Viscous Hydrodynamics for Relativistic Heavy-Ion Collisions: Riemann Solver for Quark-Gluon Plasma

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

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Chiho NONAKA

Hydrodynamic Model:

Yukinao Akamatsu, Shu-ichiro Inutsuka, Makoto Takamoto

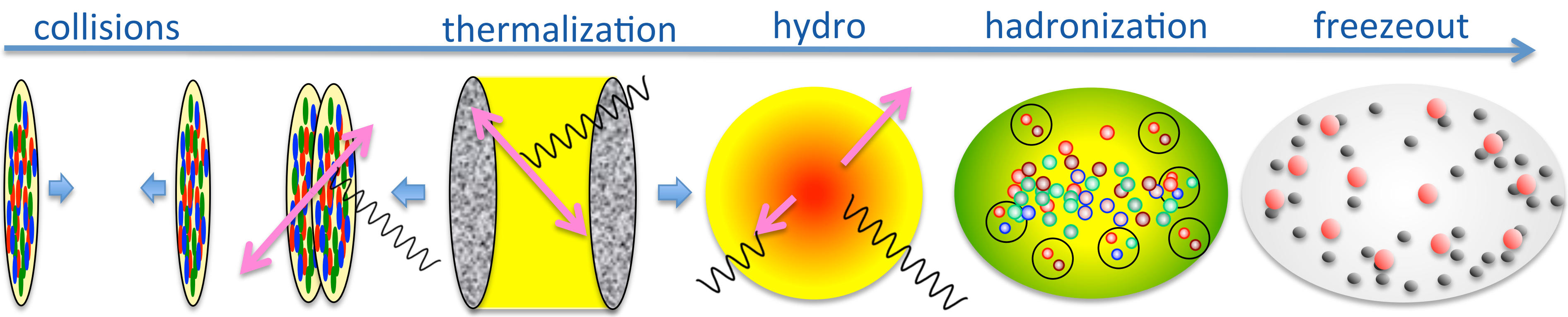
Hybrid Model:

Yukinao Akamatsu, Steffen Bass, Jonah Bernhard

September 24, 2013@RANP 2013, Rio de Janeiro, Brazil

Heavy Ion Collisions vs Model

- Current understanding



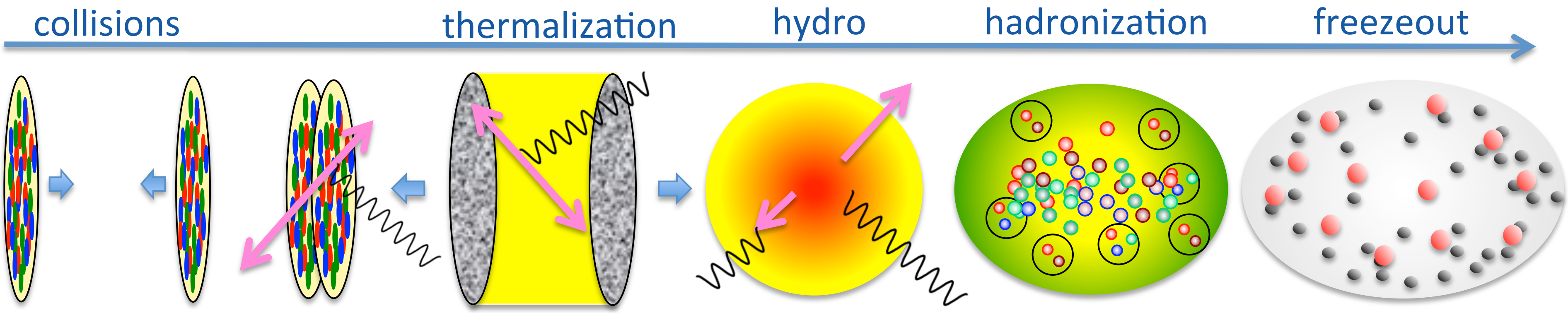
observables

strong elliptic flow @RHIC

model

Heavy Ion Collisions vs Model

- Current understanding



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hydrodynamic model

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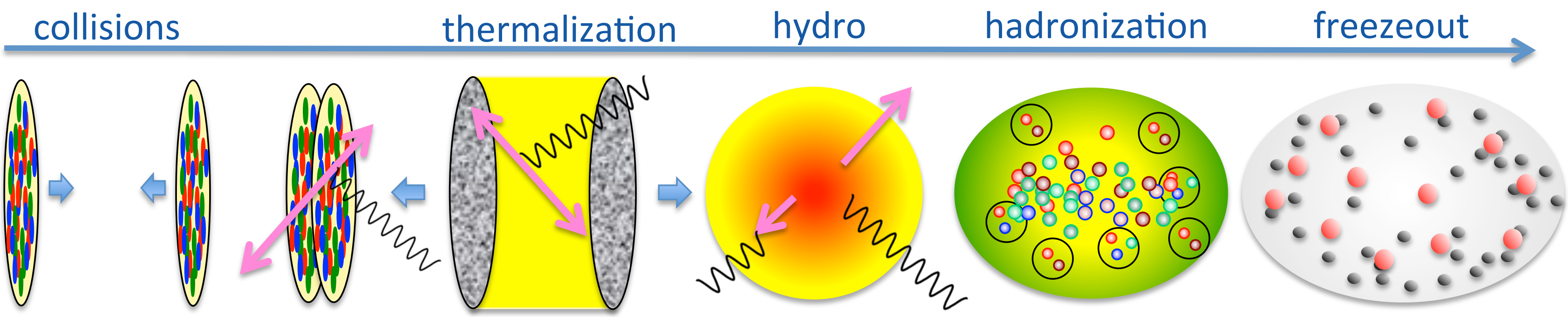


observables

strong elliptic flow @RHIC

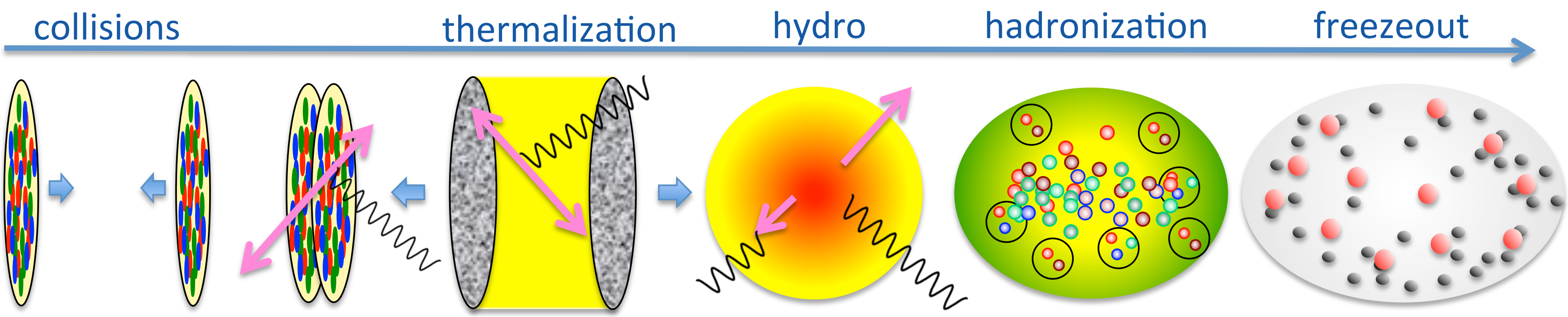
particle yields:
P_T distribution
(proton)

model

hydrodynamic model

Heavy Ion Collisions vs Model

- Current understanding



observables

strong elliptic flow @RHIC

particle yields:
P_T distribution

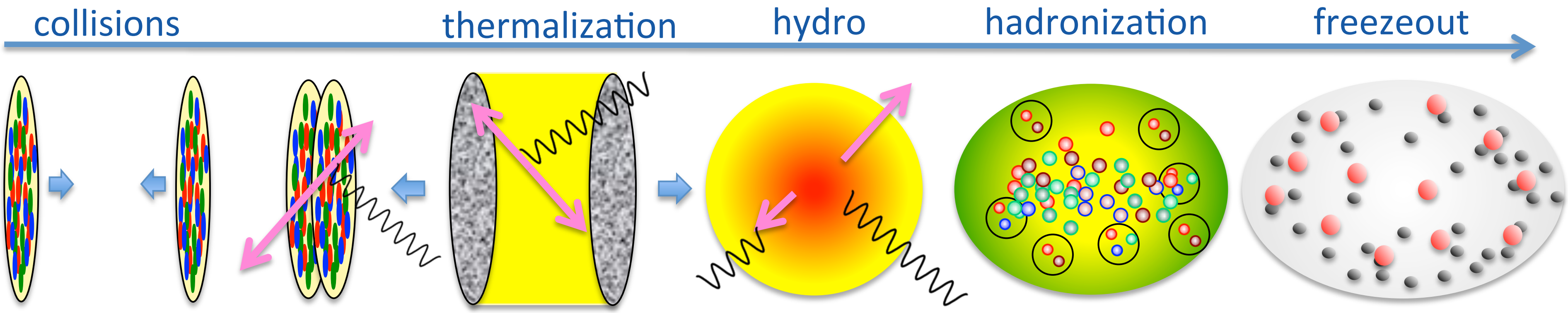
model

hydrodynamic model

- Continuous particle emission
- Partial chemical freezeout
- Hadron base event generator

Heavy Ion Collisions vs Model

- Current understanding



observables

strong elliptic flow @RHIC

particle yields:
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model

hydrodynamic model

- Continuous particle emission
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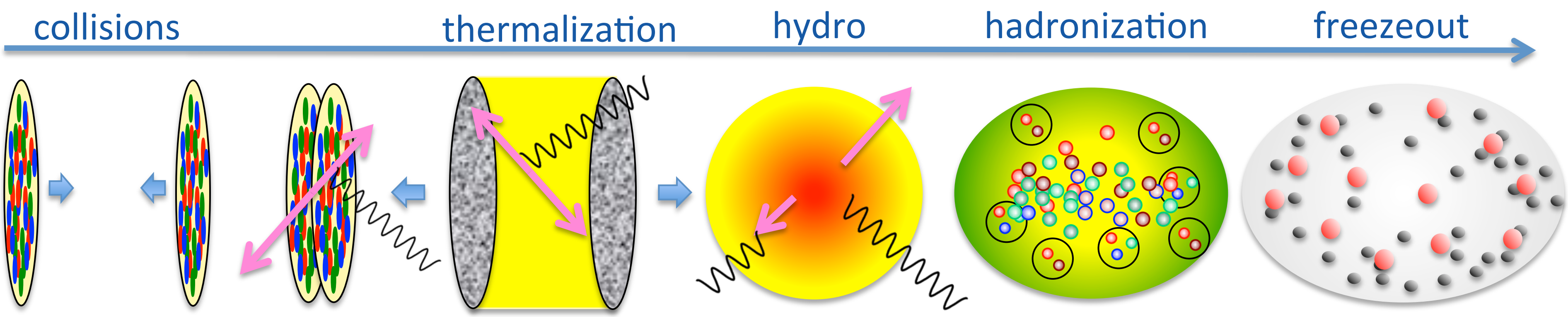


observables

strong elliptic flow @RHIC

particle yields:
P_T distribution

v_2

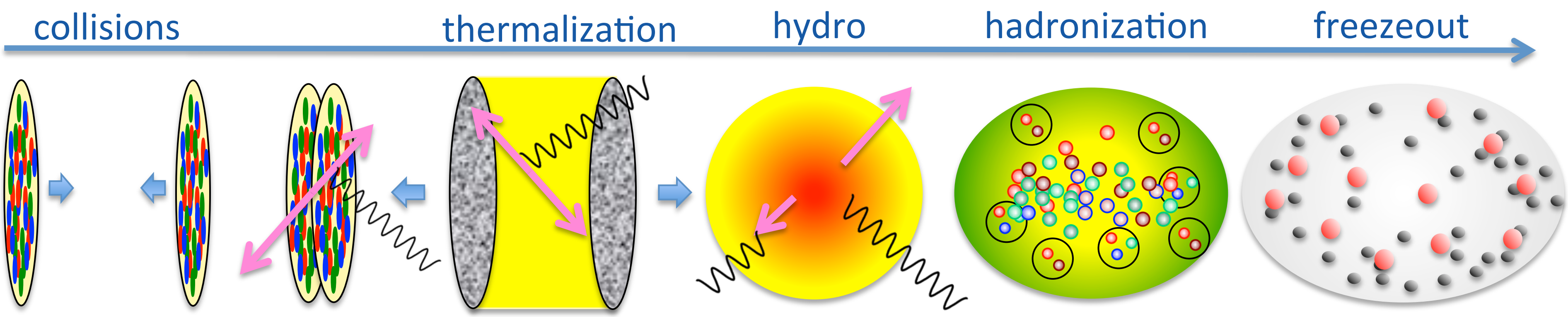
model

hydrodynamic model

- Continuous particle emission
- Partial chemical freezeout
- Hadron base event generator

Heavy Ion Collisions vs Model

- Current understanding



observables

v_2

model

fluctuating
initial conditions

strong elliptic flow @RHIC

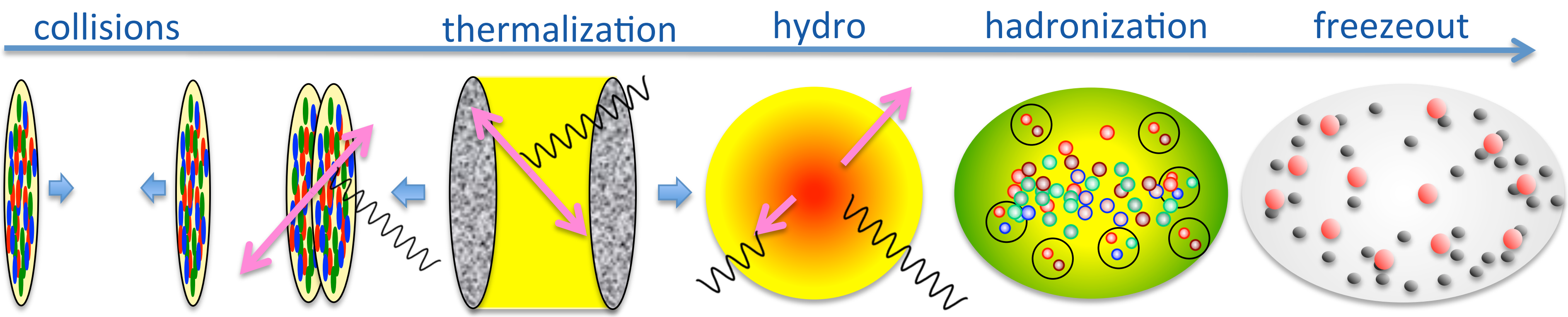
hydrodynamic model

particle yields:
P_T distribution

- Continuous particle emission
- Partial chemical freezeout
- Hadron base event generator

Heavy Ion Collisions vs Model

- Current understanding



observables

v_2, v_n

model

fluctuating
initial conditions

strong elliptic flow @RHIC

hydrodynamic model

particle yields:
P_T distribution

•Continuous particle emission
•Partial chemical freezeout
•Hadron base event generator

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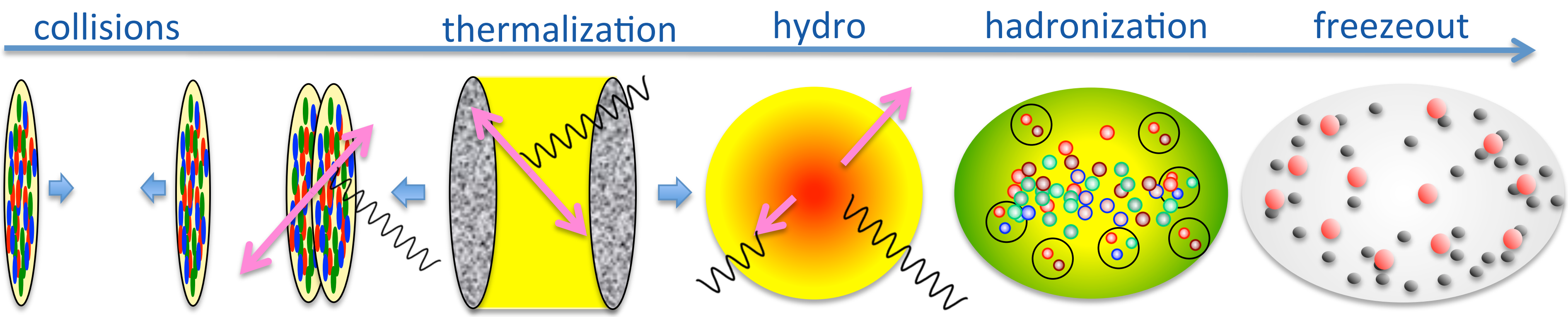
Heavy Ion Collisions vs Model

- Current understanding



observables

v_2, v_n

model

fluctuating
initial conditions

strong elliptic flow @RHIC

hydrodynamic model

shock wave, viscosity effect

particle yields:
P_T distribution

- Continuous particle emission
- Partial chemical freezeout
- Hadron base event generator

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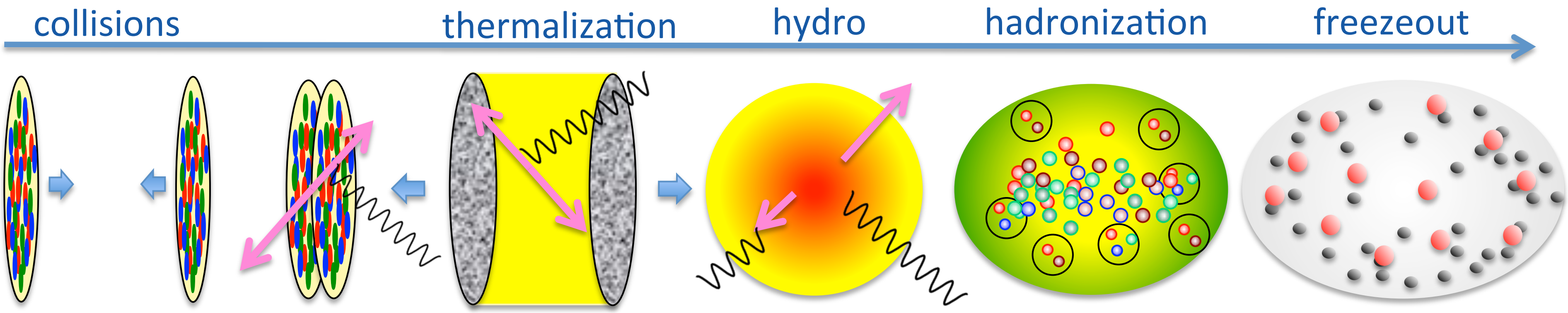
Our Hybrid Model



