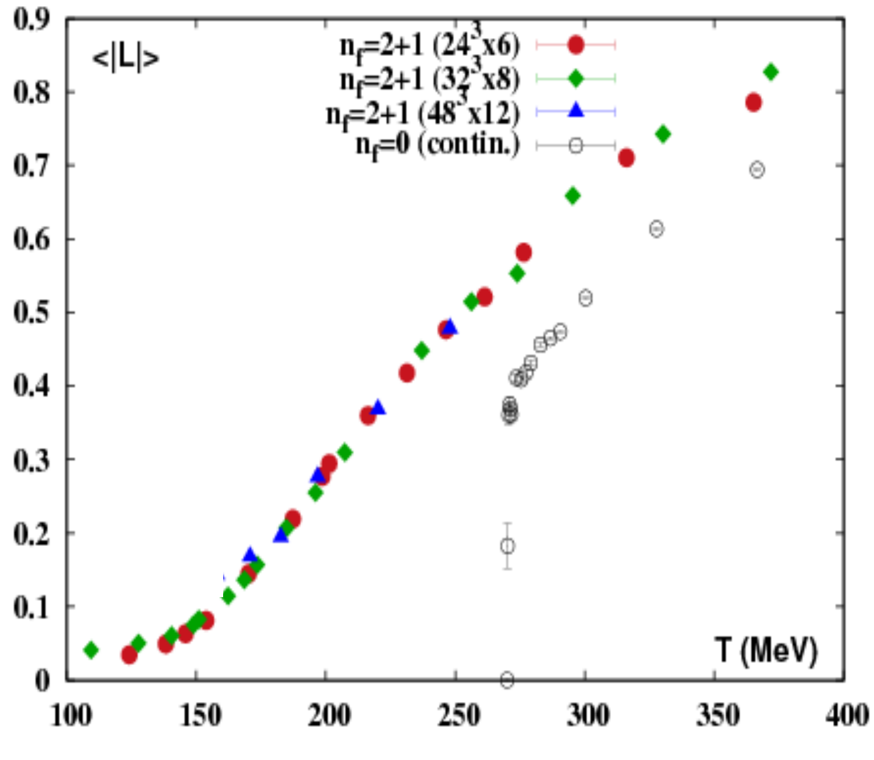
$$R_A^{SU(3)} = (2 - \frac{\pi}{2}) = 0.429$$

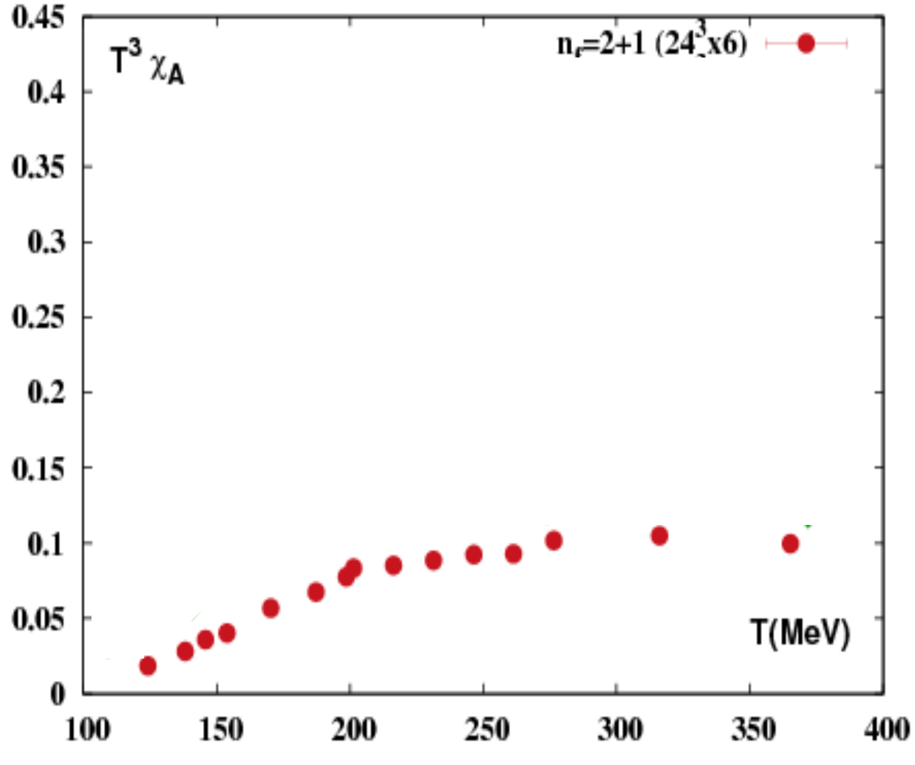
In the SU(2) case $R_A^{SU(2)} = (2 - \frac{2}{\pi}) = 0.363$ is in agreement with MC results

Polyakov loop and fluctuations in QCD

- Smooth behavior for Polyakov loop and fluctuations

  ⇒ difficult to determine where is deconfinement

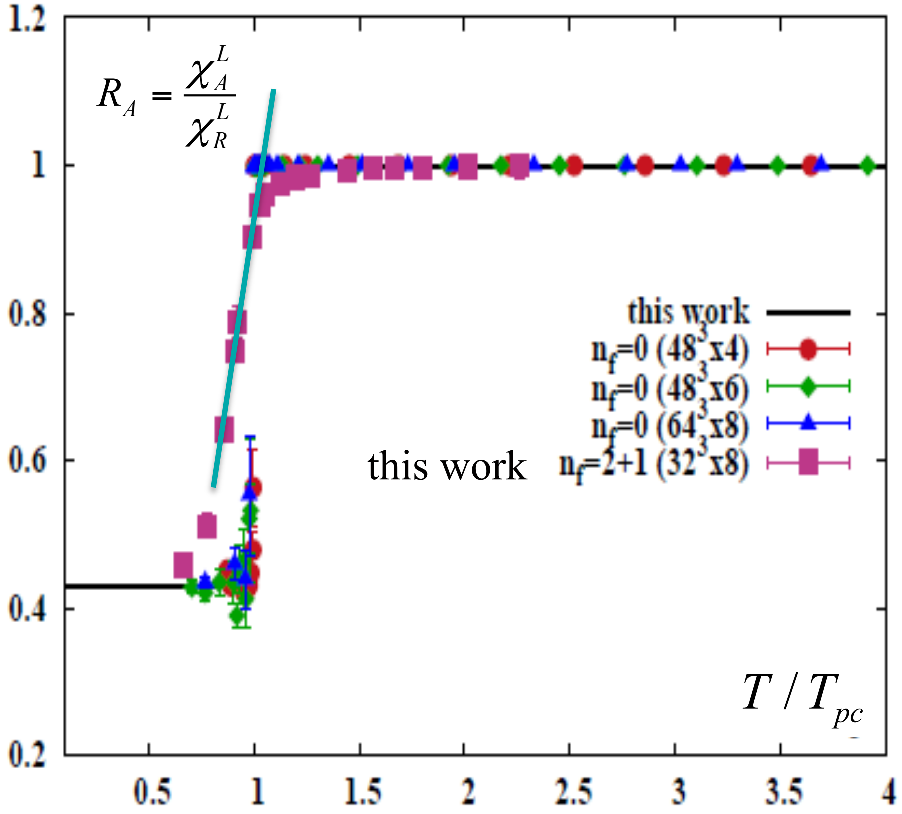




The influence of fermions on the Polyakov loop susceptibility ratio

- Z(3) symmetry broken, however ratios still showing deconfinement

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.