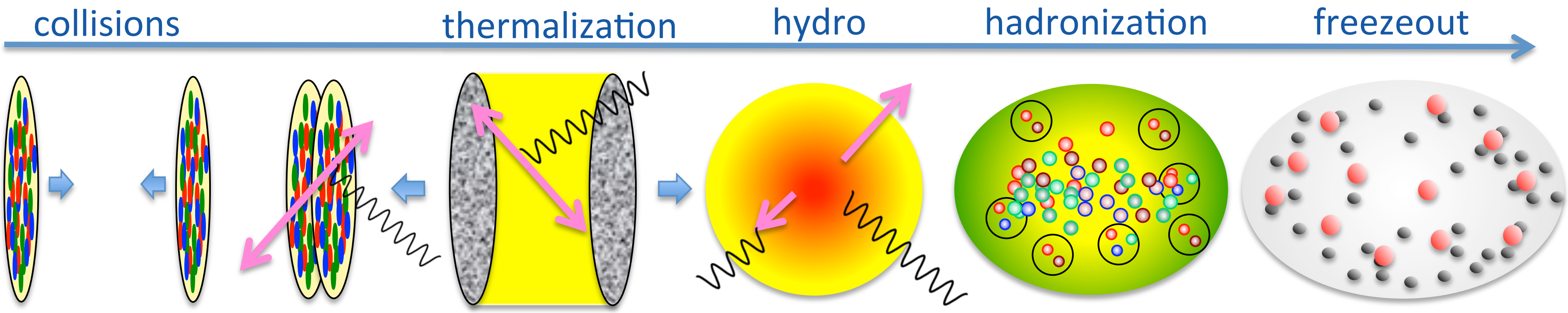
Fluctuating Initial conditions

Hydrodynamic expansion

Freezeout process

- From Hydro to particle
- Final state interactions

Our Hybrid Model



Fluctuating Initial conditions   Hydrodynamic expansion   Freezeout process

- From Hydro to particle
- Final state interactions

Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665、J. Comp. Phys. (2014)34

MC-KLN → hydrodynamic model → Cornelius → Oscar sampler → UrQMD

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http://www.aiu.ac.jp/~ynara/mckln/

Freezeout hypersurface finder
Huovinen, Petersen

Ohio group

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Akamatsu, Inutsuka, CN, Takamoto:

arXiv:1302.1665、J. Comp. Phys. (2014) 34

HYDRODYNAMIC MODEL

Viscous Hydrodynamic Model

- Relativistic viscous hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0$$

  - First order in gradient: acausality
  - Second order in gradient:
    - Israel-Stewart, Ottinger and Grmela, AdS/CFT, Grad's 14-momentum expansion, Renomarization group
- Numerical scheme
  - Shock-wave capturing schemes: Riemann problem
    - Godunov scheme: analytical solution of Riemann problem
    - SHASTA: the first version of Flux Corrected Transport algorithm, Song, Heinz, Chaudhuri
    - Kurganov-Tadmor (KT) scheme, McGill

Our Approach

Takamoto and Inutsuka, arXiv:1106.1732
Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665

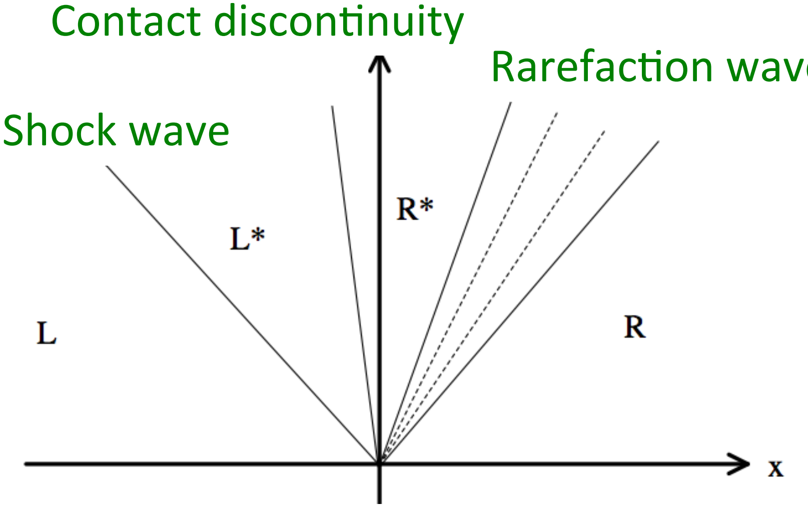
- Israel-Stewart Theory

(ideal hydro)

1. dissipative fluid dynamics = advection + dissipation

Riemann solver: Godunov method

exact solution



Two shock approximation

Mignone, Plewa and Bodo, Astrophys. J. S160, 199 (2005)

Rarefaction wave $\longrightarrow$ shock wave

2. relaxation equation = advection + stiff equation

Numerical Scheme

- Israel-Stewart Theory

Takamoto and Inutsuka, arXiv:1106.1732

1. Dissipative fluid equation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}$$
$$= T_{\text{ideal}} + T_{\text{dissip}}$$

$$\partial_t U + \nabla \cdot F(U) = 0 \qquad U = U_{\text{ideal}} + U_{\text{dissip}}$$

2. Relaxation equation

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$

$$\hat{D}\pi^{\mu\nu} = \frac{1}{\tau_\pi}(\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}) - I^{\mu\nu}_\pi,$$

$$\hat{D}q^\mu = \frac{1}{\tau_q}(q^\mu_{NS} - q^\mu) - I^\mu_q,$$

$$\hat{D} = u^\mu \partial_u$$

$$\left(\frac{\partial}{\partial t} + v^j \frac{\partial}{\partial x^j}\right)\Pi = -\frac{I_\Pi}{\gamma}, \quad + \quad \frac{\partial}{\partial t}\Pi = \frac{1}{\gamma\tau_\Pi}(\Pi_{NS} - \Pi),$$

advection — stiff equation

$$\Delta t < \tau_{\text{relax}} << \tau_{\text{fluid}}$$

I: second order terms

$$\tau^{\mu\nu} = \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Relaxation Equation

Takamoto and Inutsuka, arXiv:1106.1732

- Numerical scheme

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + v^j \frac{\partial}{\partial x^j}\right)\Pi = -\frac{I_\Pi}{\gamma}, \quad + \quad \frac{\partial}{\partial t}\Pi = \frac{1}{\gamma\tau_\Pi}(\Pi_{NS} - \Pi),$$

advection

up wind method

stiff equation

$$\Delta t < \tau_{\mathrm{relax}} << \tau_{\mathrm{fluid}}$$

- during Δt Π_{NS} ~constant

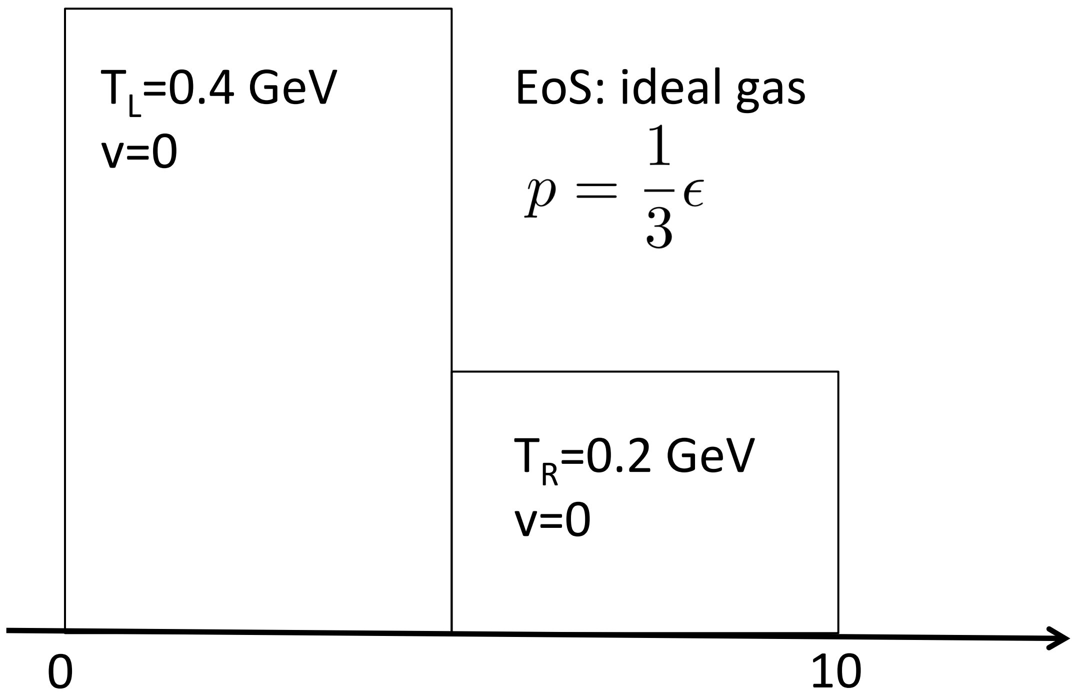
Piecewise exact solution

$$\Pi = (\Pi_0 - \Pi_{NS})\exp\left[-\frac{t - t_0}{\tau_\Pi}\right] + \Pi_{NS}$$

fast numerical scheme

Comparison

- Shock Tube Test : *Molnar, Niemi, Rischke*, Eur.Phys.J.C65,615(2010)

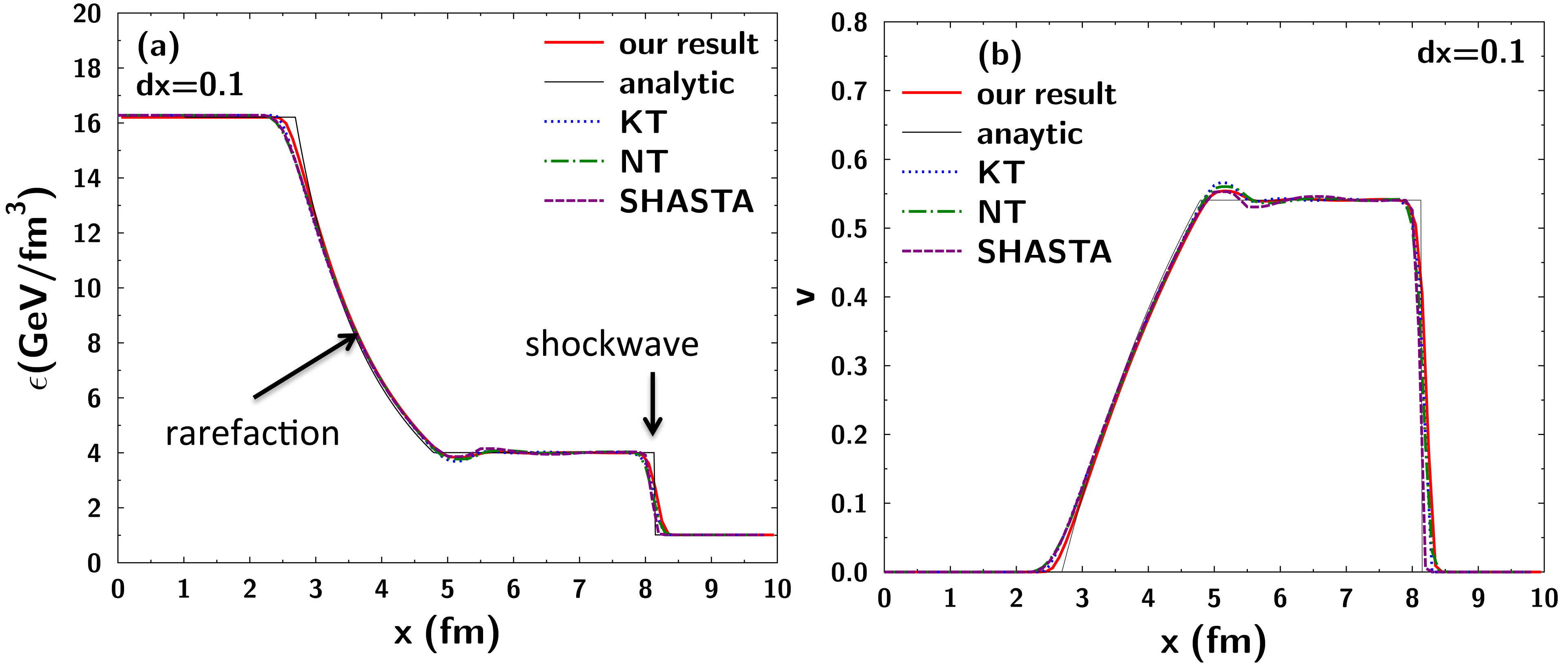


Nx=100, dx=0.1, dt=0.04

- Analytical solution
- Numerical schemes
  SHASTA, KT, NT
  Our scheme

Shocktube problem

- Ideal case



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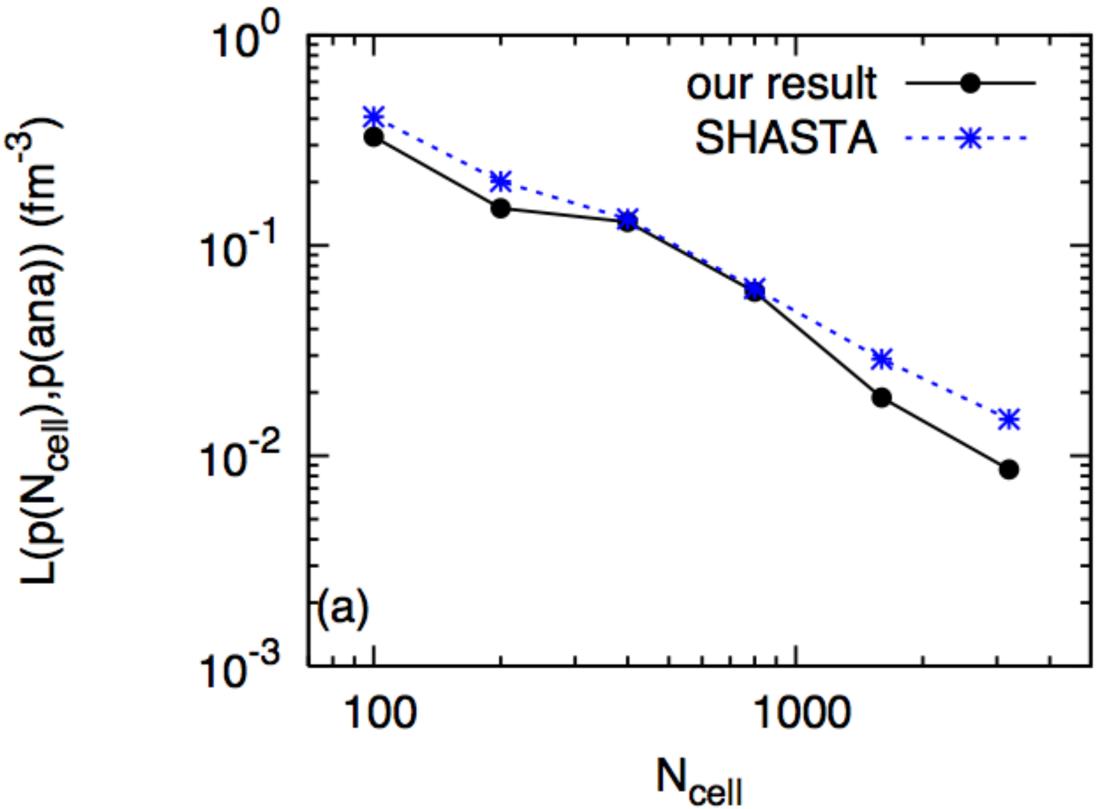
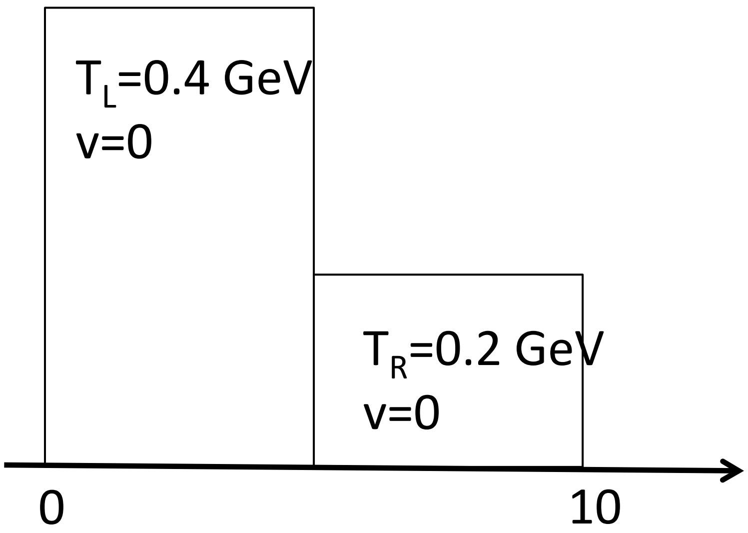
L1 Norm

- Numerical dissipation: deviation from analytical solution





For analysis of heavy ion collisions

N_{cell}=100: dx=0.1 fm

$$L(p(N_{\rm cell}), p({\rm anaytic})) = \sum_{i=1}^{N_{\rm cell}} |p(N_{\rm cell}) - p({\rm analytic})| \frac{\lambda}{N_{\rm cell}}$$

λ=10 fm

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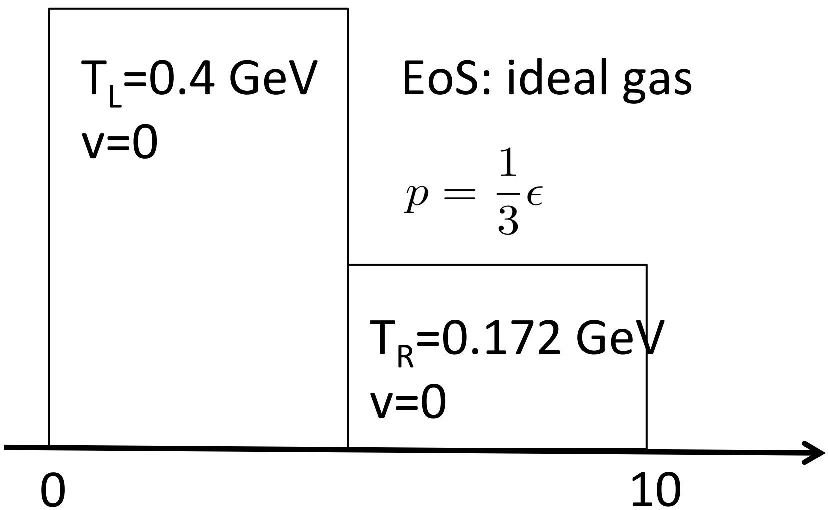
Large ΔT difference