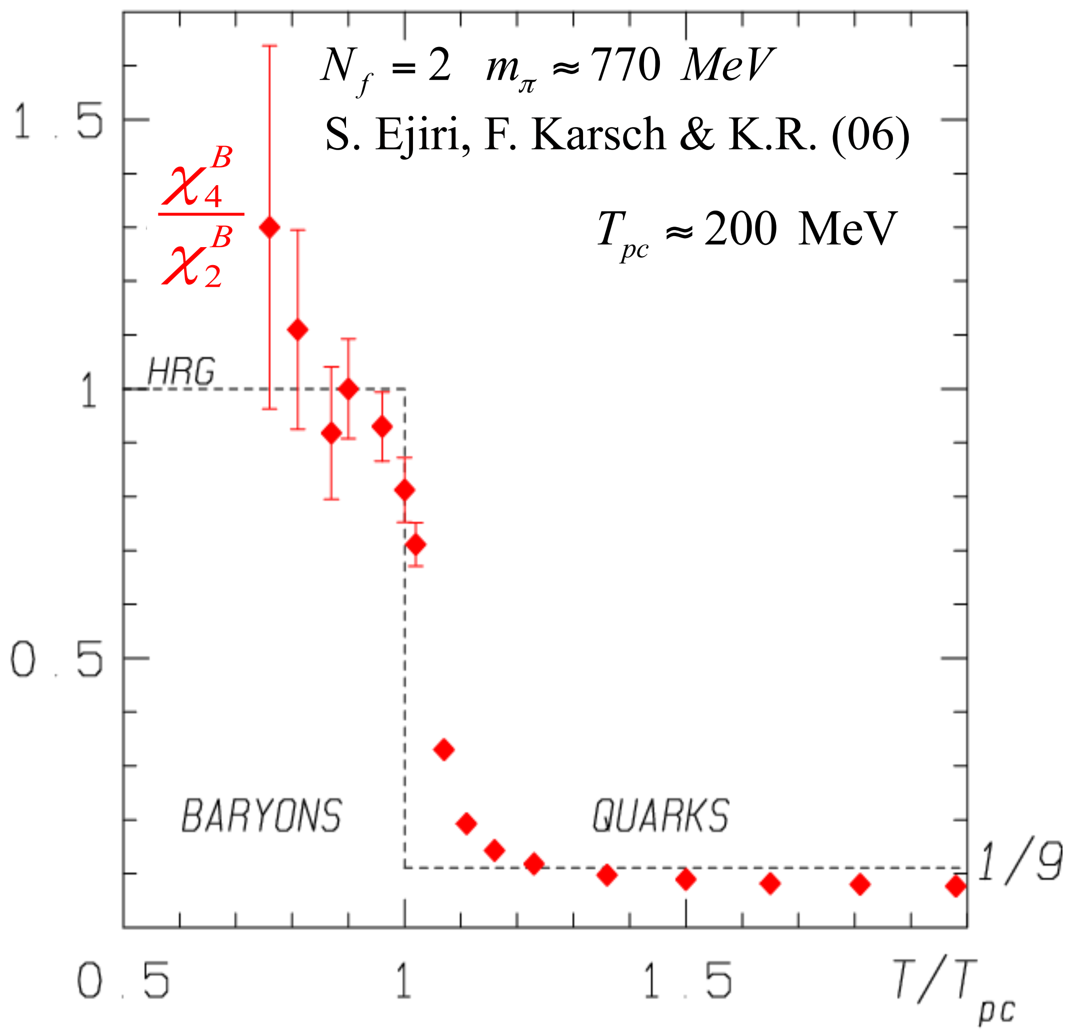


- Change of the slope in the narrow temperature range signals color deconfinement

- Dynamical quarks imply smoothening of the susceptibilities ratio, between the limiting values as in the SU(3) pure gauge theory

Probing deconfinement in QCD

$16^3 \times 4$ lattice with p4 fermion action



- HRG factorization of pressure:

$$\boxed{P^B(T,\mu_q) = F(T)\cosh(B\mu_B / T)}$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Kurtosis of net quark number density in PQM model

V. Skokov, B. Friman &K.R.

- For $T < T_c$

  the assymptotic value

  due to „confinement" properties

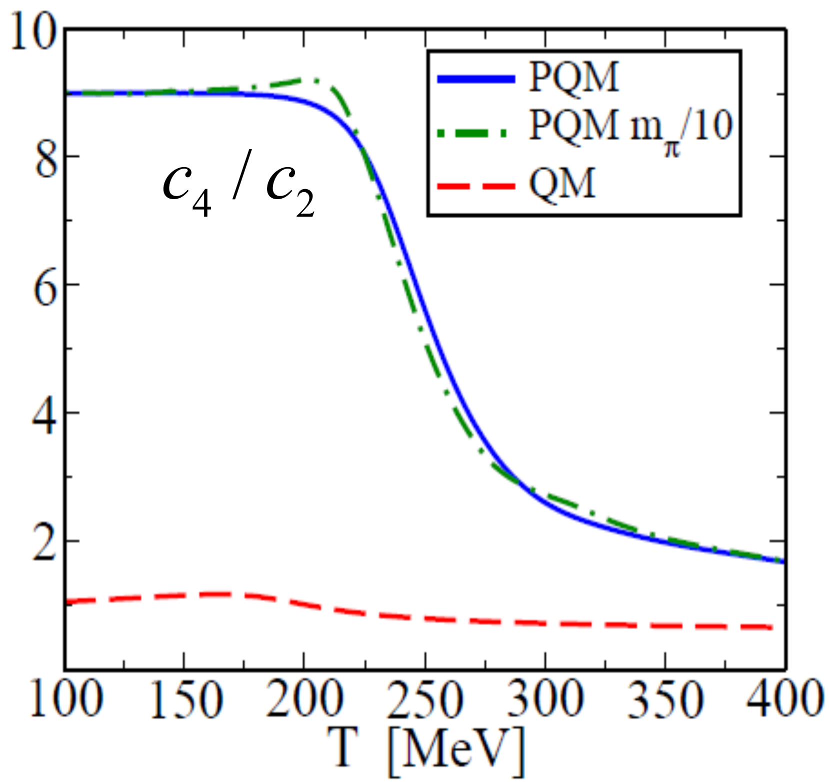
$$\frac{P_{q\bar{q}}(T)}{T^4} \approx \frac{2N_f}{N_c^{\,2}}\left(\frac{3m_q}{T}\right)^2 K_2\left(\frac{3m_q}{T}\right)\cosh\frac{3\mu_q}{T}$$

$$\Rightarrow \quad c_4 / c_2 = 9$$

- For $T >> T_c$

$$\frac{P_{q\bar{q}}(T)}{T^4} = N_f N_c\left[\frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu}{T}\right)^2 + \frac{7\pi^2}{180}\right]$$

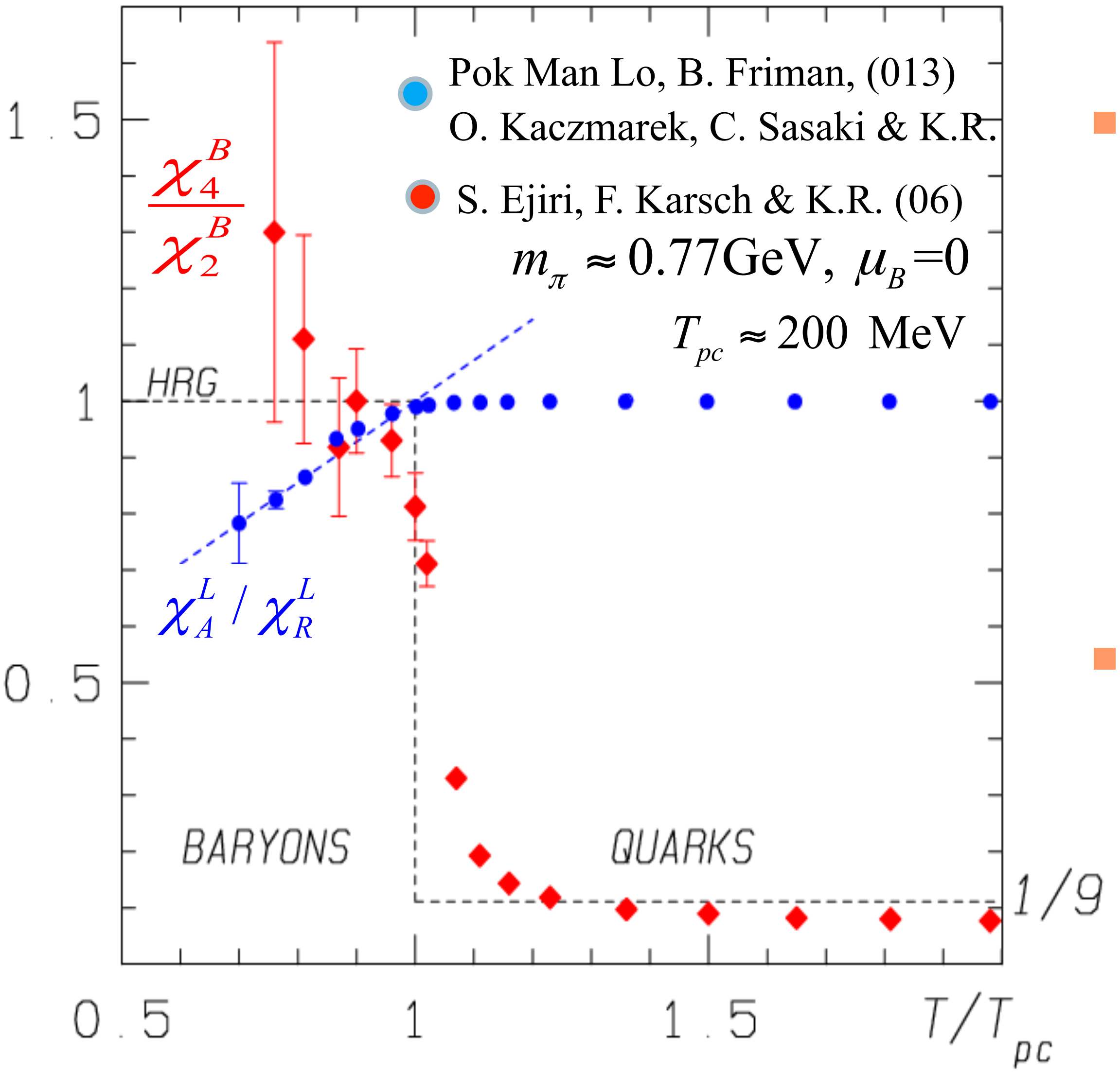
$$\Rightarrow c_4 / c_2 = 6 / \pi^2$$



- Smooth change with a very weak dependence on the pion mass

Probing deconfinement in QCD

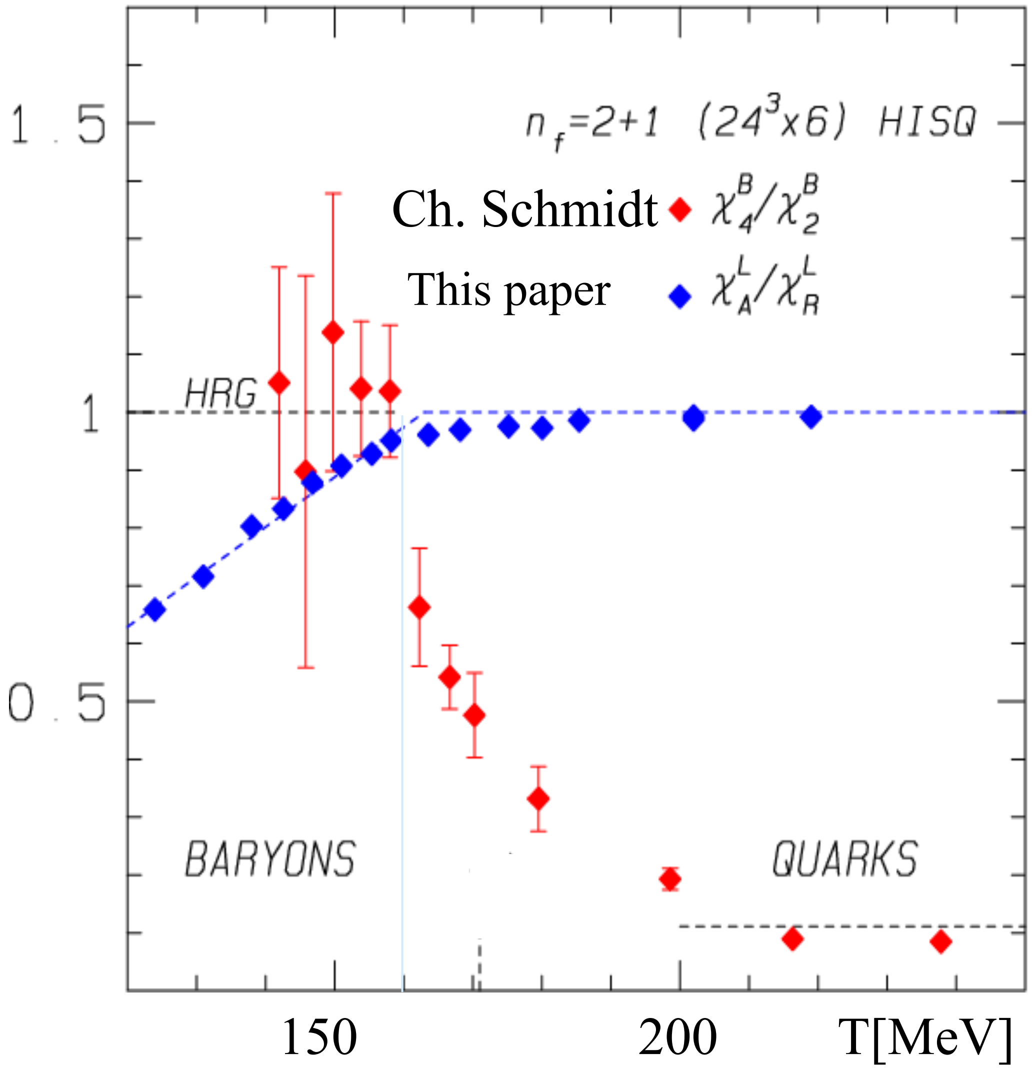
$16^3 \times 4$ lattice with p4 fermion action



- The change of the slope of the ratio of the Polyakov loop susceptibilities χ_A^L / χ_R^L appears at the same T where the kurtosis drops from its HRG asymptotic value
- In the presence of quarks there is "remnant" of Z(N) symmetry in the χ_A^L / χ_R^L ratio, indicating deconfinement of quarks

Probing deconfinement in QCD

Pok Man Lo, B. Friman, O. Kaczmarek, C. Sasaki & K.R.



Change of the slope of the ratio of the Polyakov loop susceptibilities χ_A^L / χ_R^L appears at the same T where the kurtosis drops from its HRG asymptotic value

- In the presence of quarks there is "remnant" of Z(N) symmetry in the χ_A^L / χ_R^L ratio, indicating deconfinement

Remnant of the O(4) chiral phase transtion in QCD

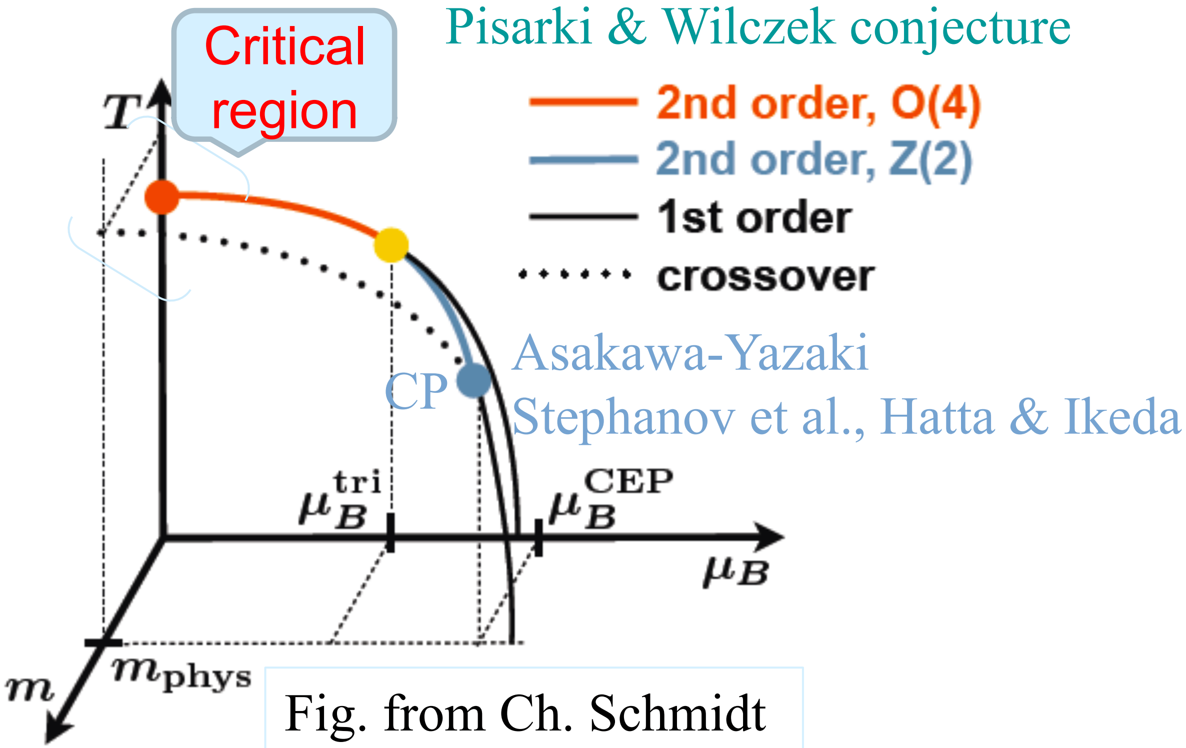


Fig. from Ch. Schmidt

At the CP:
Divergence of Fluctuations, Correlation length and specific heat

- The QCD crossover line can appear in the O(4) critical region!