- T_L=0.4 GeV, T_R=0.172 GeV
  - SHASTA becomes unstable.
  - Our algorithm is stable.
- SHASTA: anti diffusion term, A_{ad}
  - A_{ad} = 1 : default value, unstable
  - A_{ad} =0.99: stable,

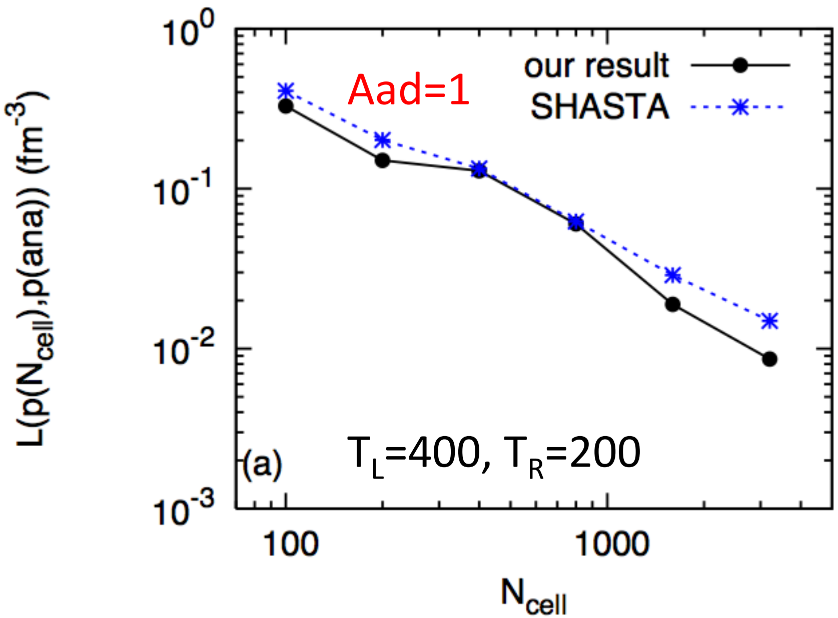
    more numerical dissipation

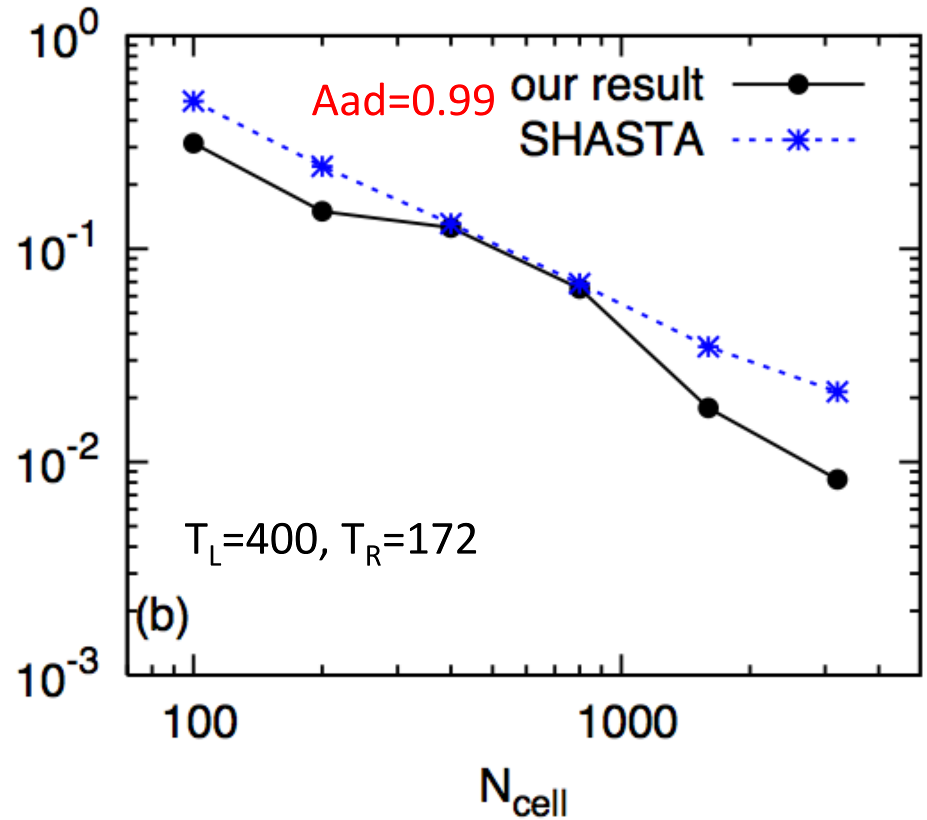


Nx=100, dx=0.1, dt=0.04

L1 norm

- SHASTA with small A_{ad} has large numerical dissipation



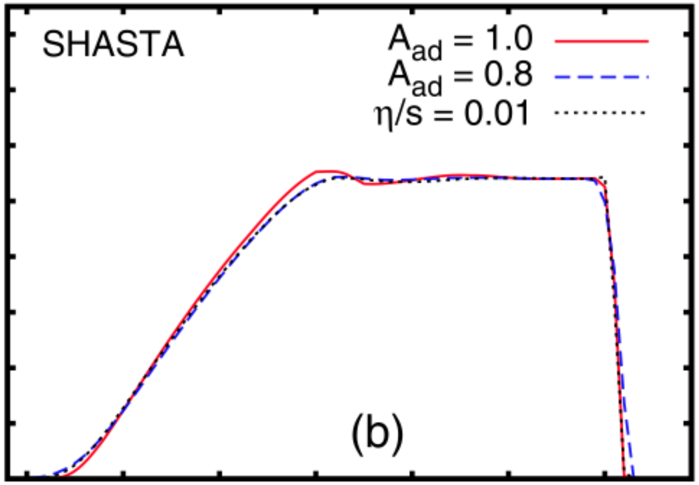


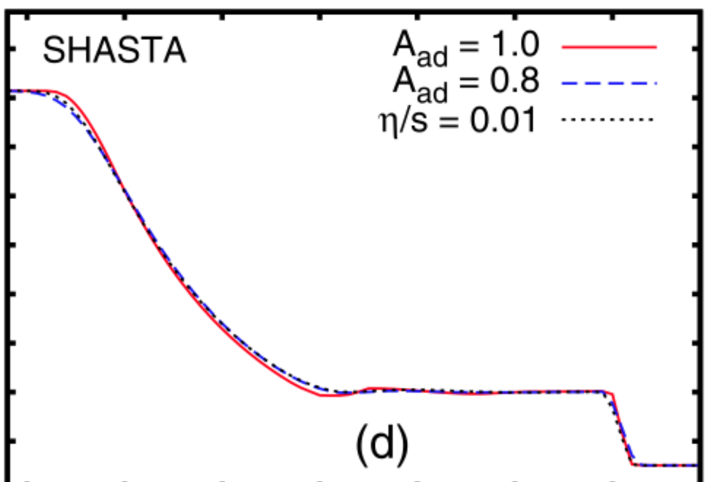
$$L(p(N_{\rm cell}), p({\rm anaytic})) = \sum_{i=1}^{N_{\rm cell}} |p(N_{\rm cell}) - p({\rm analytic})| \frac{\lambda}{N_{\rm cell}}$$

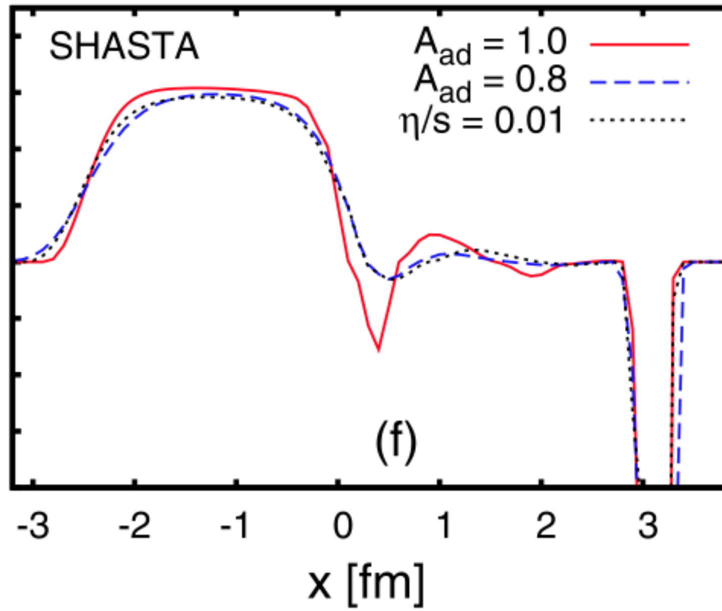
λ=10 fm

Artificial and Physical Viscosities

Molnar, Niemi, Rischke, Eur.Phys.J.C65,615(2010)







Antidiffusion terms : artificial viscosity stability

$$U_i^{n+1} = \tilde{U}_i - \tilde{A}_i + \tilde{A_{i-1}}$$
$$A_i = A\mathrm{ad}\tilde{\Delta}_i/8$$

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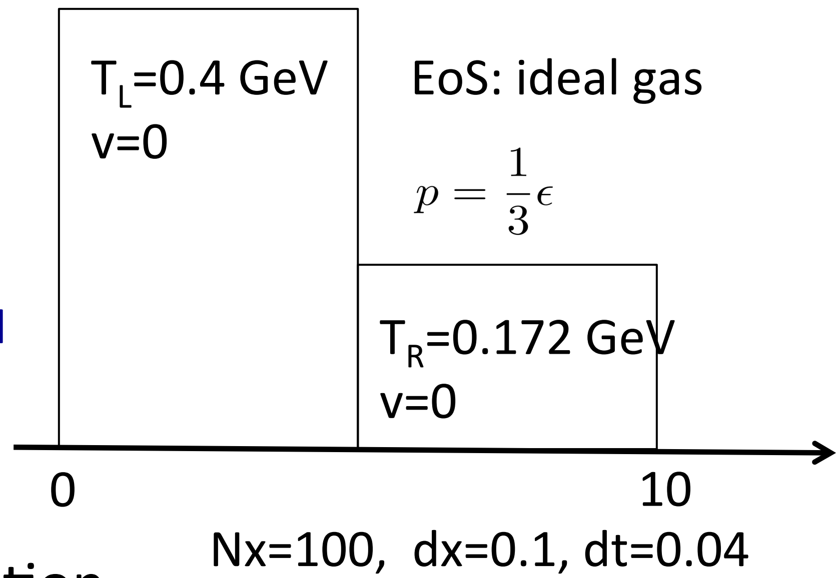
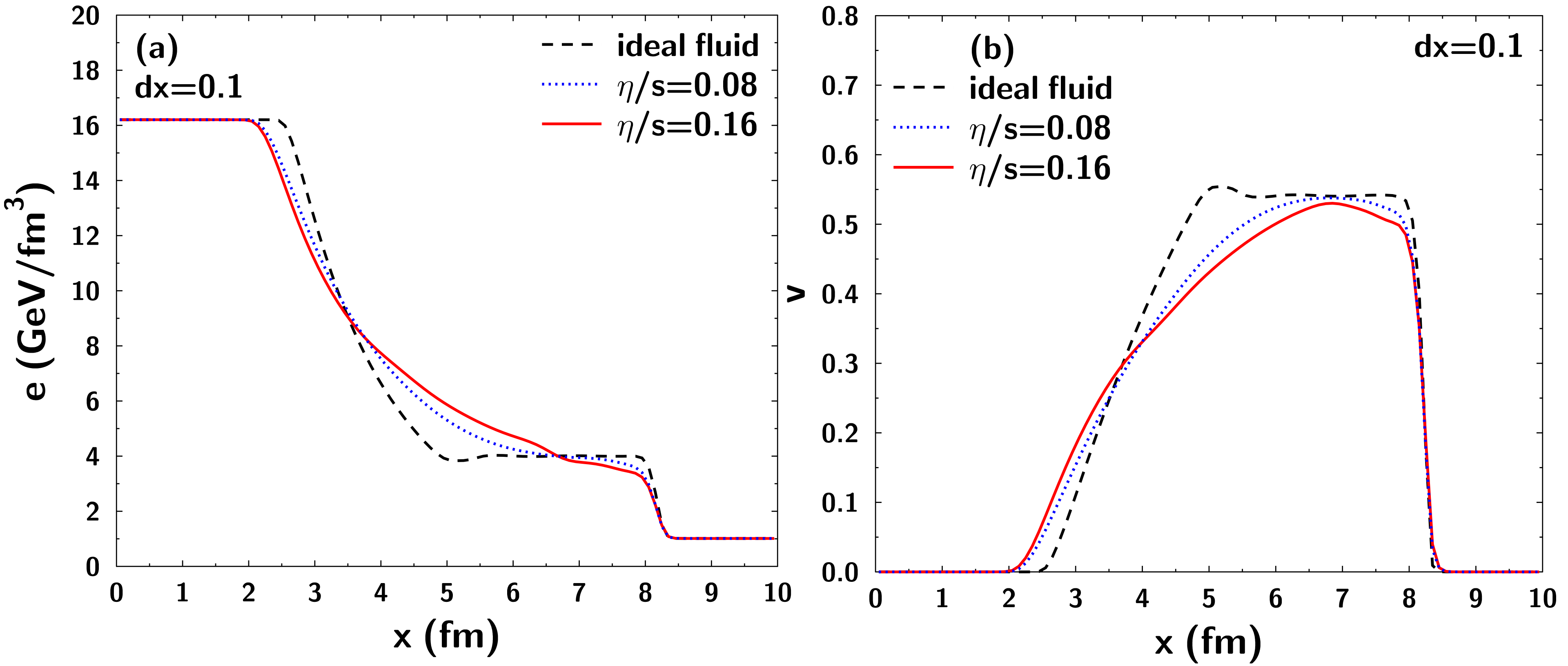
  - Our algorithm is stable.
- SHASTA: anti diffusion term, A_{ad}
  - A_{ad} = 1 : default value
  - A_{ad} =0.99: stable,

    more numerical dissipation
- Large fluctuation (ex initial conditions)
  - Our algorithm is stable even with small numerical dissipation.



Shocktube problem

- Viscous case



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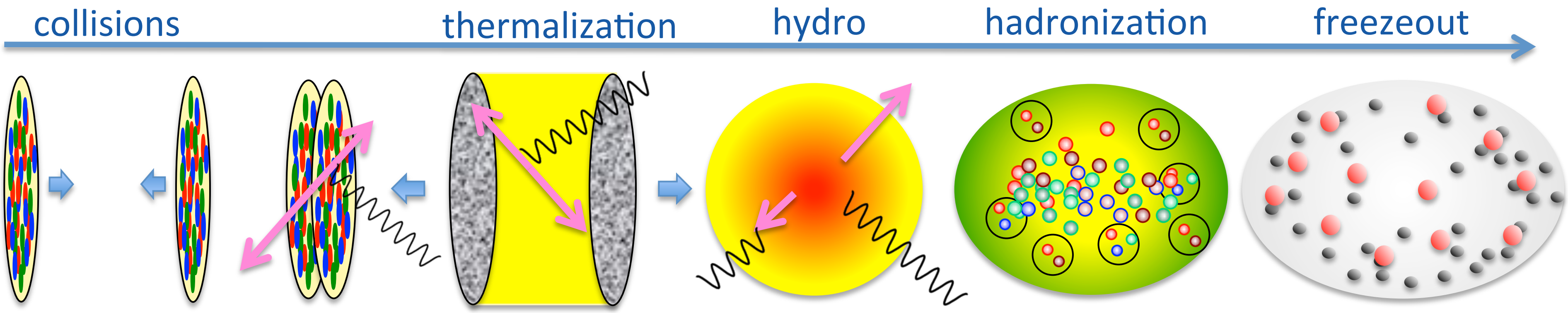
HYBRID MODEL