This has been indeed shown in LQCD calculations by:

BNL-Bielefeld group

Phys. Rev. D83, 014504 (2011)

Phys. Rev. D80, 094505 (2009)

O(4) scaling and magnetic equation of state

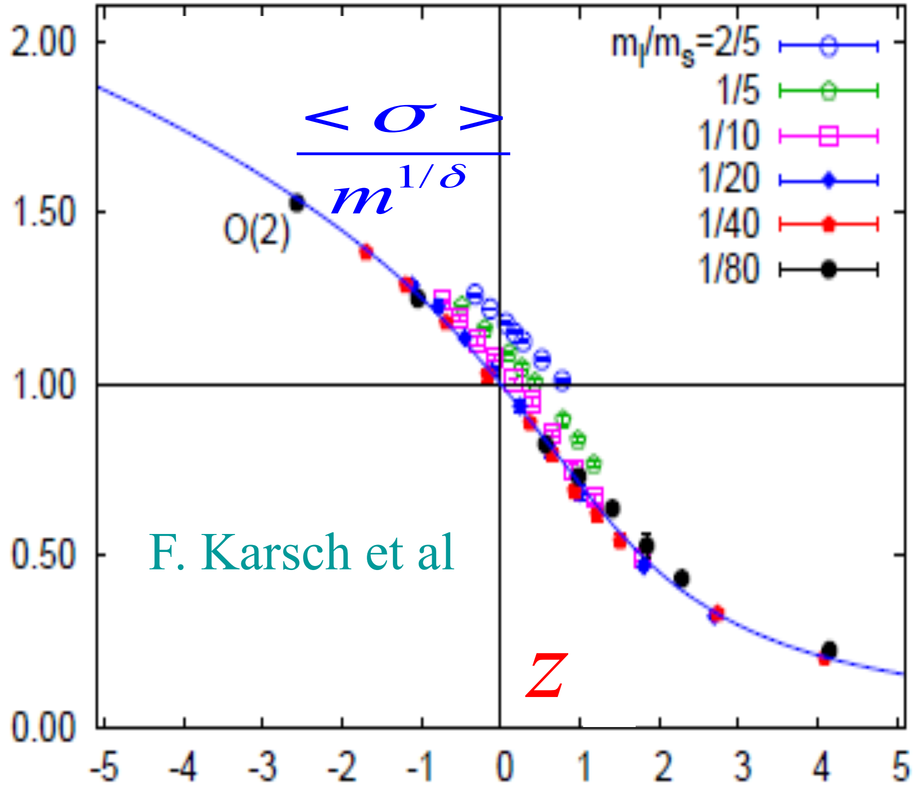
$$F = F_R(T,\mu_q,\mu_I) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)$$

- Phase transition encoded in the magnetic equation of state

$$<\sigma> = -\frac{\partial P}{\partial m} \Rightarrow \text{pseudo-critical lin}$$

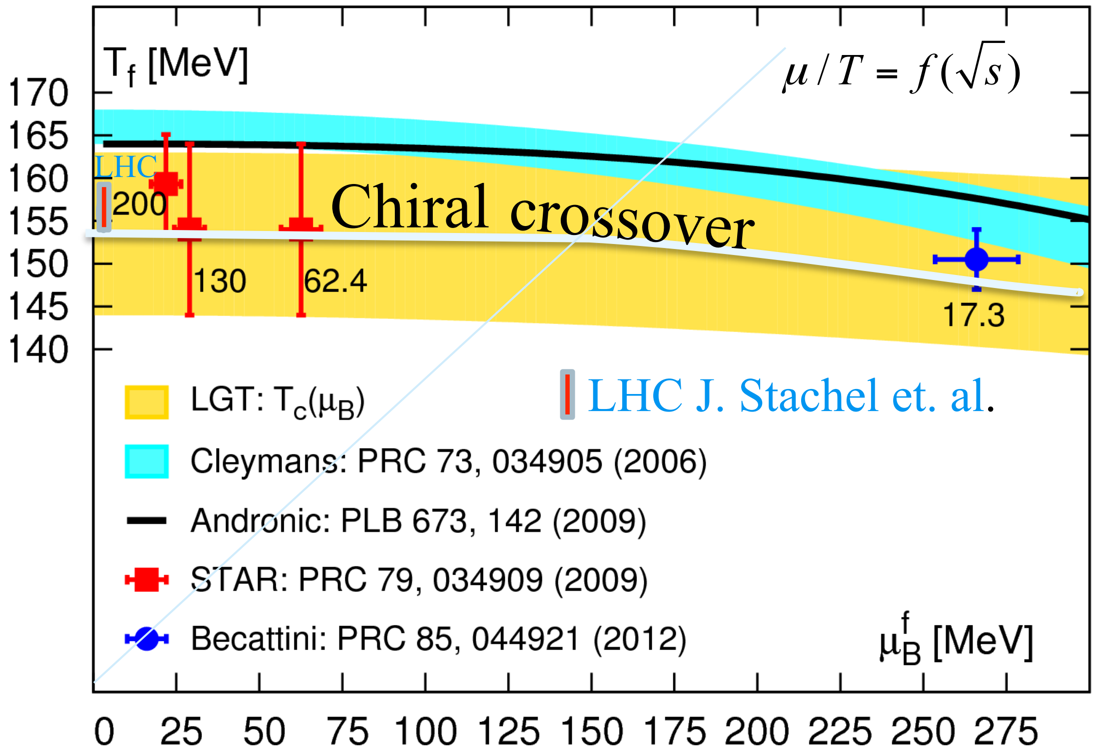
$$\boxed{\frac{<\sigma>}{m^{1/\delta}} = f_s(z)\ , \quad z = tm^{-1/\beta\delta}}$$

QCD chiral crossover transition in the critical region of the O(4) 2nd order



universal scaling function common for all models belonging to the O(4) universality class: known from spin models J. Engels & F. Karsch (2012)

Chemical freezeout and the QCD chiral crossover



Chiral crossover Temperature from LGT

$$T_c = 154 \pm 9 \text{ MeV}$$

HotQCD Coll. (QM'12)

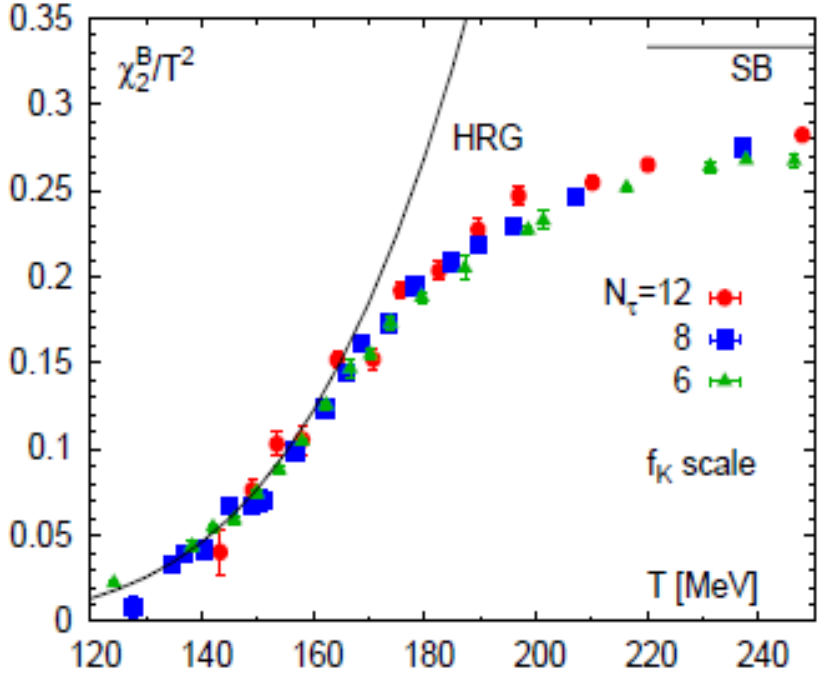
Chemical Freezeout LHC (ALICE)

$$T_f = 156 - 158 \text{ MeV}$$

Peter Braun-Munzinger

- Is there a memory that the system has passed through a region of QCD O(4)-chiral crossover transition?

HotQCD Collaboration



Quark fluctuations and O(4) universality class

- Due to the expected O(4) scaling in QCD the free energy:

$$\boxed{F = F_R(T,\mu_q,\mu_I) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t, b^{\beta\delta/\nu} h)}$$

- Consider generalized susceptibilities of the net-quark number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad c_s^{(n)} = d\ h^{(2-\alpha-n)/\beta\delta}\ f_\pm^{(n)}(z)$$

- Since for $T < T_{pc}$, $c_R^{(n)}$ are well described by the HRG

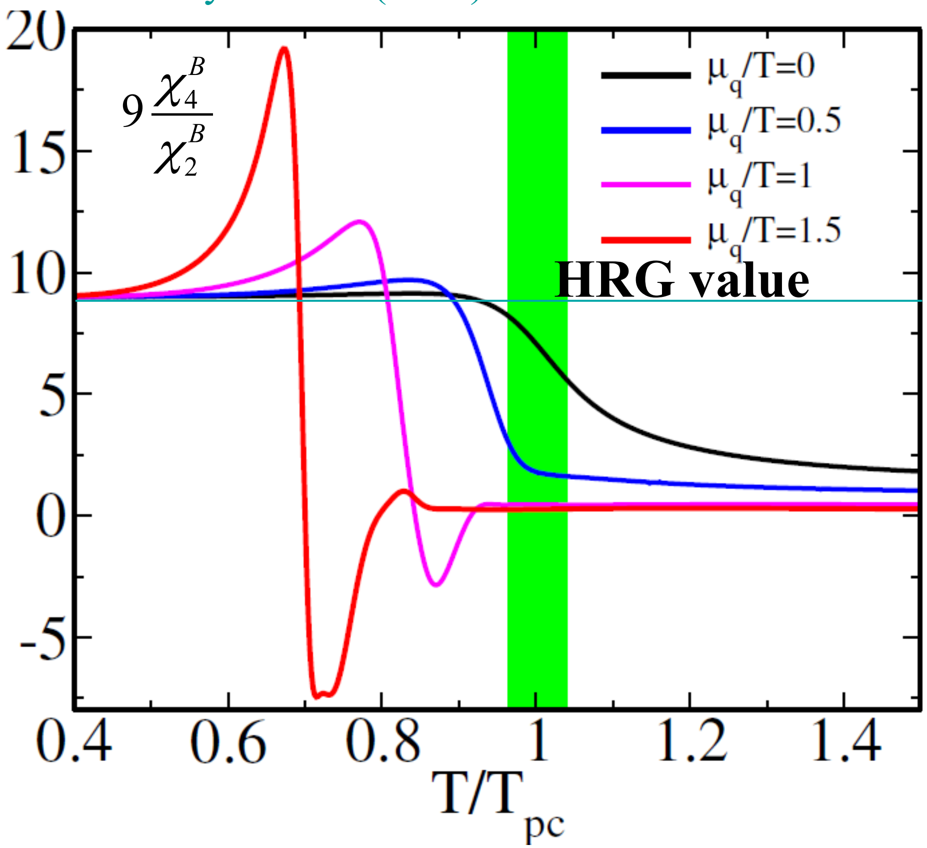
search for deviations (in particular for a larger n) from HRG

⇨ to quantify the contributions of $c_S^{(n)}$, i.e. the O(4) criticality

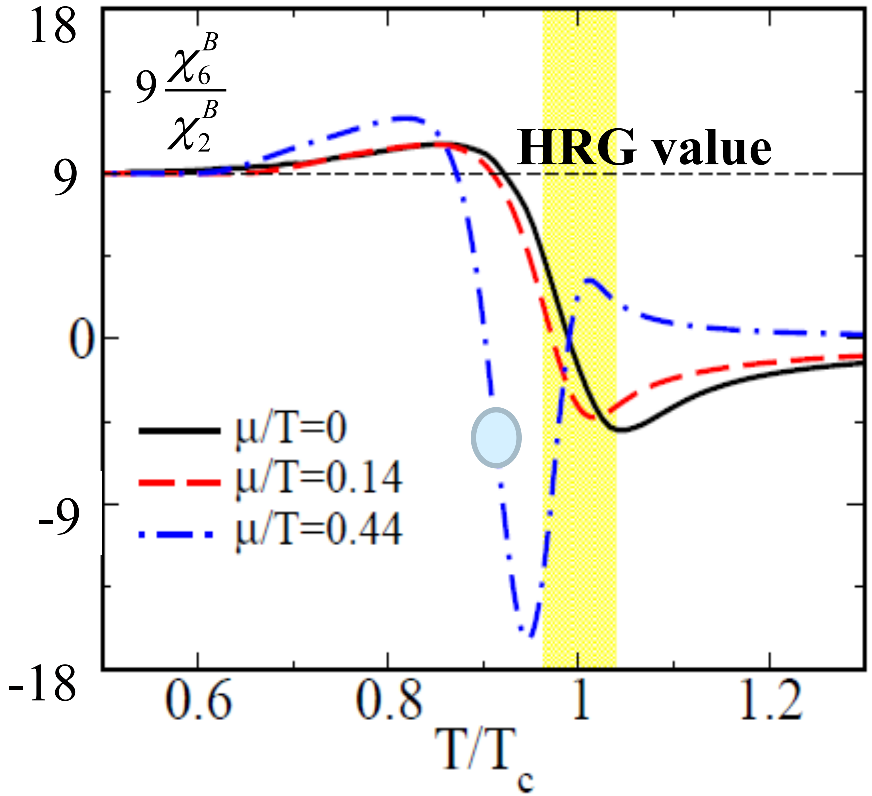
S. Ejiri, F. Karsch & K.R. Phys. Lett. B633, (2006) 275
M. Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103 (2009) 262301
V. Skokov, B. Stokic, B. Friman &K.R. Phys. Rev. C82 (2010) 015206
F. Karsch & K. R. Phys.Lett. B695 (2011) 136
B. Friman, et al. . Phys.Lett. B708 (2012) 179, Nucl.Phys. A880 (2012) 48

Ratios of cumulants at finite density in PQM model with FRG

B. Friman, F. Karsch, V. Skokov &K.R.
Eur.Phys.J. C71 (2011) 1694

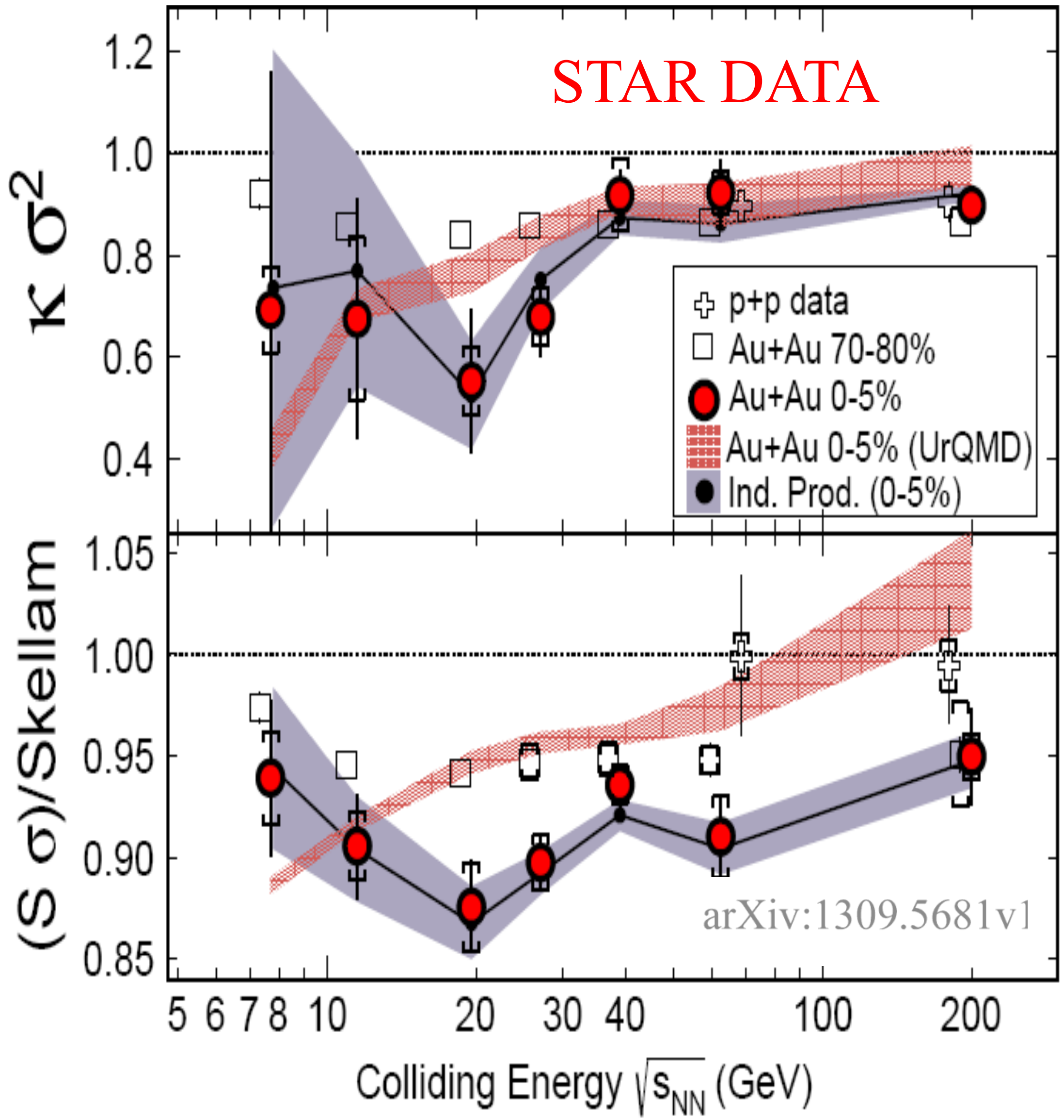


B. Friman, V. Skokov &K.R.
Phys.Rev. C83 (2011) 054904



Deviations from low -T HRG values are increasing with μ / T **and the cumulant order . Negative fluctuations near the chiral crossover.**

STAR data on the first four moments of net baryon number



Deviations from the HRG

$$S\sigma = \frac{\chi_B^{(3)}}{\chi_B^{(2)}} \quad , \quad \kappa\sigma^2 = \frac{\chi_B^{(4)}}{\chi_B^{(2)}}$$