Our Hybrid Model



Fluctuating Initial conditions

Hydrodynamic expansion

Freezeout process
- From Hydro to particle
- Final state interactions

Akamatsu, Inutsuka, CN, Takamoto,
arXiv:1302.1665、J. Comp. Phys. (2014) 34

MC-KLN → hydrodynamic model → Cornelius → Oscar sampler → UrQMD

Nara

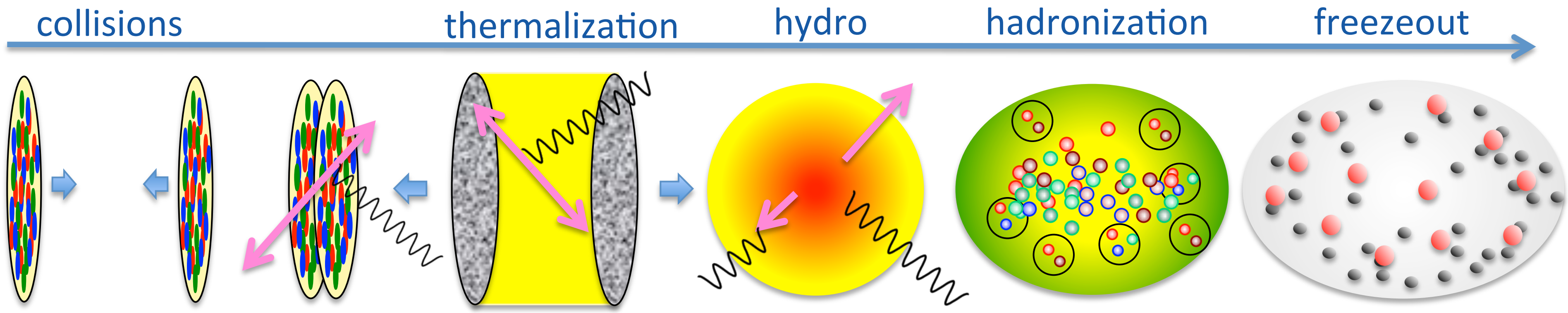
http://www.aiu.ac.jp/~ynara/mckln/

Freezeout hypersurface finder

Huovinen, Petersen

Ohio group

Our Hybrid Model



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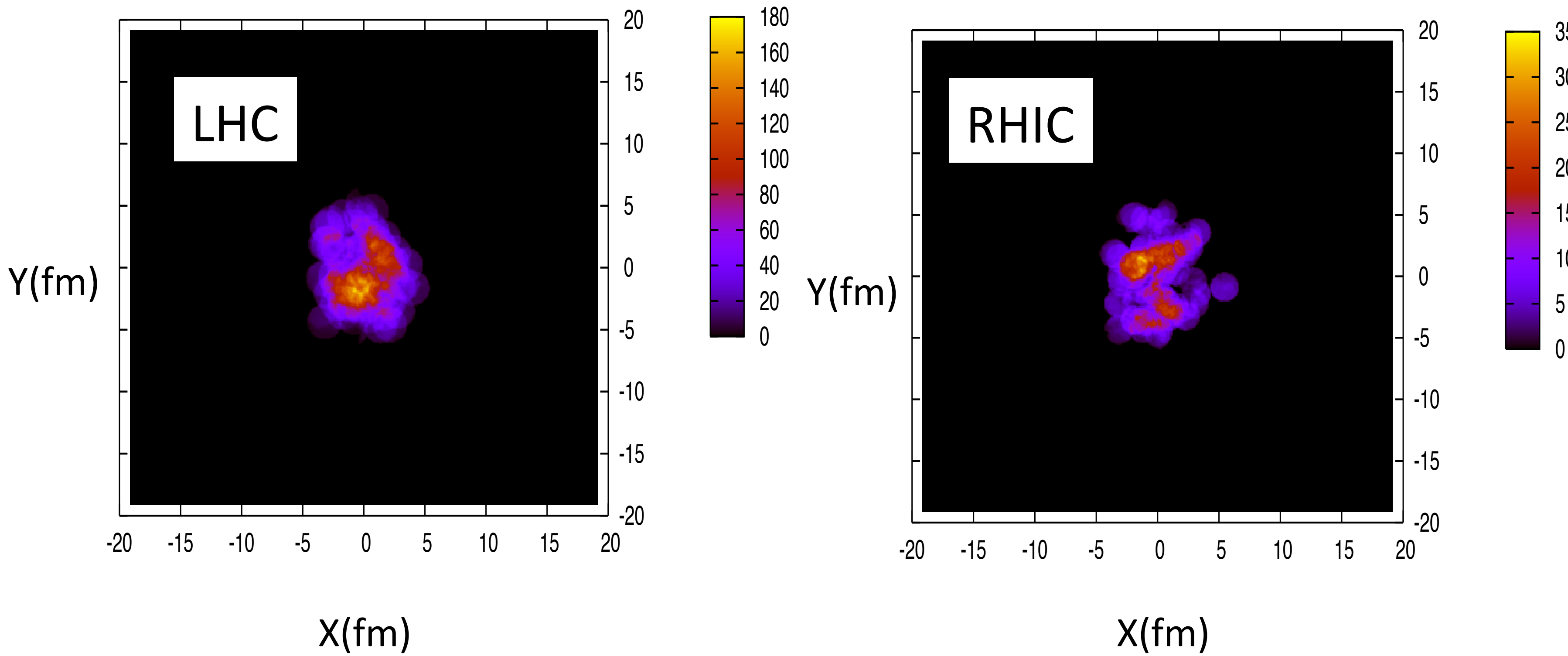
Ohio group

Simulation setups:
- Free gluon EoS
- Hydro in 2D boost invariant simulation
- UrQMD with |y|<0.5

Initial Pressure Distribution

- MC-KLN (centrality 15-20%)

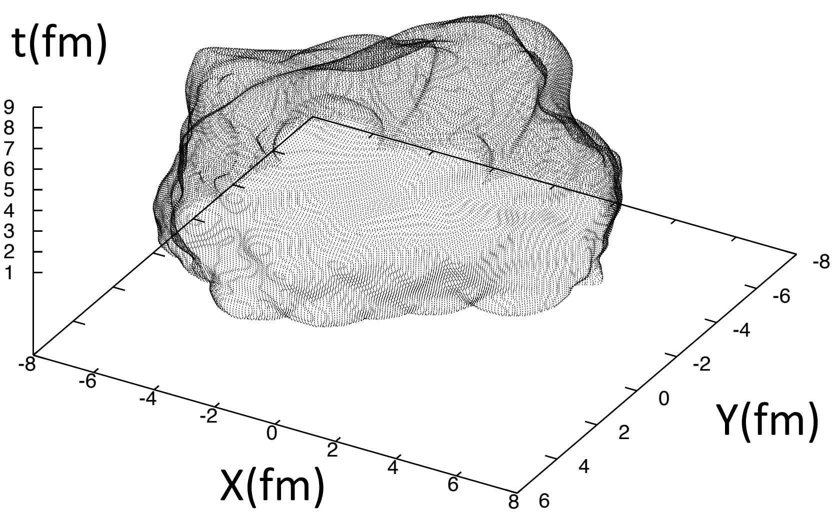
Pressure (fm^{-4})



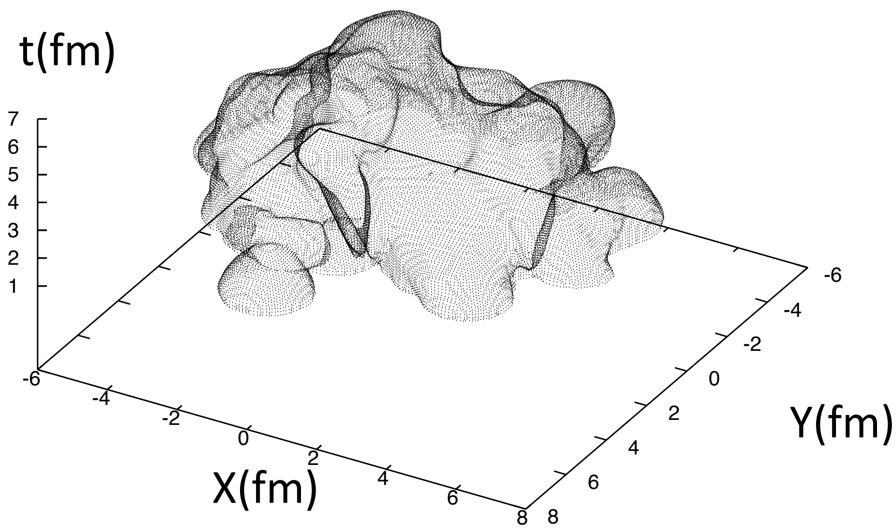
freezeout hypersurface

- Output from Cornelius

T_{SW}=155MeV



LHC



RHIC

Time Evolution of ε_n and v_n

- Eccentricity & Flow anisotropy

$$\varepsilon_n e^{in\Phi_n} = \left\langle z^n \right\rangle \Big/ \left\langle |z|^n \right\rangle, \quad z = x + iy$$

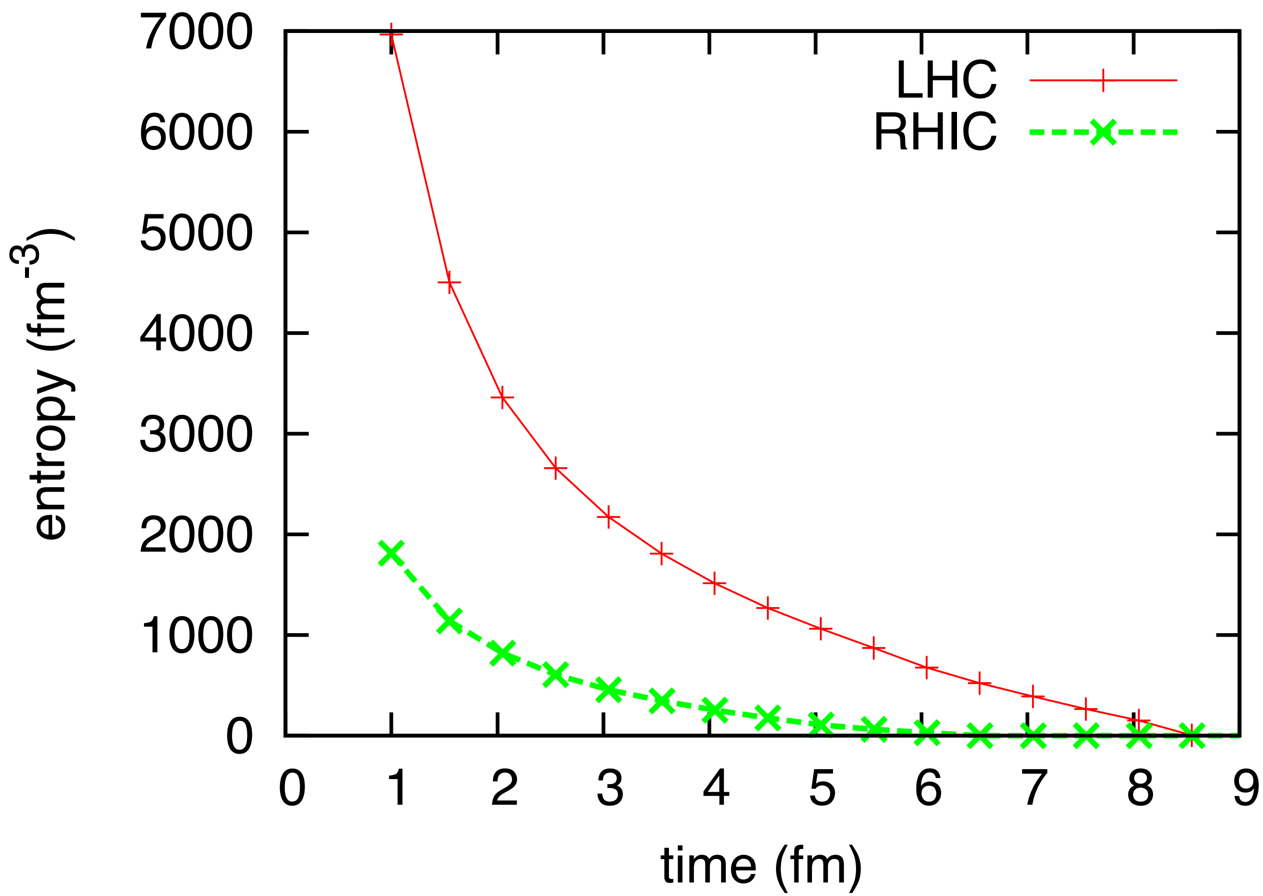
Shift the origin so that ε_1=0

$$v_n e^{in\psi_n} = \left\langle v^n \right\rangle, \quad v = v_x + iv_y, \quad (0 \le \varepsilon_n, v_n \le 1)$$

$$\left\langle \cdots \right\rangle = \int_{T>T_f=155\mathrm{MeV}} d^2x \, \cdots S^0(x,y) \Big/ \int_{T>T_f=155\mathrm{MeV}} d^2x \, S^0(x,y)$$

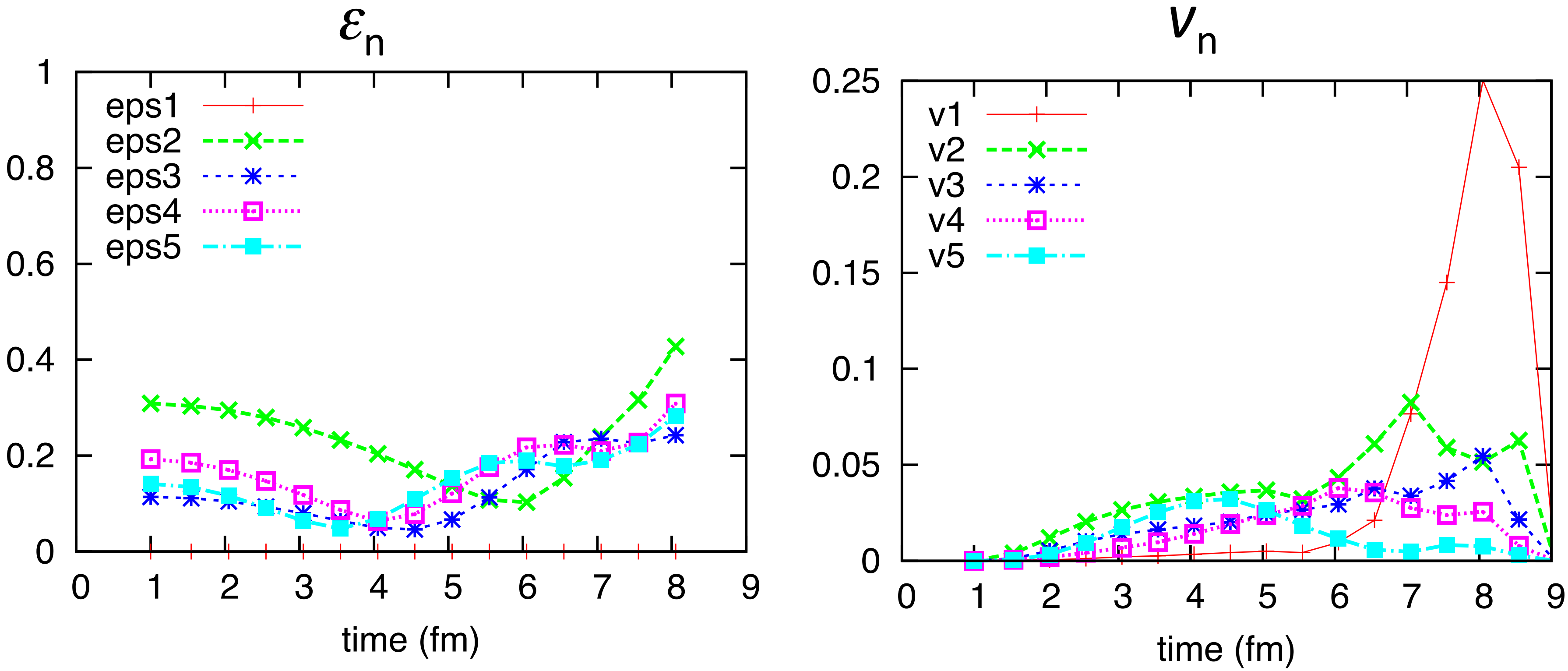
Time Evolution of Entropy

- Entropy of hydro ($T>T_{SW}$=155MeV)



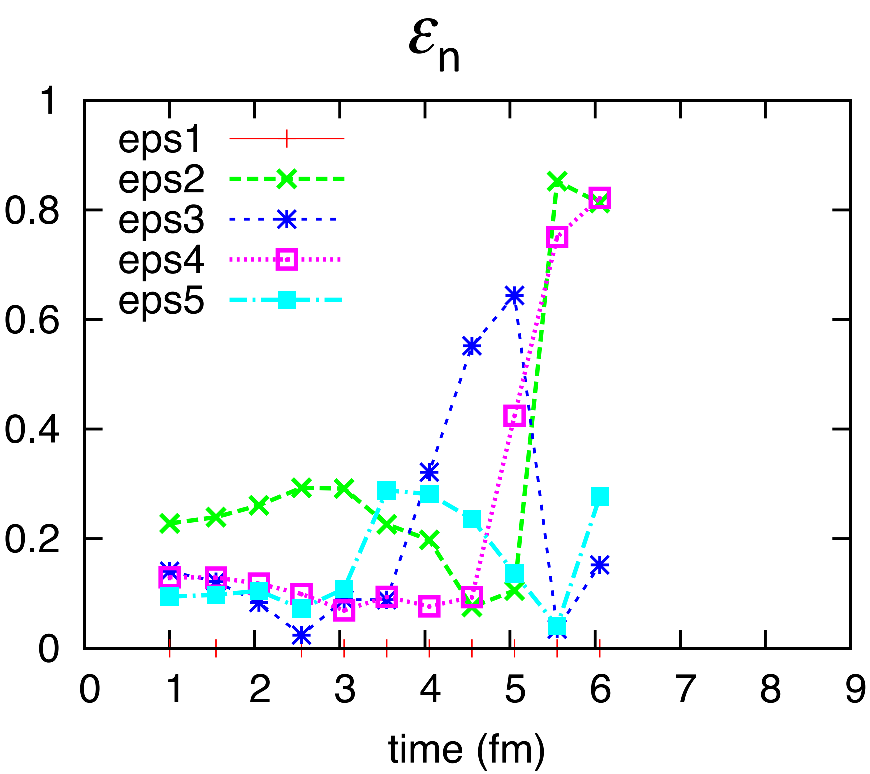
Eccentricities vs higher harmonics

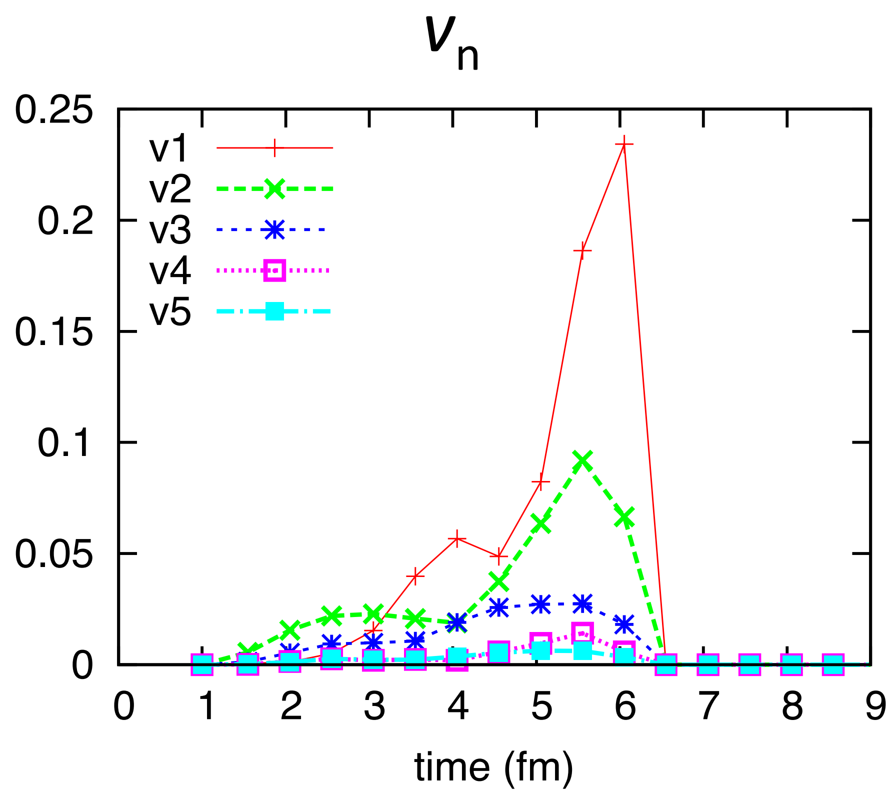
- LHC (one event)



Eccentricities vs higher harmonics

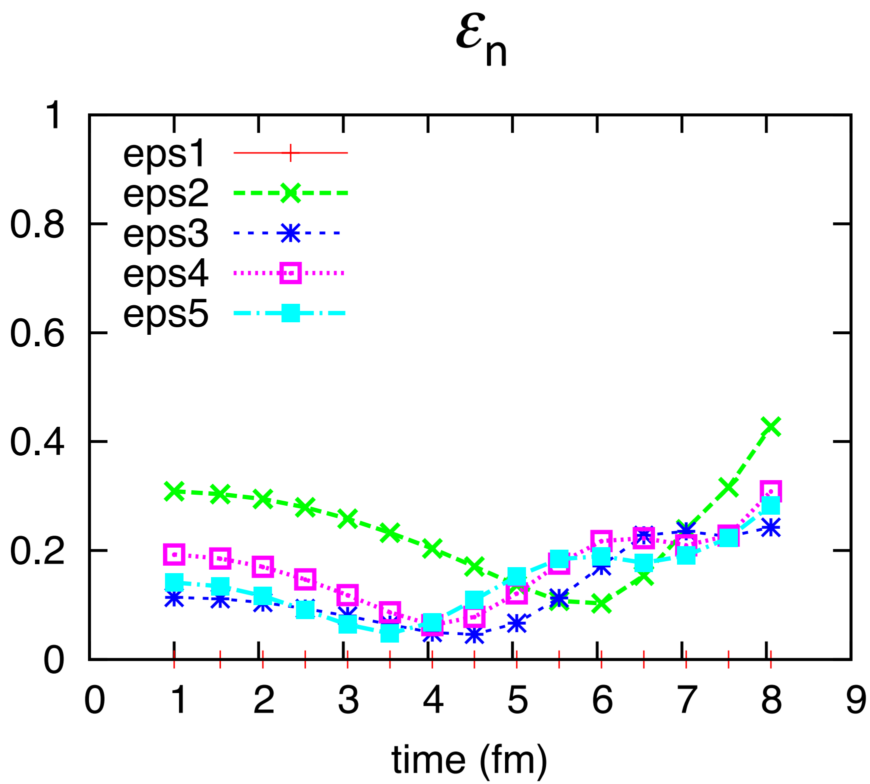
- RHIC (one event)

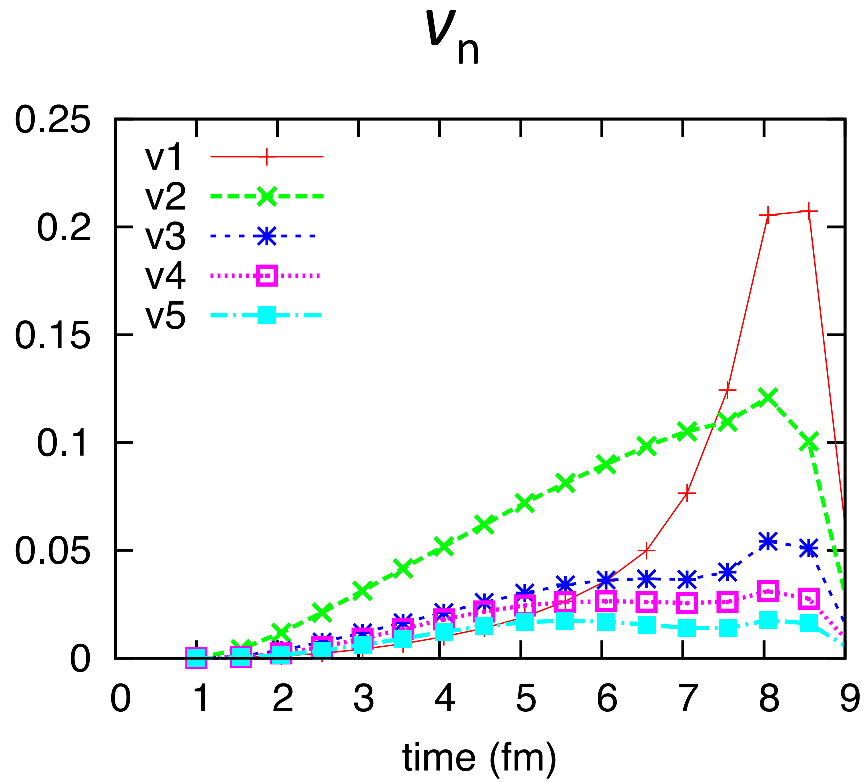




Eccentricities vs higher harmonics

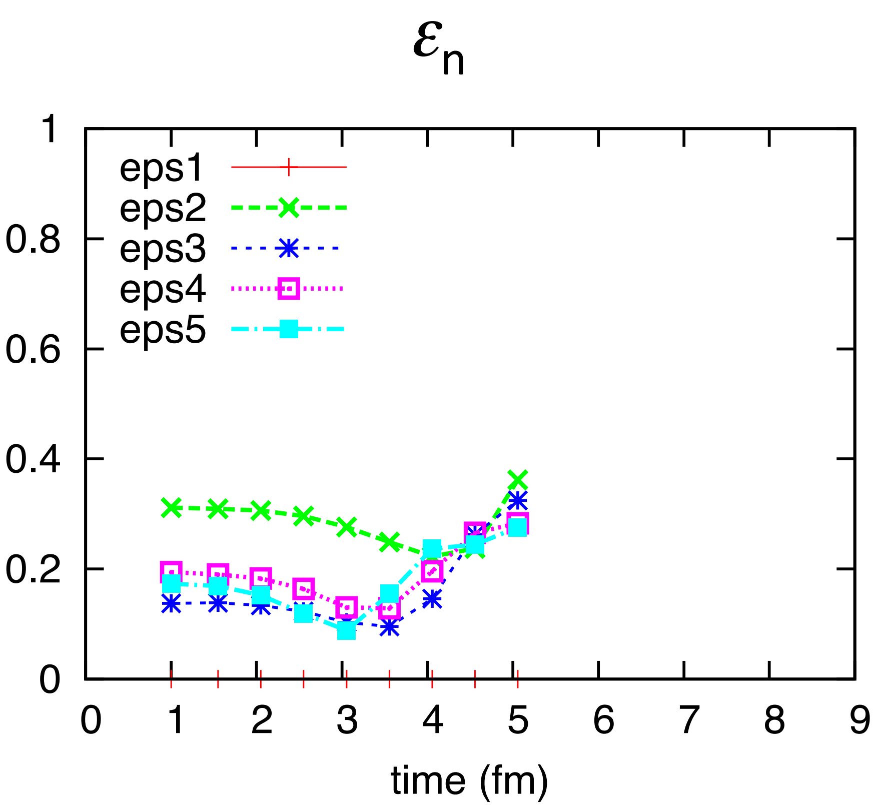
- LHC (200 events)

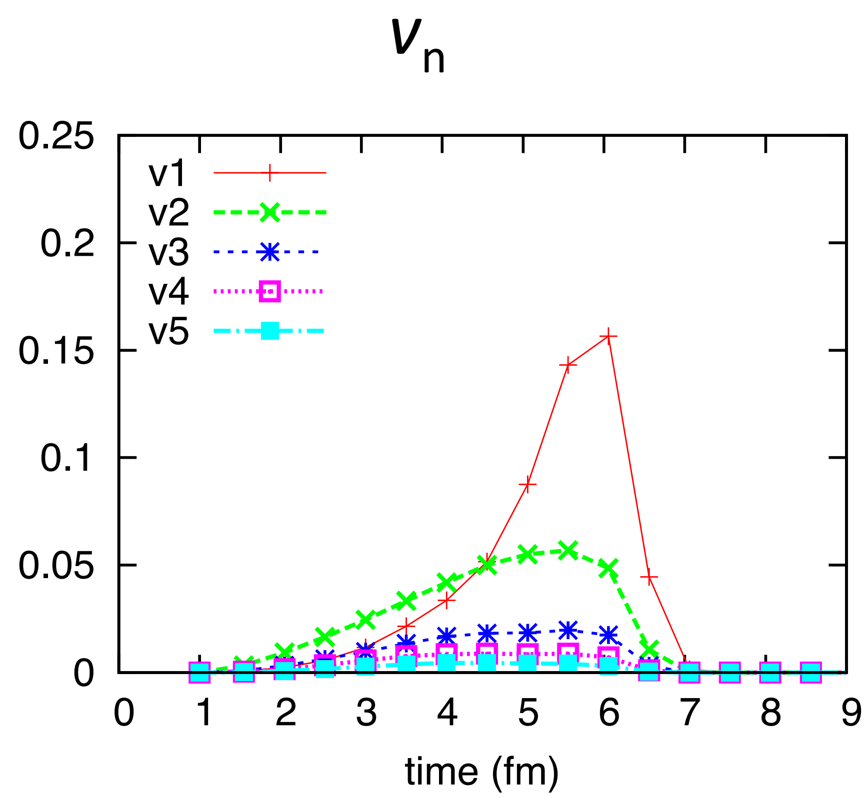




Eccentricities vs higher harmonics

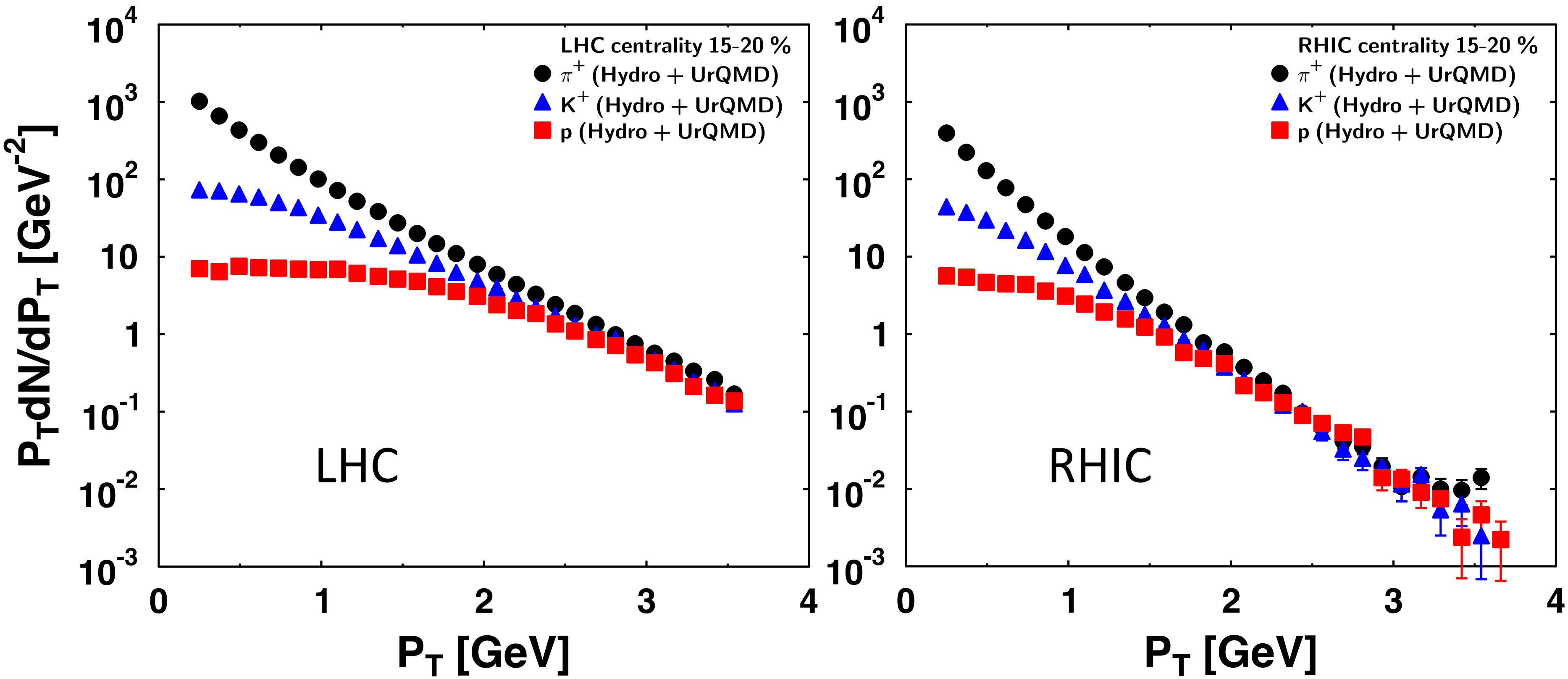
- RHIC (200 events)





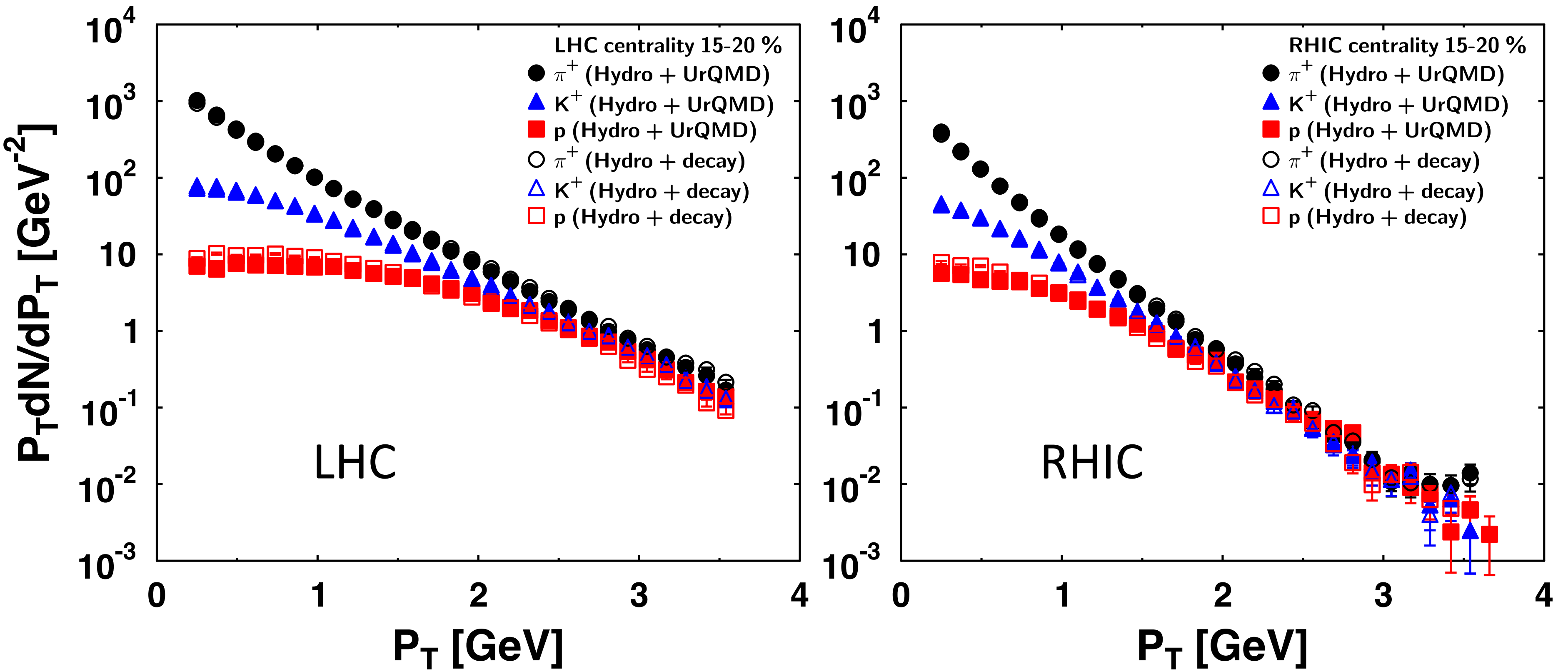
Hydro + UrQMD

- Transverse momentum spectrum



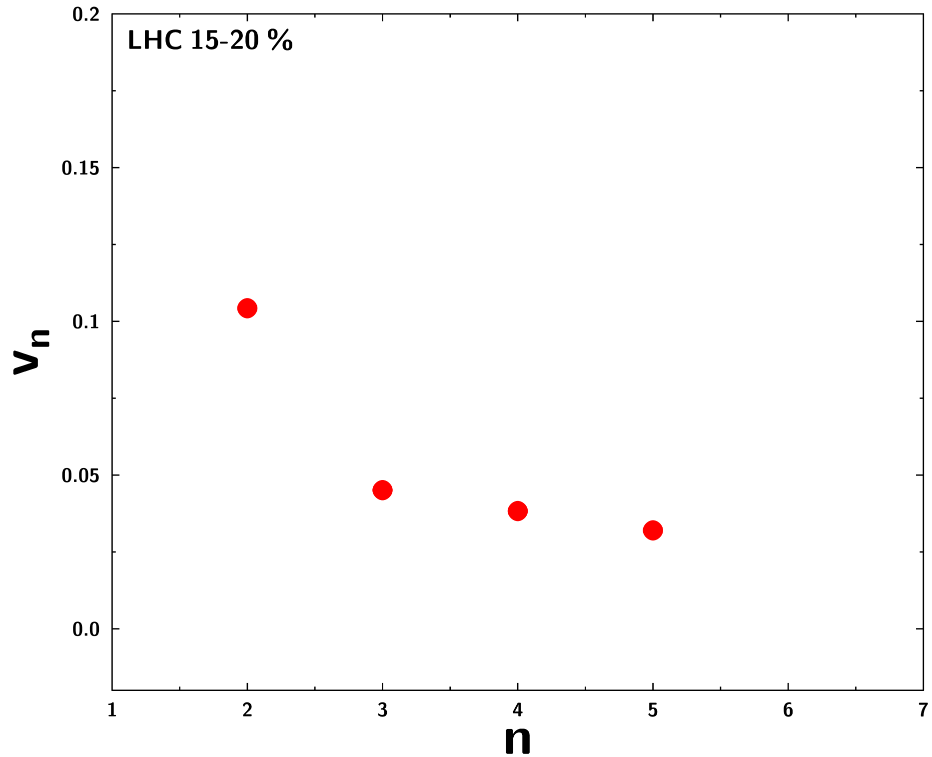
Effect of Hadronic Interaction

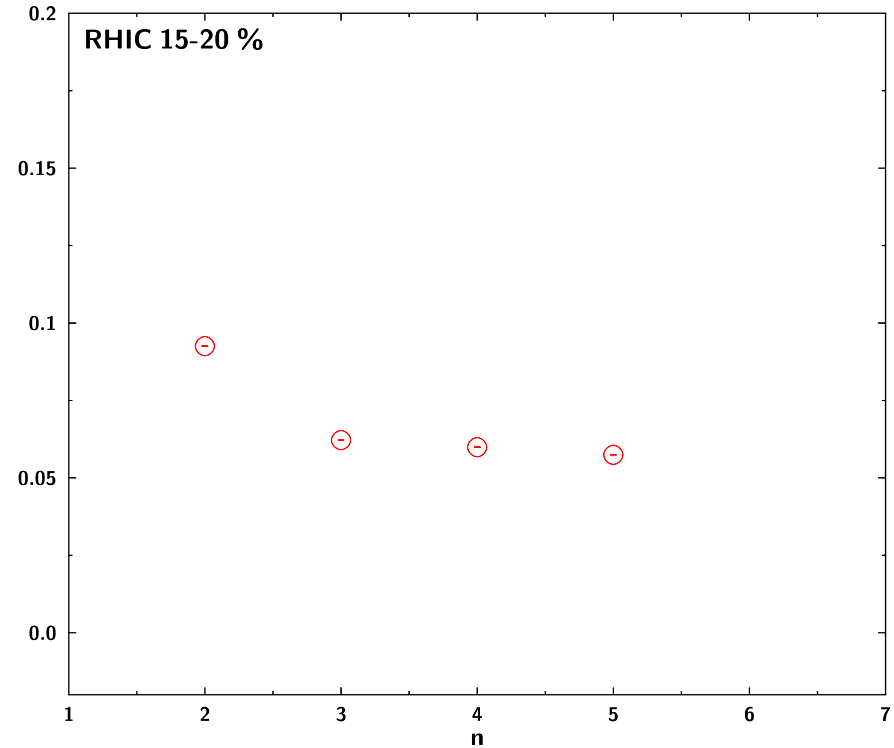
- Transverse momentum distribution



Higher harmonics from Hydro + UrQMD

- Effect of hadronic interaction





Summary

- We develop a state-of-the-art numerical scheme
  - Viscosity effect
  - Shock wave capturing scheme: Godunov method