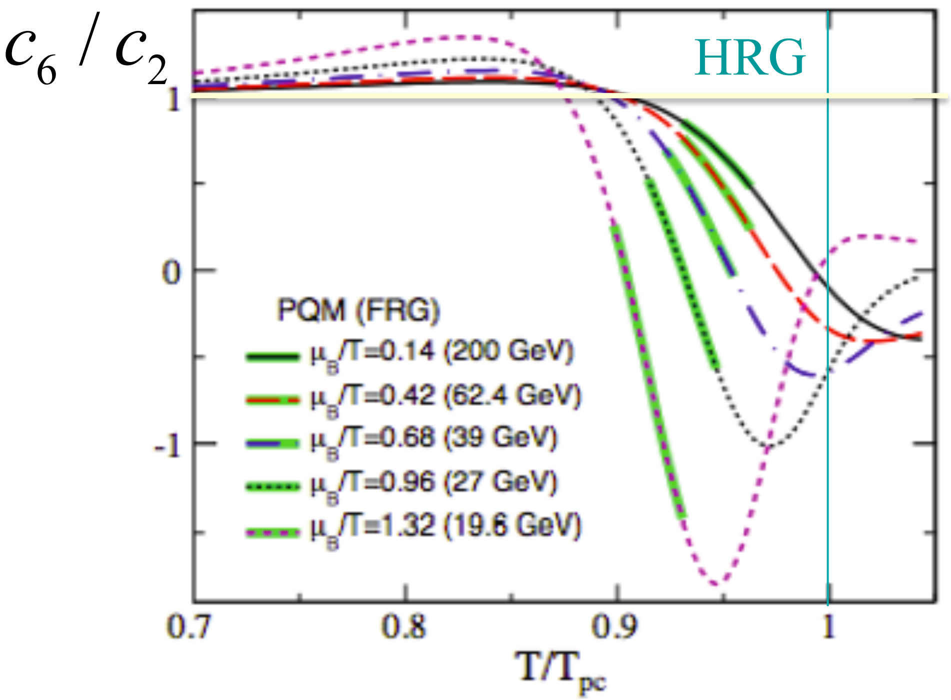
$$S\sigma|_{HRG} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}} \quad , \kappa\sigma|_{HRG} = 1$$

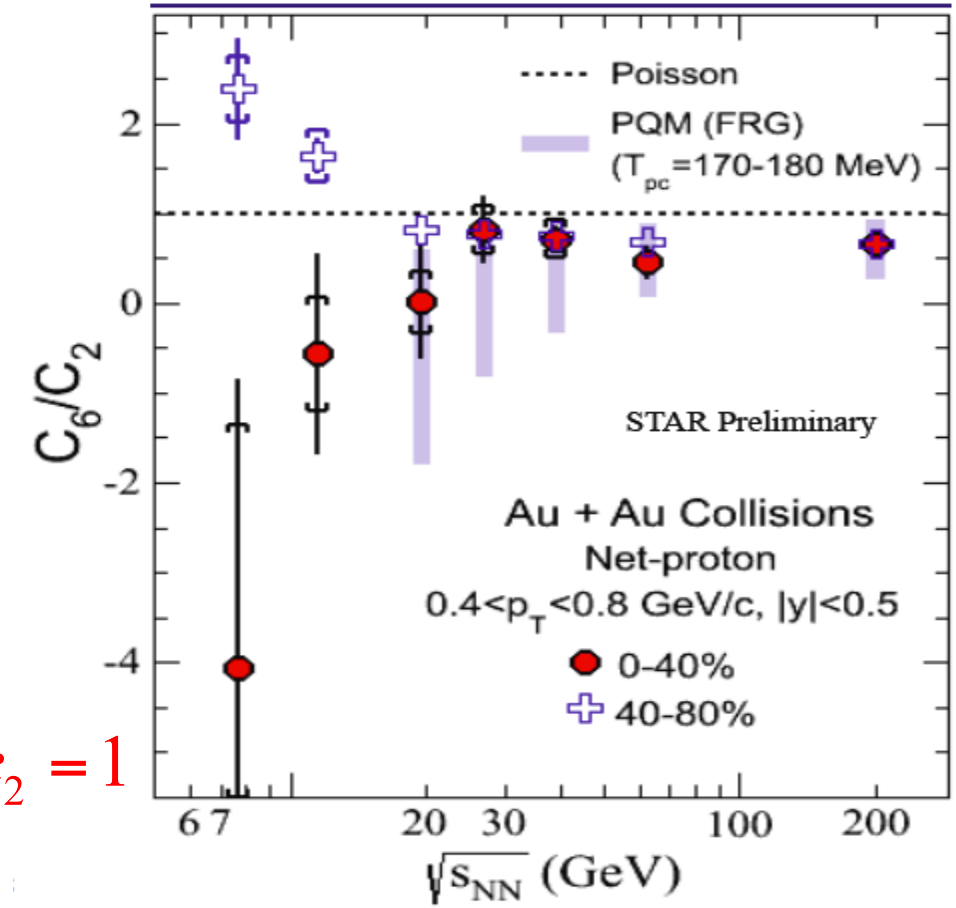
Data qualitatively consistent with the change of these ratios due to the contribution of the O(4) singular part to the free energy

STAR DATA Presented at QM'12

Lizhu Chen for STAR Coll.

V. Skokov, B. Friman & K.R., F. Karsch et al.





The HRG reference predicts: $c_6 / c_2 = 1$

O(4) singular part contribution:

strong deviations from HRG: negative structure already at vanishing baryon density

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}}$$

What is the influence of O(4) criticality on P(N)?

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- For the net baryon number use the Skellam distribution (HRG baseline)

$$P(N) = \left(\frac{B}{\bar{B}}\right)^{N/2} I_N(2\sqrt{B\bar{B}})\exp[-(B+\bar{B})]$$

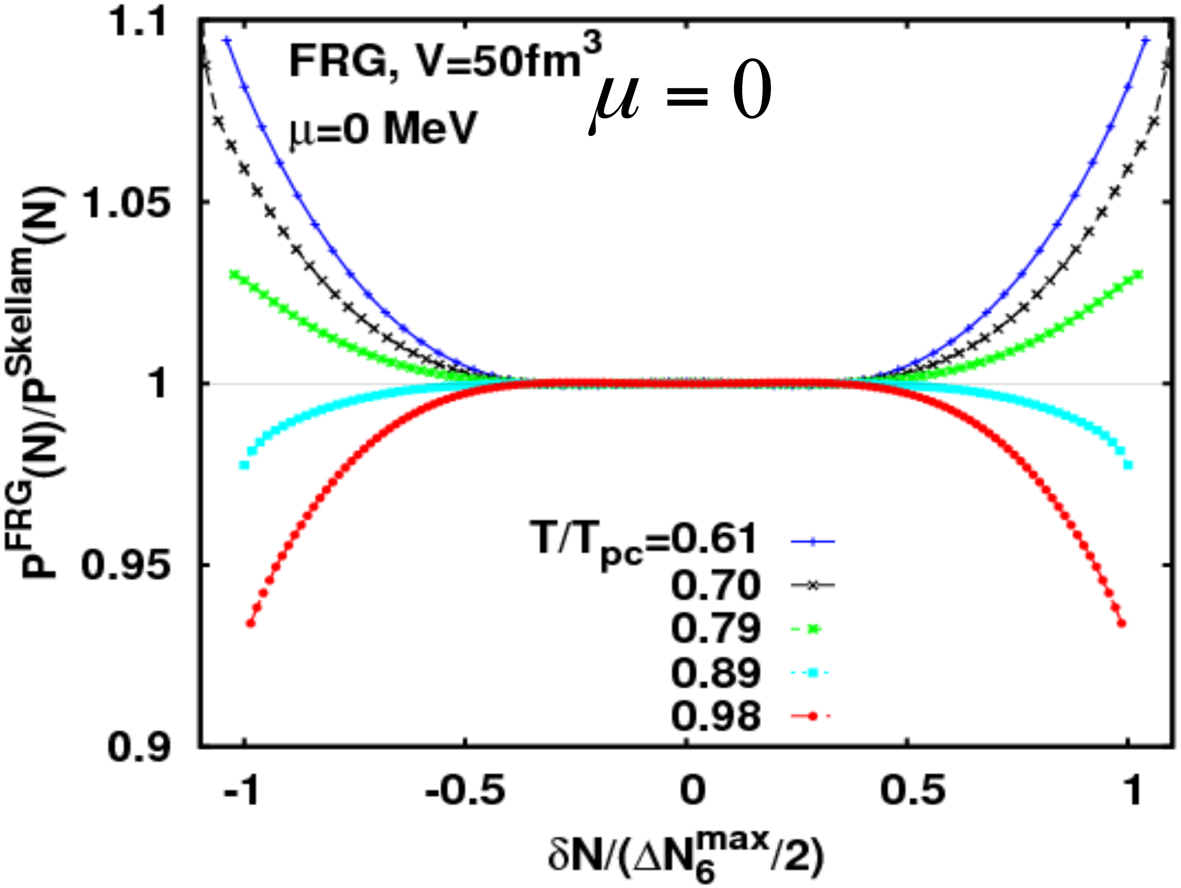
as the reference for the non-critical behavior

- Calculate P(N) in an effective chiral model which exhibits O(4) scaling and compare to the Skellam distribtuion

The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman &K.R. (PQM model within renormalization group FRG)