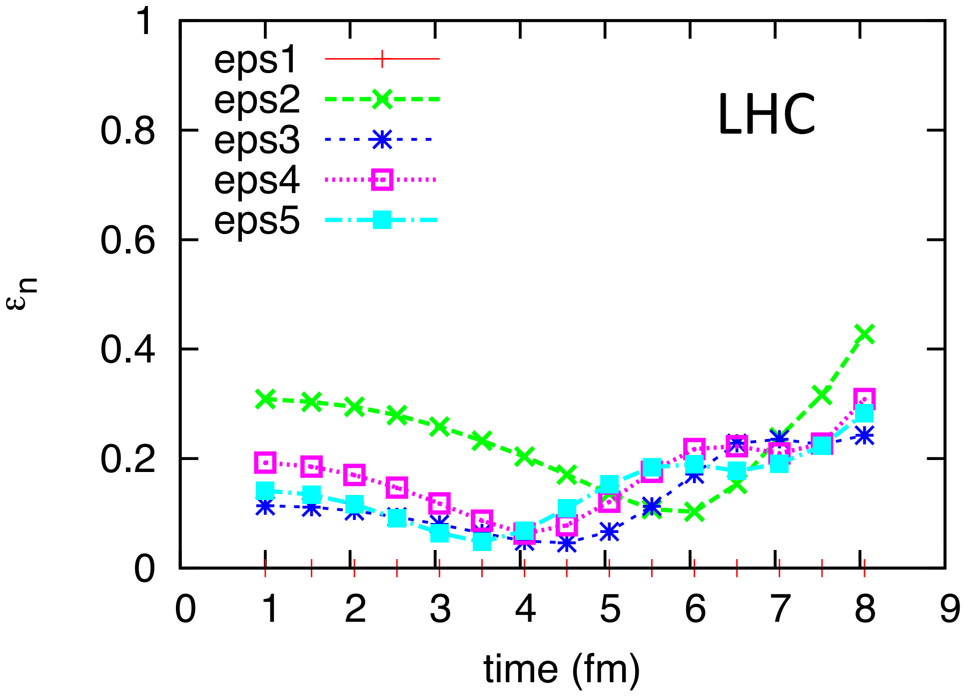
Our algorithm

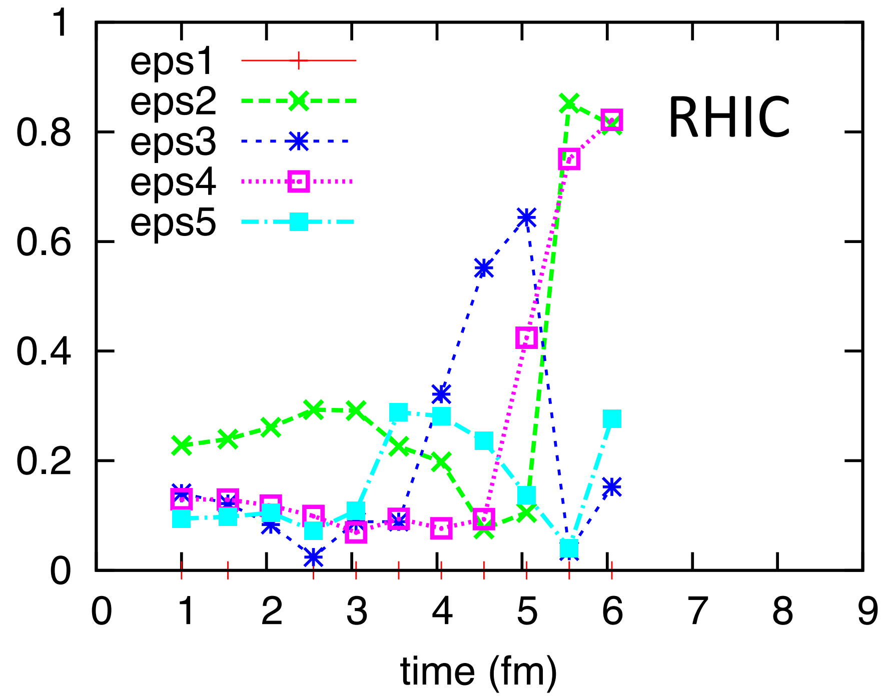
  - Less artificial diffusion: crucial for viscosity analyses
  - Stable for strong shock wave

- Construction of a hybrid model
  - Fluctuating initial conditions + Hydrodynamic evolution + UrQMD
- Higher Harmonics
  - Time evolution, hadron interaction

Time Evolution of ε_n

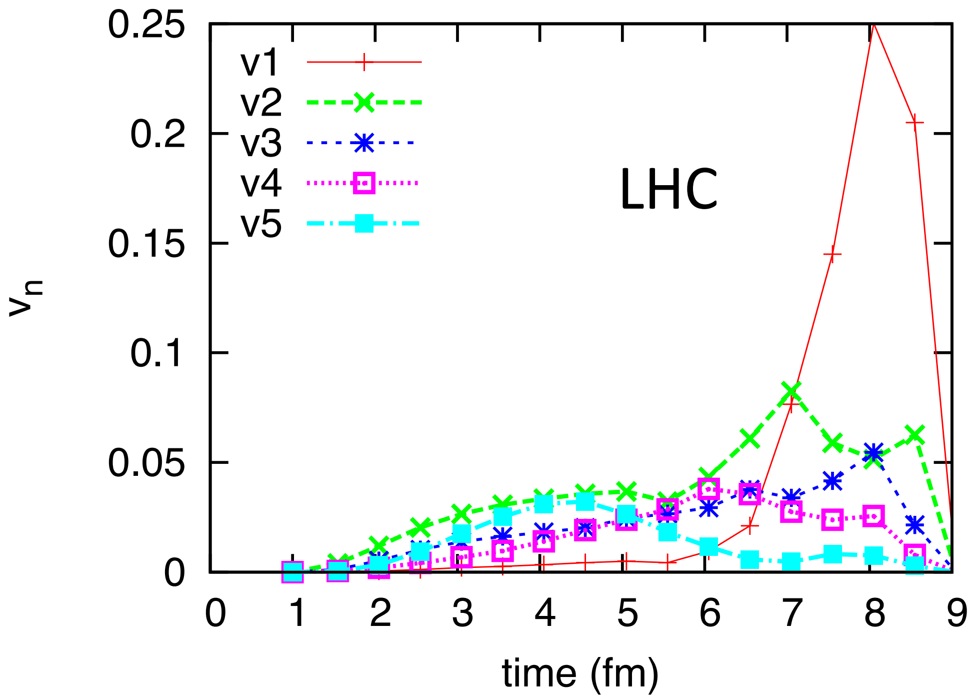
- Eccentricities

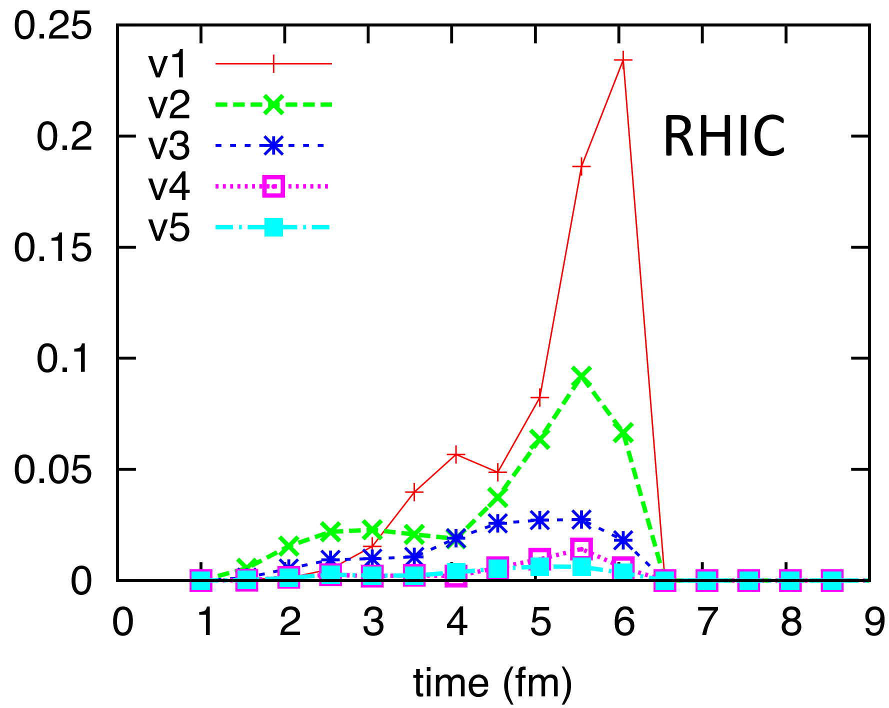




Time Evolution of v_n

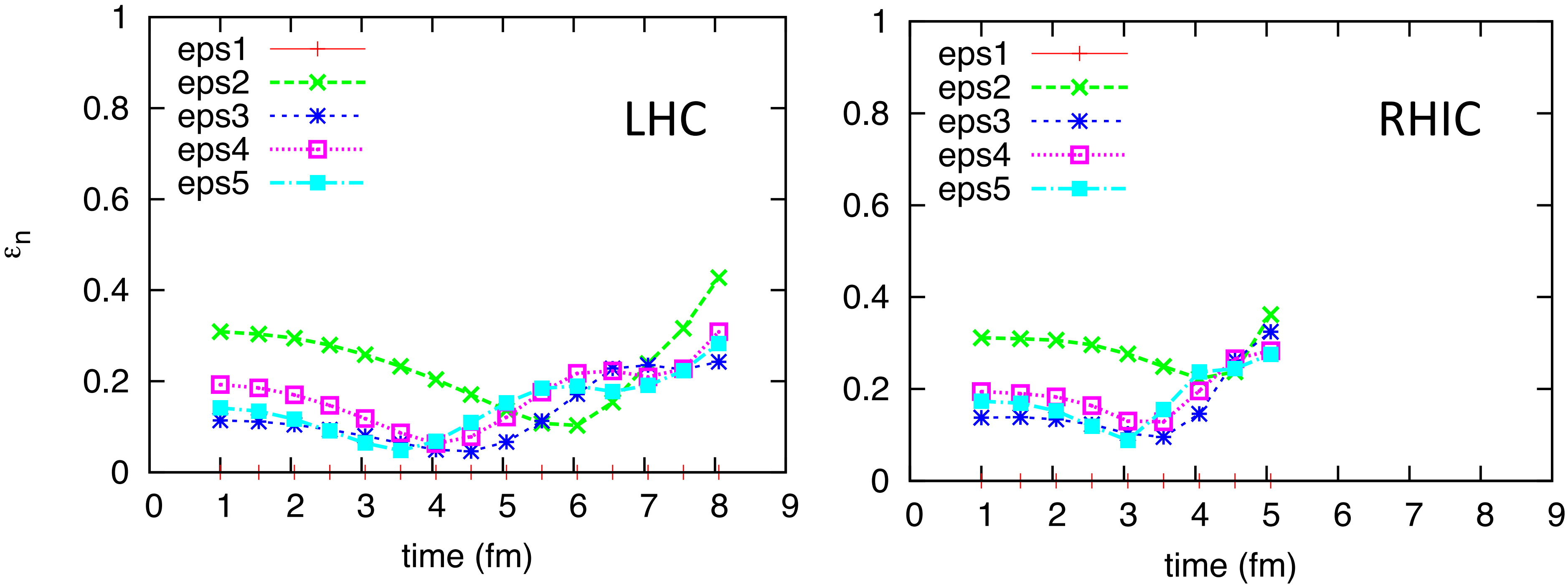
- Flow anisotropies





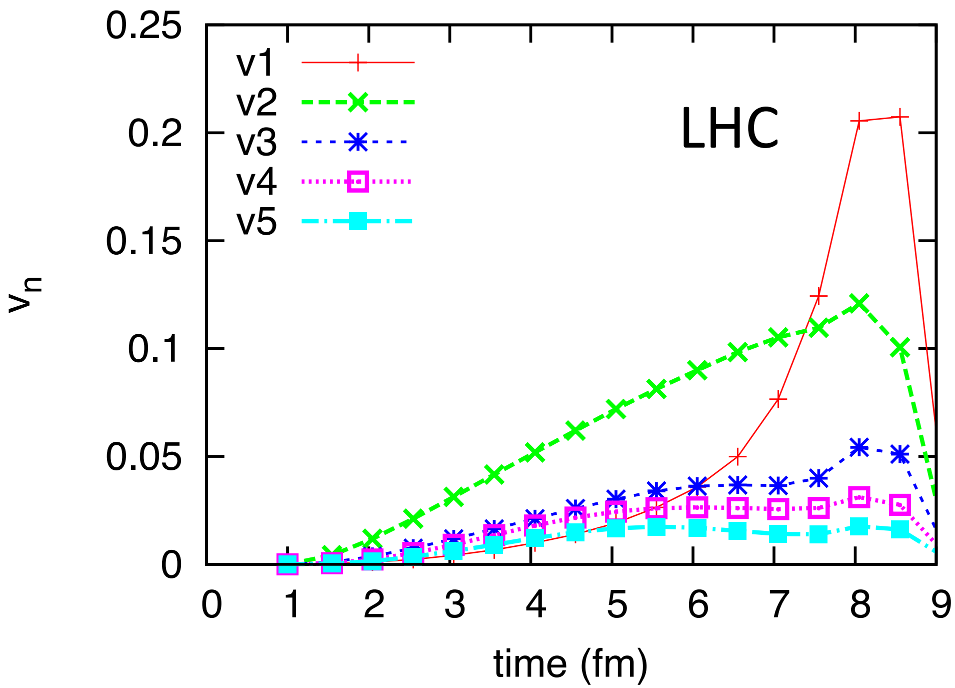
Time Evolution of ε_n

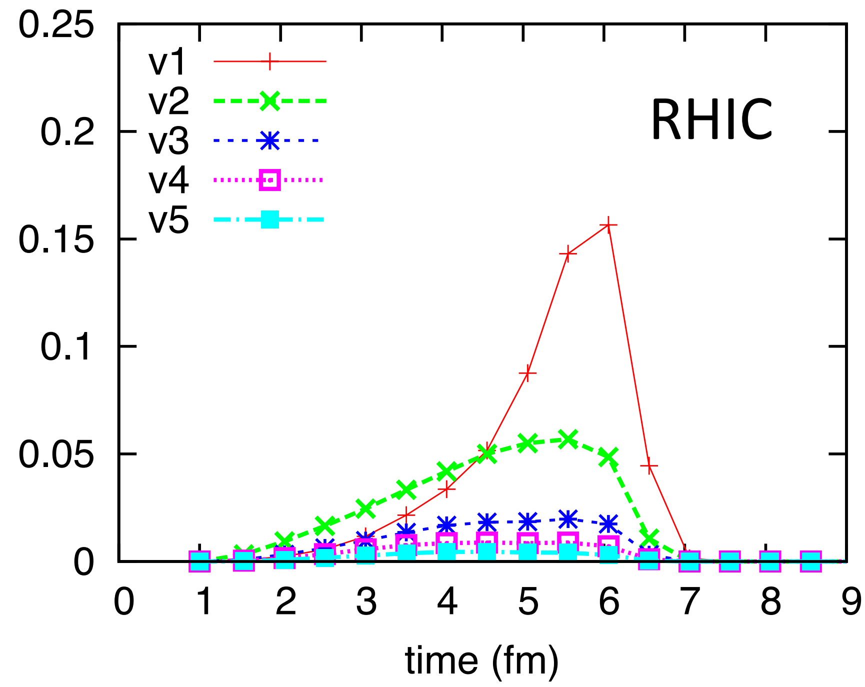
- Eccentricities from 200 events



Time Evolution of v_n

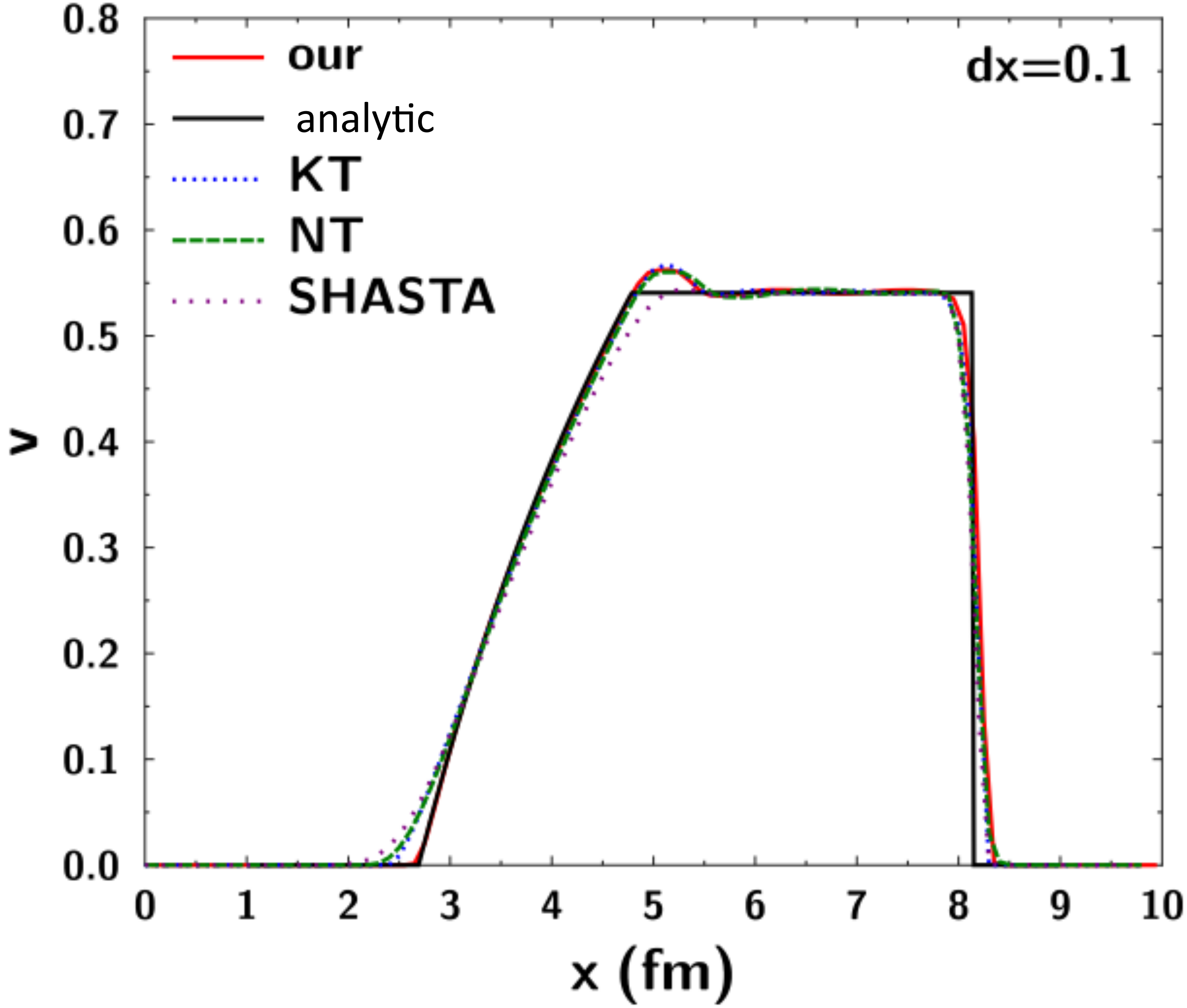
- Flow anisotropies from 200 events





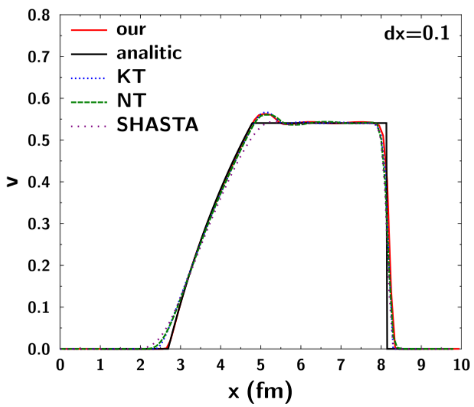
Velocity

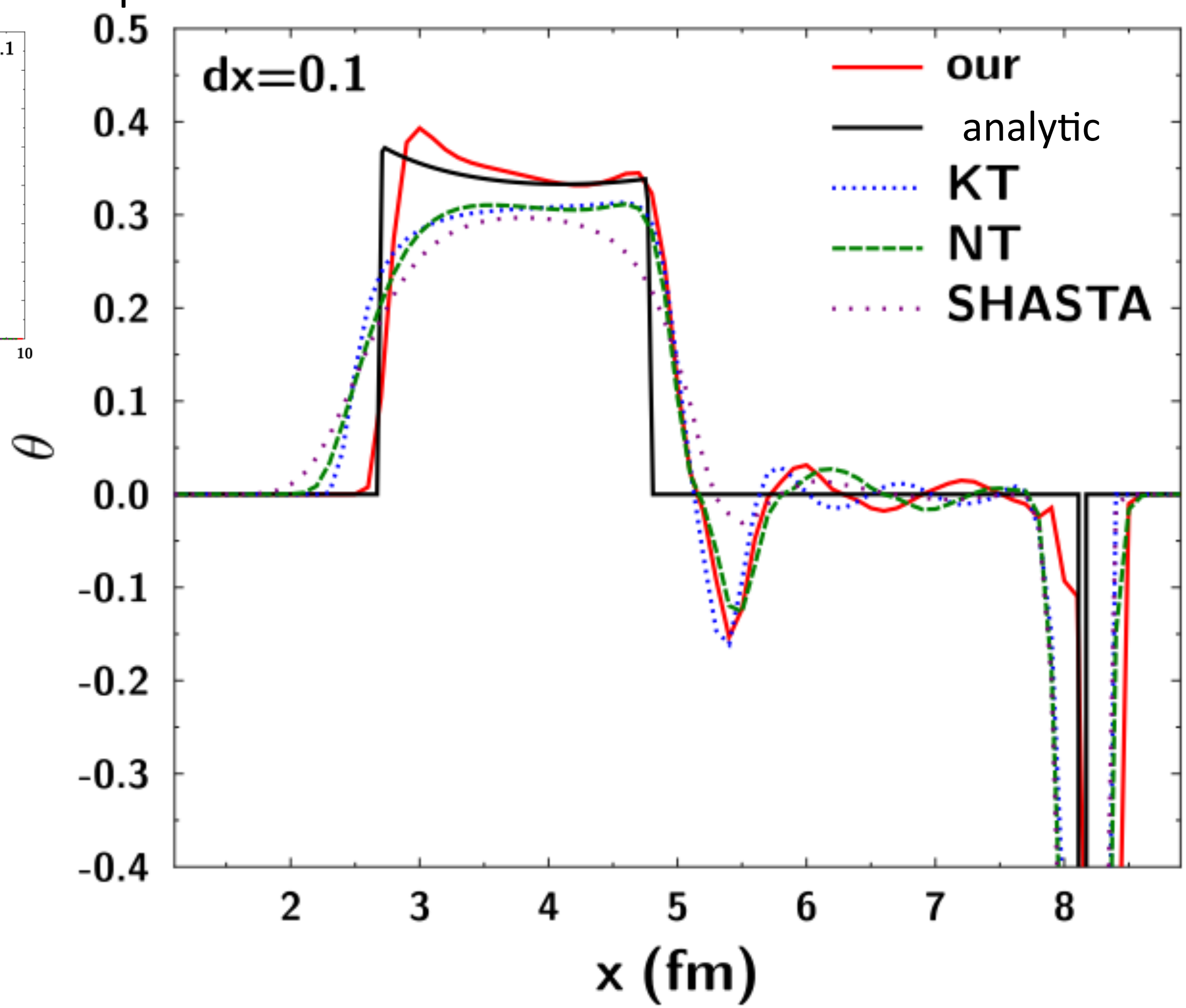
t=4.0 fm dt=0.04, 100 steps



θ

t=4.0 fm dt=0.04, 100 steps





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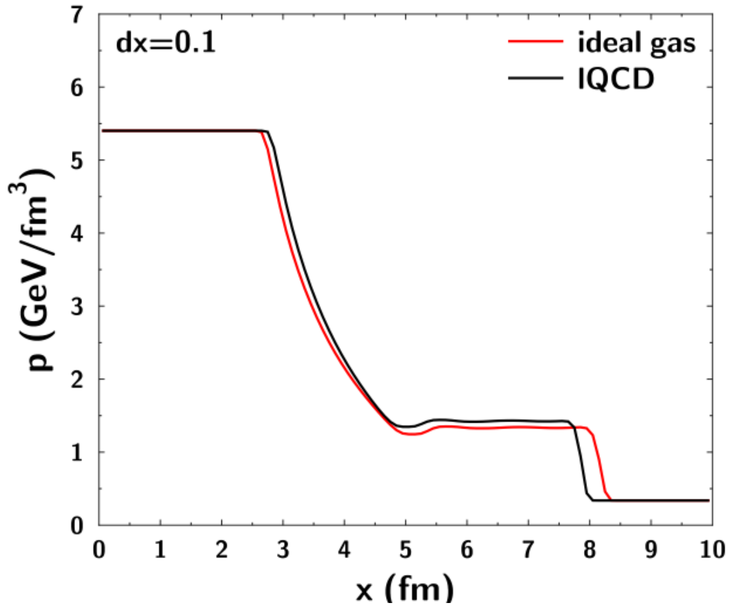
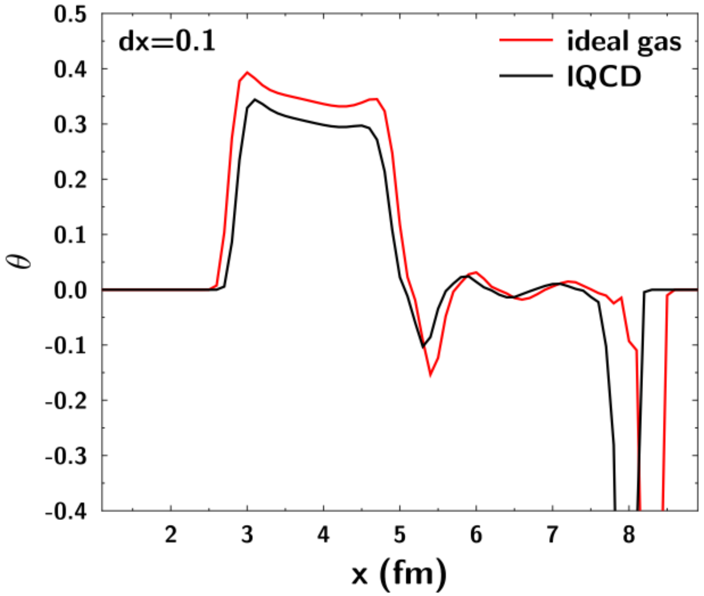
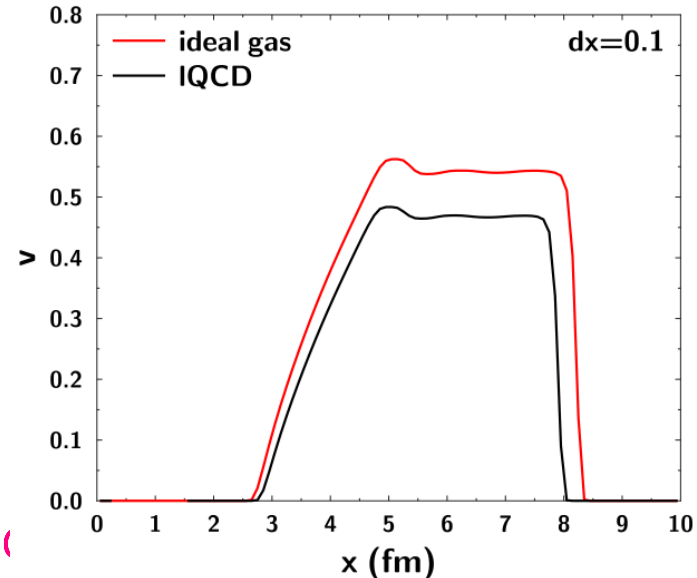
EoS Dependence

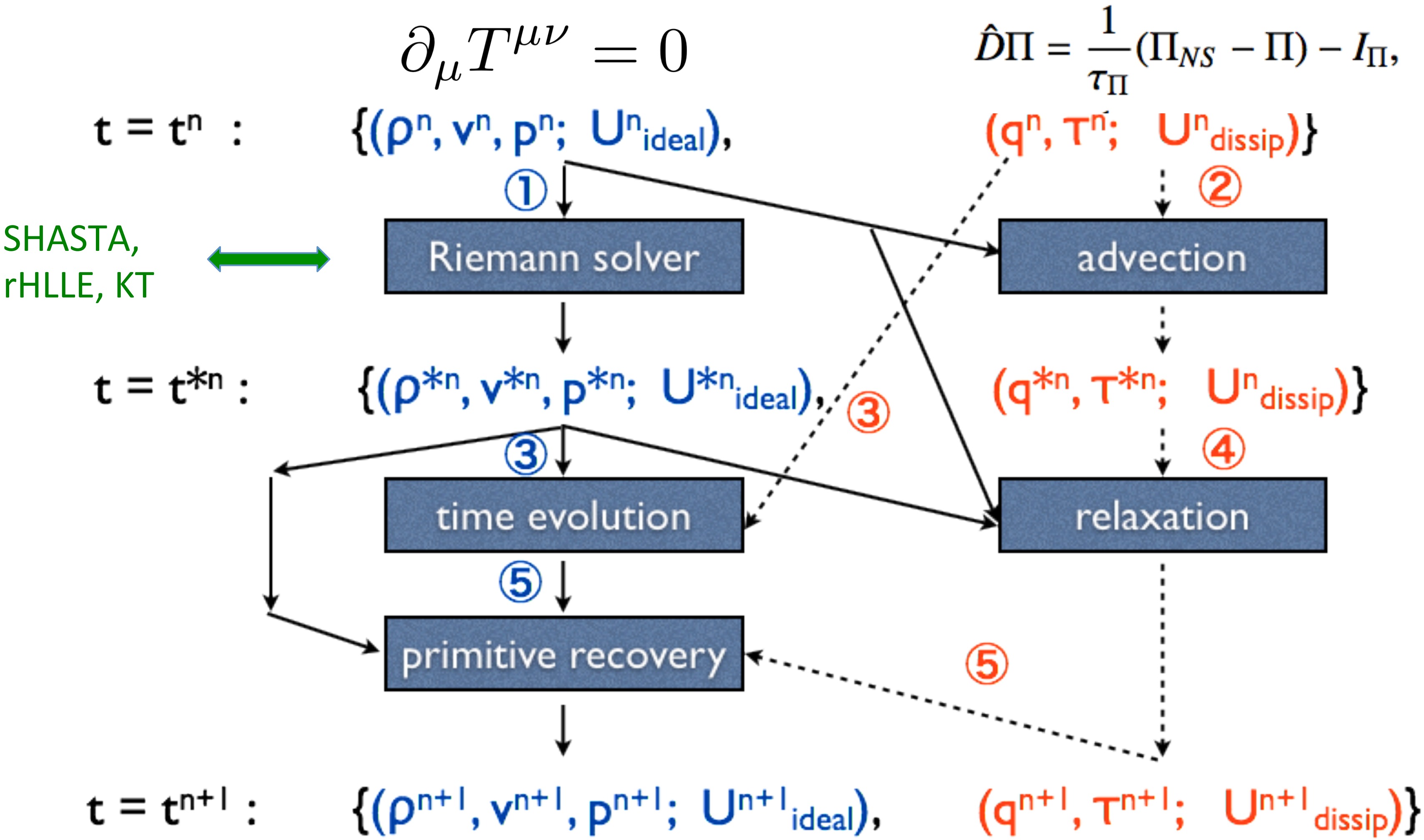






Numerical Method

Takamoto and Inutsuka, arXiv:



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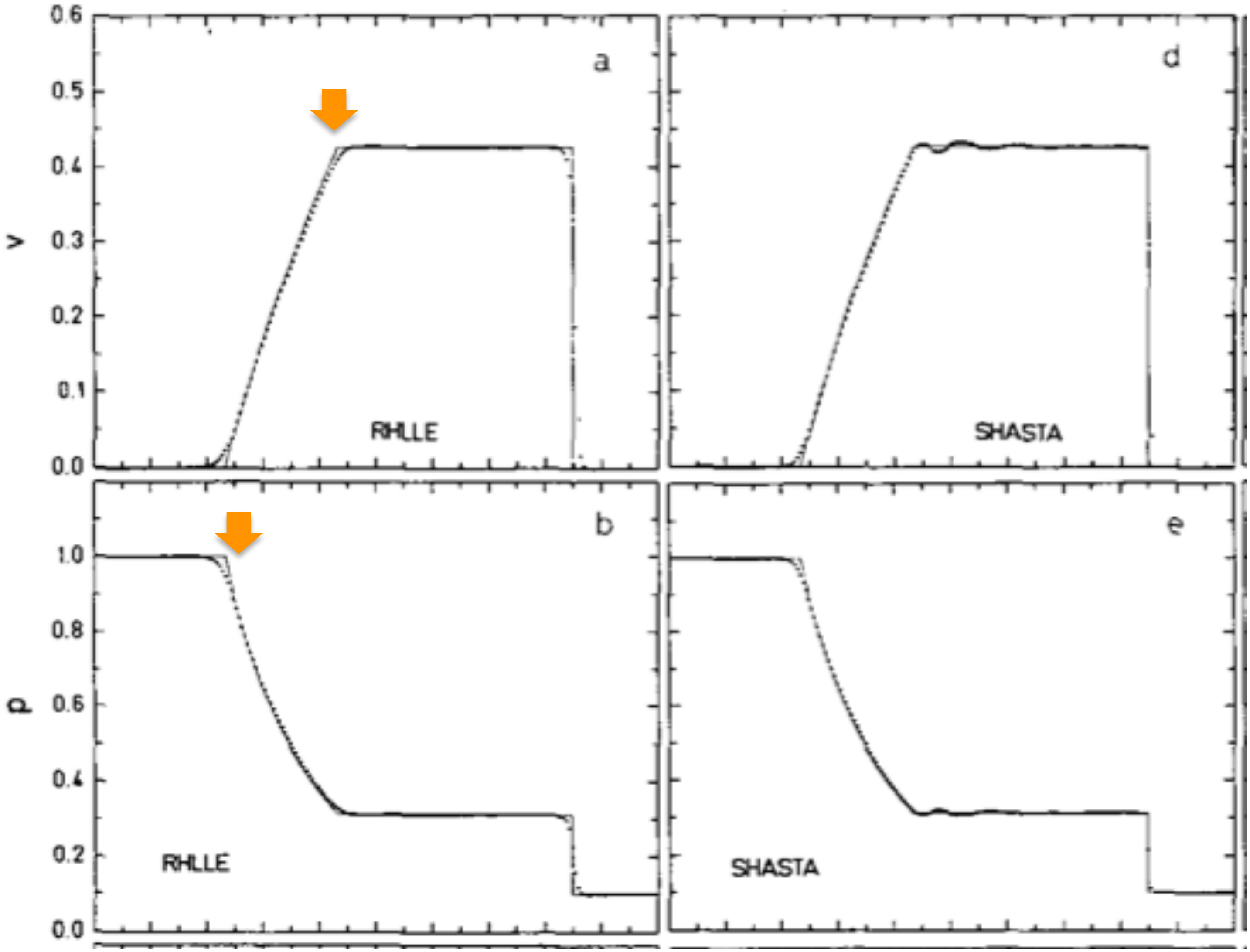
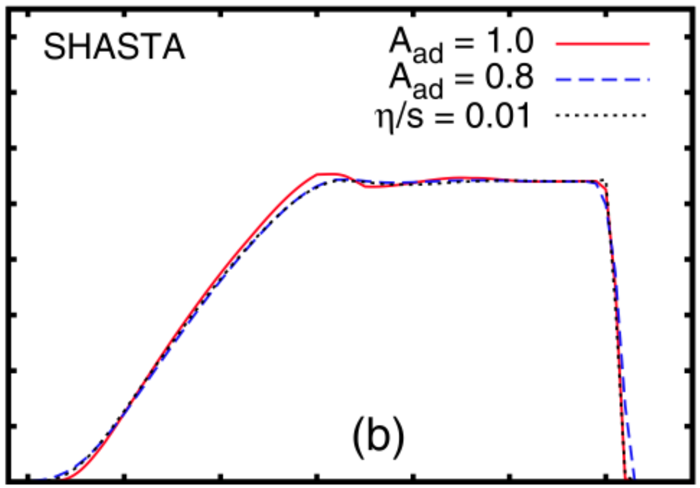
rHLLE vs SHASTA