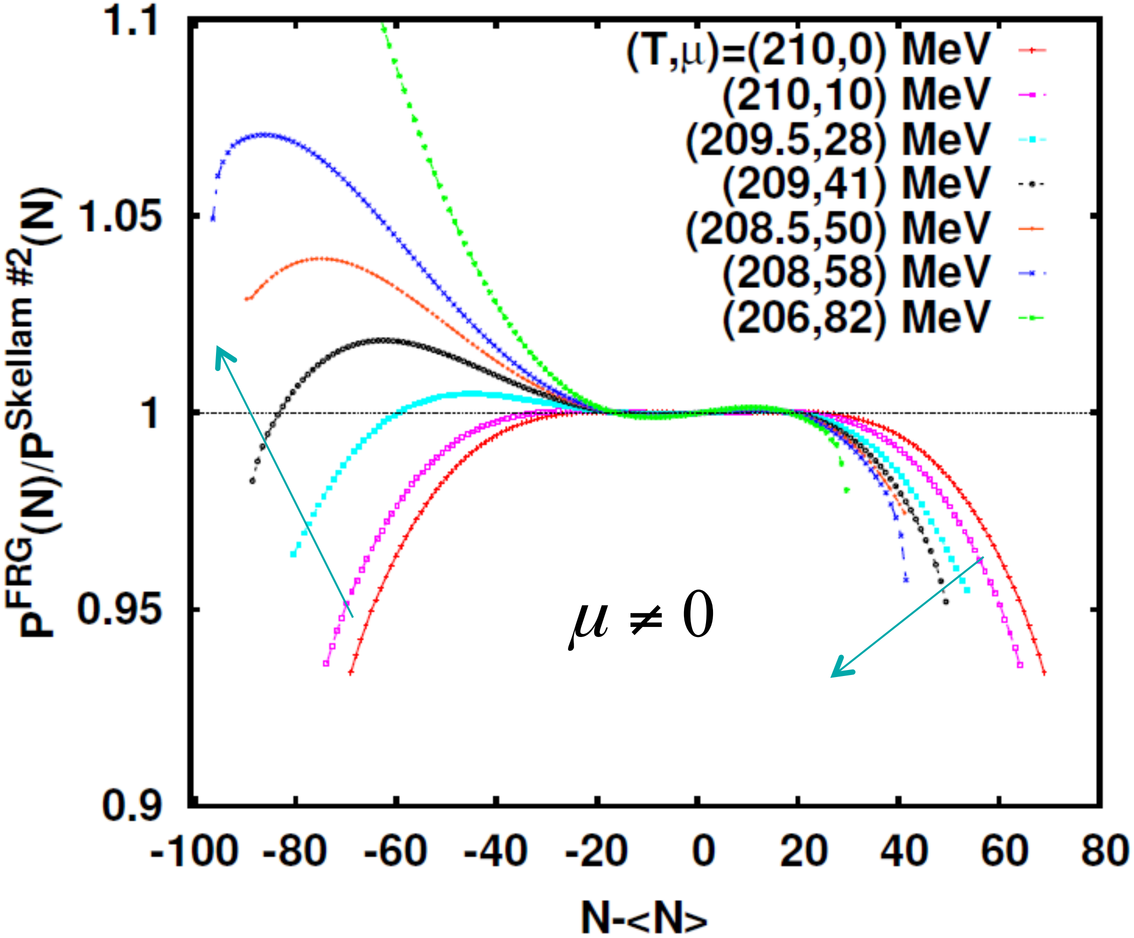


- Ratios less than unity near the chiral crossover, indicating the contribution of the O(4) criticality to the thermodynamic pressure

The influence of O(4) criticality on P(N) for $\mu \neq 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance near $T_{pc}(\mu)$

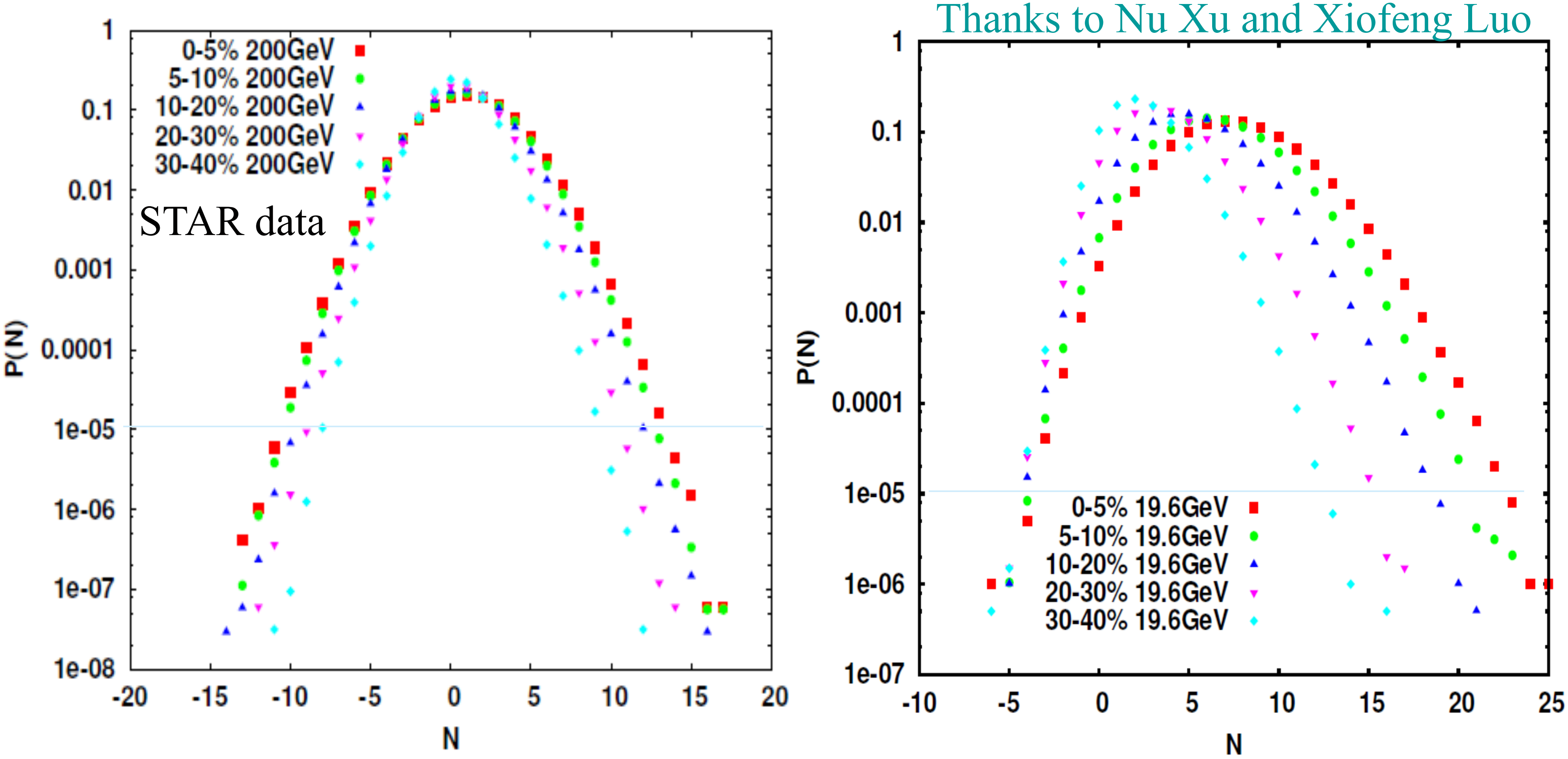
K. Morita, B. Friman et al.



- Asymmetric P(N)
- Near $T_{pc}(\mu)$ the ratios less than unity for $N > \langle N \rangle$
- For sufficiently large μ the for $P^{FRG}(N)/P^{Skellam(N)} > 1$ $N < \langle N \rangle$