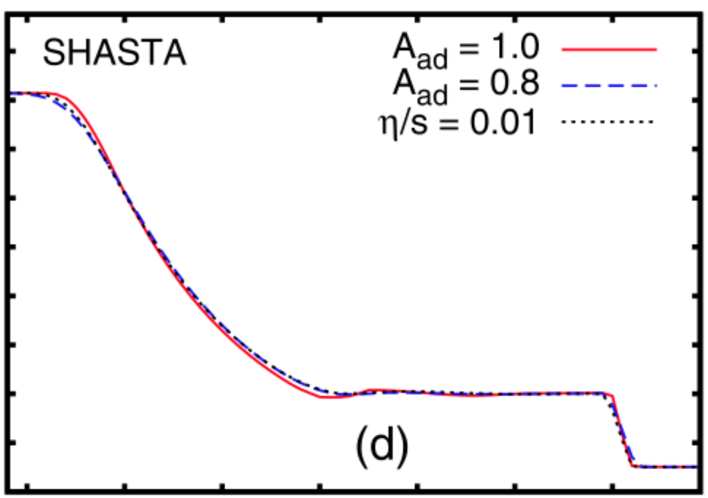
Schneider et al. J. Comp.105(1993)92

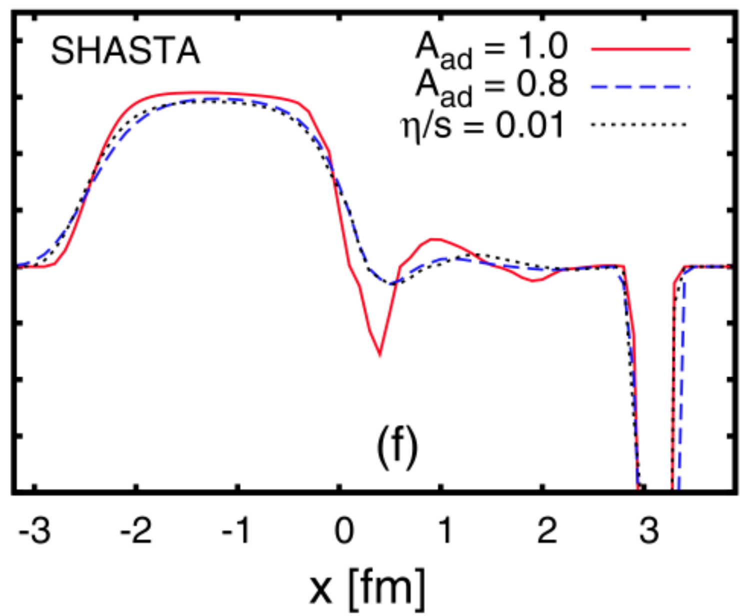


Artificial and Physical Viscosities

Molnar, Niemi, Rischke, Eur.Phys.J.C65,615(2010)







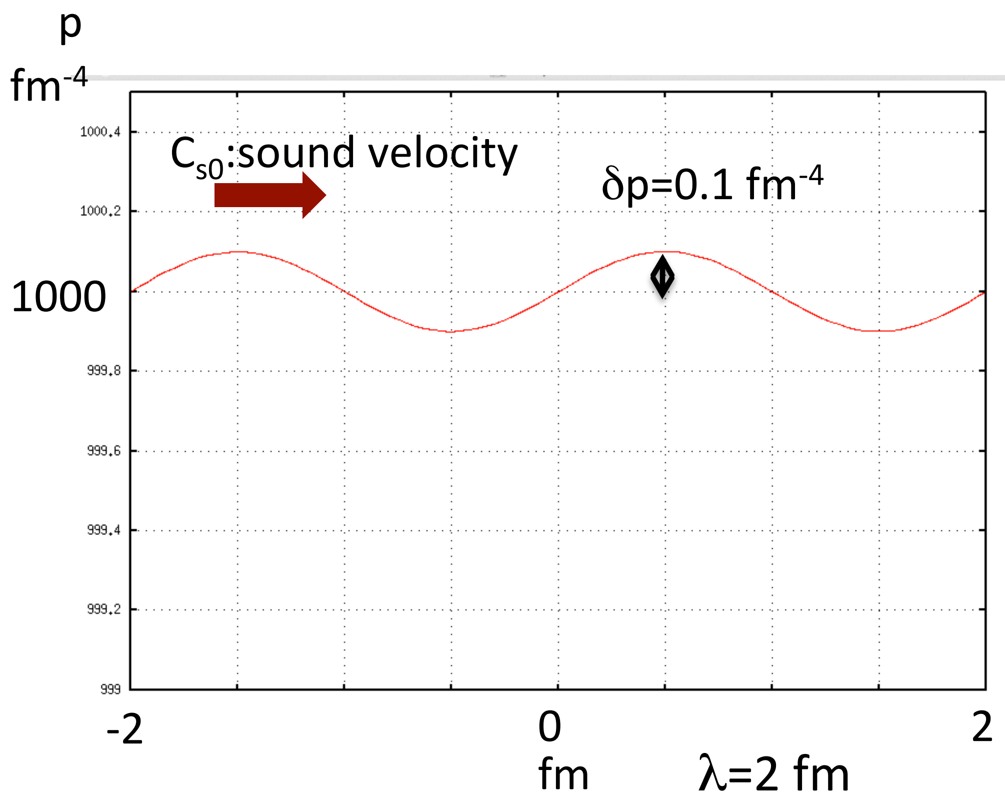
Antidiffusion terms : artificial viscosity   stability

$$U_i^{n+1} = \tilde{U}_i - \tilde{A}_i + \tilde{A}_{i-1}$$

$$A_i = A\mathrm{ad}\tilde{\Delta}_i/8$$

Numerical Dissipation

- Sound wave propagation



periodic boundary condition

If numerical dissipation does not exist

After one cycle: $t=\lambda/c_{s0}$

$V_s(x,t)=V_{init}(x-c_{s0}t)$

With finite numerical dissipation

$V_s(x,t)\neq V_{init}(x-c_{s0}t)$

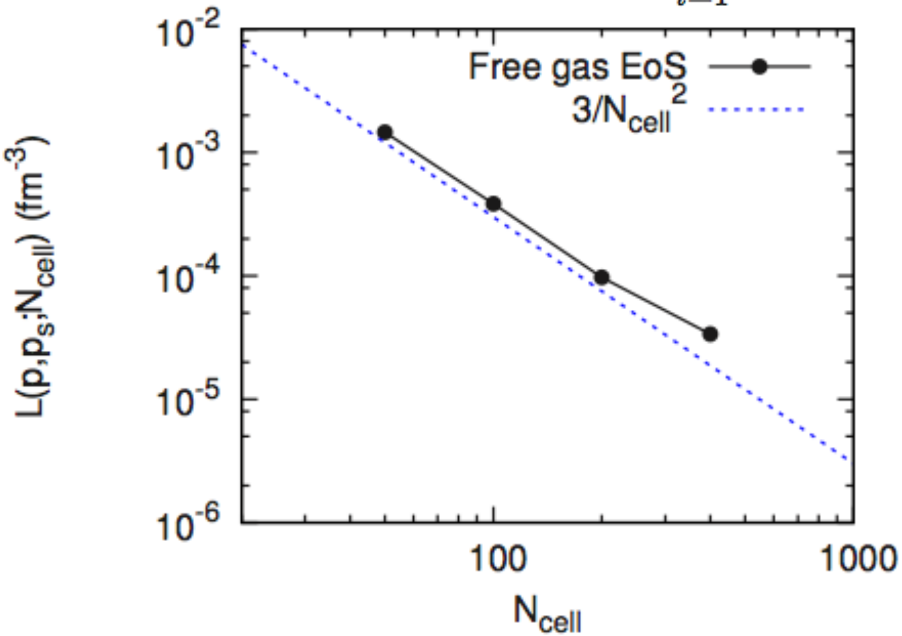
L1 norm

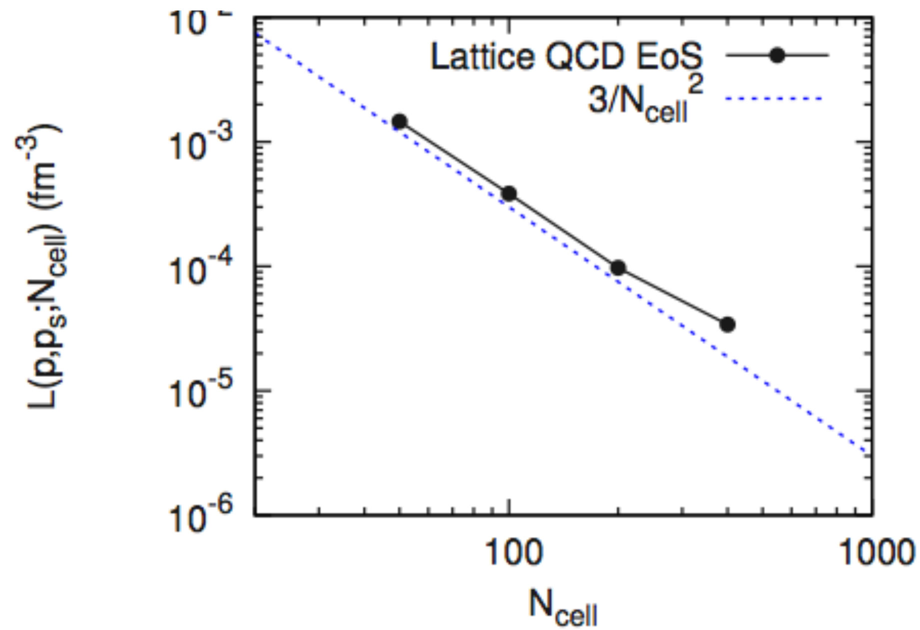
$$L(p,p_s;N_{\rm cell}) = \sum_{i=1}^{N_{\rm cell}} \mid p(x_i,\lambda/c_{s0}) - p_s(x_i,\lambda/c_{s0}) \mid \frac{\lambda}{N_{\rm cell}}$$

after one cycle

Convergence Speed

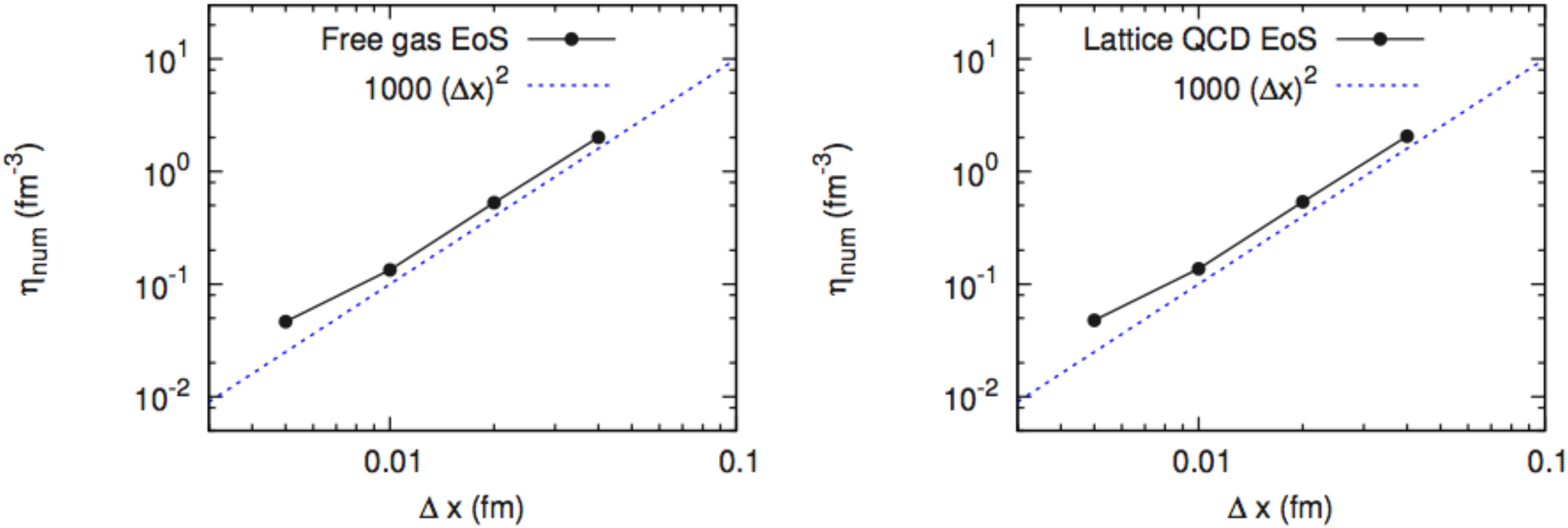
$$L(p,p_s;N_{\rm cell}) = \sum_{i=1}^{N_{\rm cell}} \mid p(x_i,\lambda/c_{s0}) - p_s(x_i,\lambda/c_{s0}) \mid \frac{\lambda}{N_{\rm cell}}$$





$L(p,p_s;N_{\rm cell}) \propto 1/N_{\rm cell}^2$ ➡ Space and time discretization
Second order accuracy

Numerical Dissipation



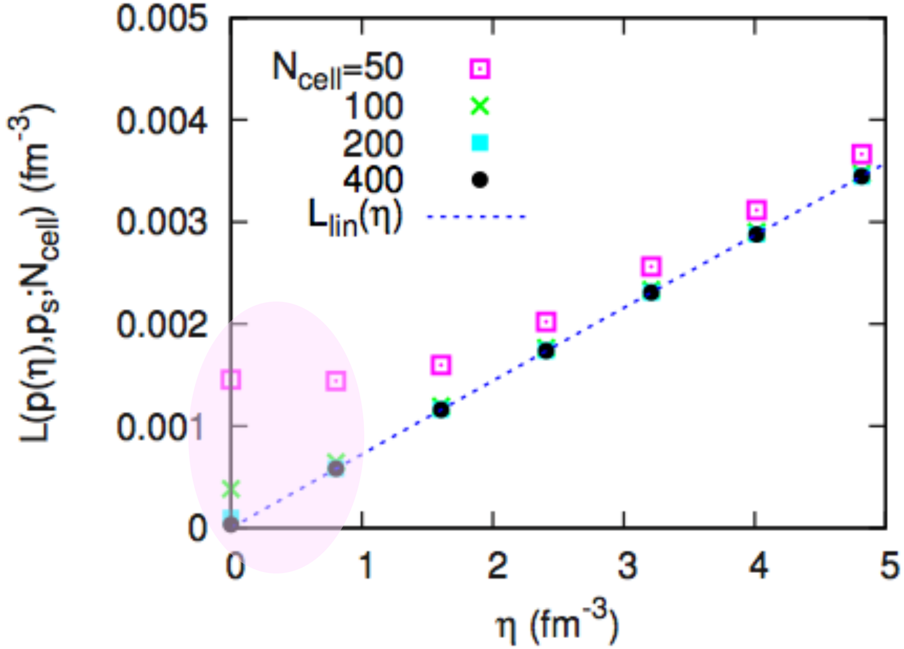
- numerical dissipation: $$\eta_{\rm num} = -\frac{3\lambda}{8\pi^2} c_{s0}(e_0+p_0) \ln\left[1-\frac{\pi}{2\lambda\delta p}L(p,p_s;N_{\rm cell})\right]$$

- from fit of calculated data

$$\eta_{\rm num} \approx 1000\,(\Delta x)^2 \quad \longrightarrow \quad \eta_{\rm num} \approx 1 \cdot \frac{c_{s0}(e_0+p_0)}{\lambda}(\Delta x)^2$$

$$L(p,p_s;N_{\rm cell}) \propto \lambda\delta p / N_{\rm cell}^2 = (\delta p/\lambda)\cdot(\Delta x)^2$$

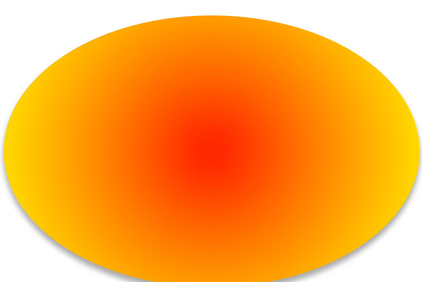
η_{num} vs Grid Size



Numerical dissipation:
Deviation from linear analyses (L_{lin})

$$\eta_{\rm num} \approx 1 \cdot \frac{c_{s0}(e_0+p_0)}{\lambda}(\Delta x)^2$$

Ex. Heavy Ion Collisions



λ ~ 10 fm

0.1<η/s<1

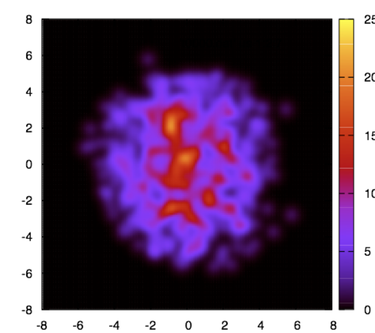
T=500 MeV

Δx << 0.8 – 2.6 fm

Fluctuating initial condition

λ ~ 1 fm

Δx << 0.25 – 0.82 fm



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