Probability distribution of net proton number STAR Coll. data at RHIC

Thanks to Nu Xu and Xiofeng Luo

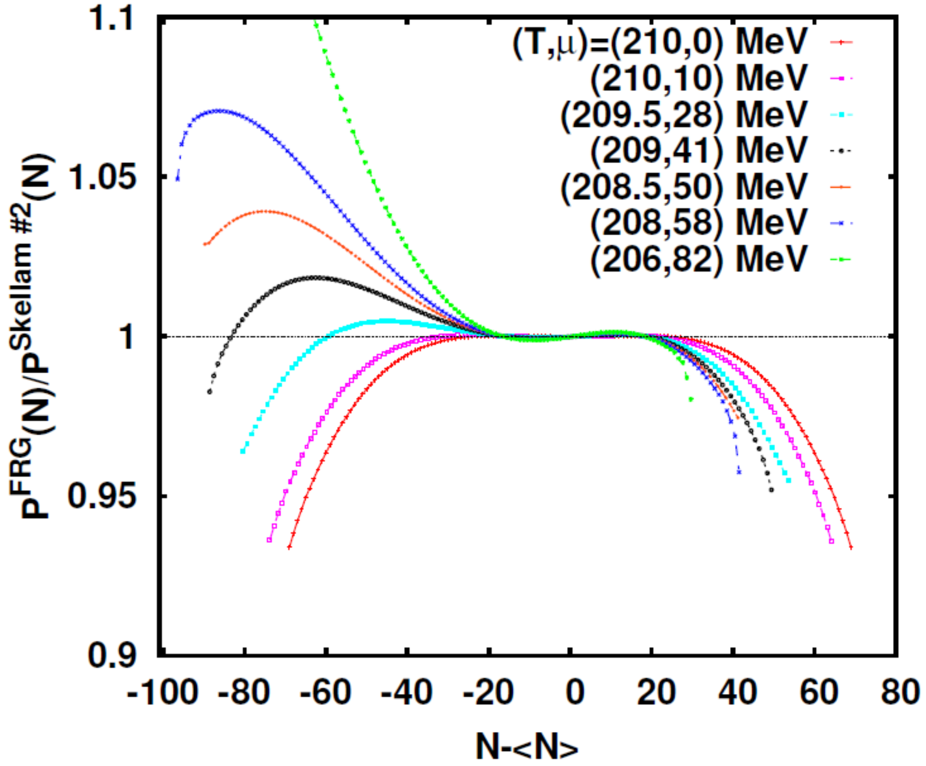


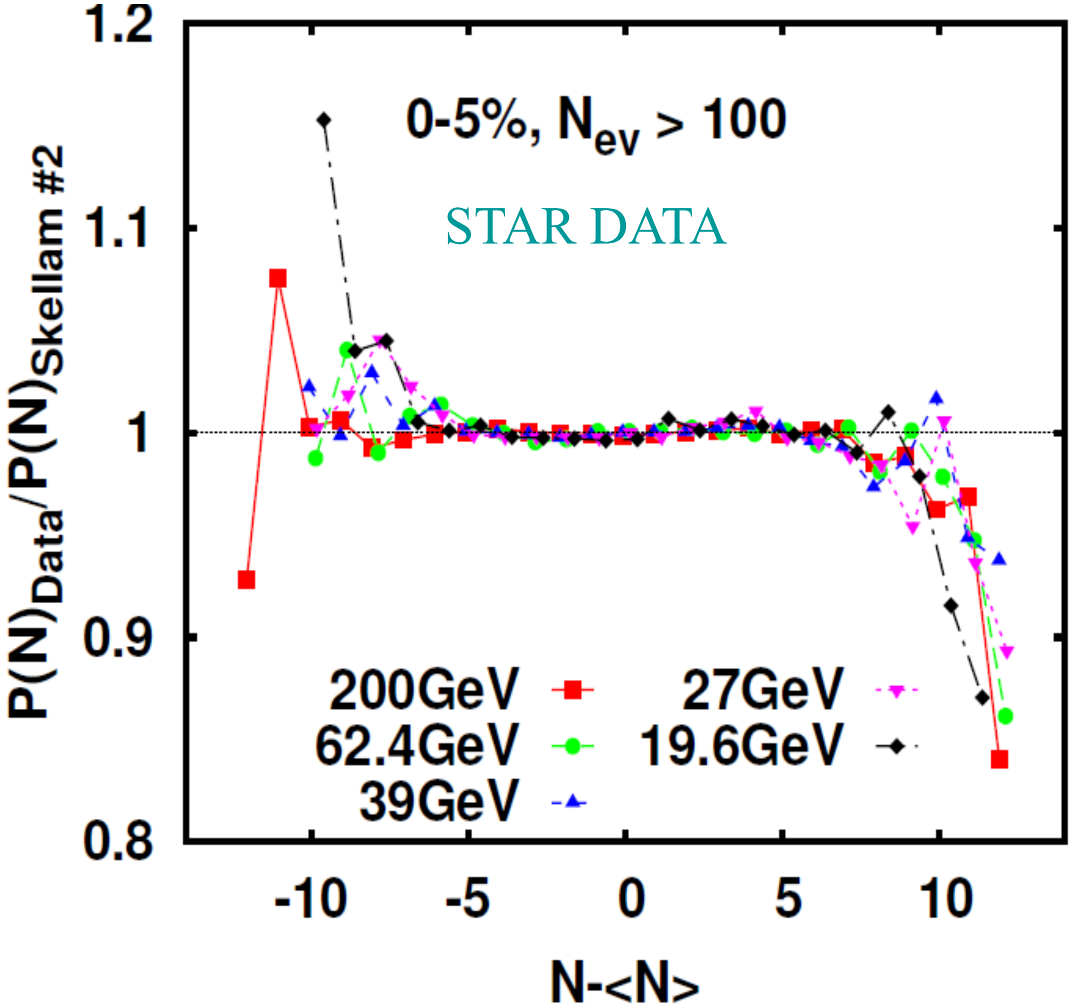
Do we also see the O(4) critical structure in these probability distributions ?

The influence of O(4) criticality on P(N) for $\mu \neq 0$

- In central collisions the probability behaves as being influenced by the chiral transition

K. Morita, B. Friman & K.R.





Conclusions:

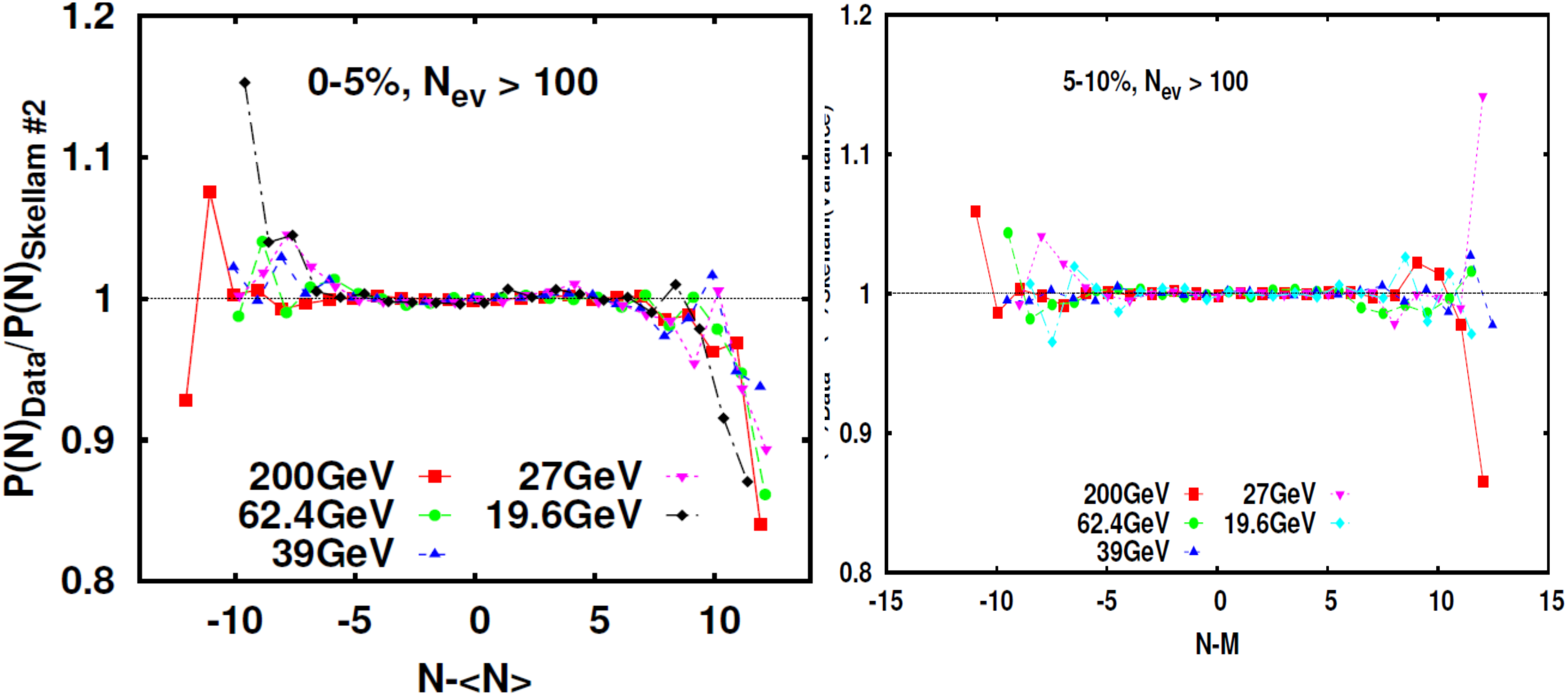
- Ratios of the Polyakov loop and the Net-charge susceptibilities are excellent probes of deconfinement and/or the O(4) chiral crossover transition in QCD
- Systematics of the net-proton fluctuations and their probability distributions measured by STAR are qualitatively consistent with the expectations that they are influenced by the O(4) criticality.

quantitative consistency will be possible by comparing data with LGT results

However, other effects could possibly also influence data:

- Exact charge conservation (Koch, Bzdak, Skokov)
- Acceptance corrections (Bzdak & Koch)
- Effects of final state interactions (Ono, Asakawa & Kitazawa)
- Non-equilibrium effects (Kitazawa, Asakawa & Ono)
- Volume fluctuations (Friman, Skokov & K.R.)
- Etc.

Energy dependence for different centralities



- Ratios at central collisions show properties expected near O(4) chiral pseudocritical line
- For less central collisions the critical structure is lost