An effective model of QCD thermodynamics[1]

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RANP2013 - Takeshi Kodama's Fest
Rio de Janeiro 2013
September 23 - September 28, 2013

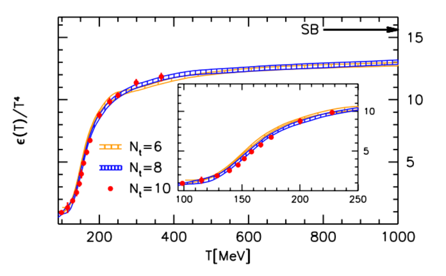
[1]based on paper by L.T., D. Blaschke, D. Prorok, and J. Berdermann - Acta Phys. Polon. Supp. **5**, 485 (2012) *and in further progress*

To achieve

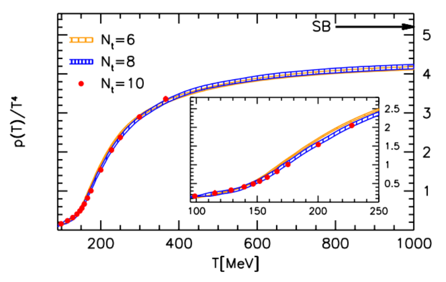
A compound and consistent effective model reproducing:

- The equation of state of hadronic matter as obtained in lattice QCD simulations.
- Basic physical characteristics of processes encountered in the dense hadronic matter:
  - Modified in medium properties of hadrons as different from those in the vacuum.
  - Modification of the notion of the mas shell.
  - Dissolving of hadrons into quarks and gluons phase,
  - Physical processes present in the full QCD treatment.

Theoretical laboratory of QCD



The energy density normalized by T^4 as a function of the temperature on N_t =6,8 and 10 lattices.



The pressure normalized by T^4 as a function of the temperature on N_t =6,8 and 10 lattices.

S. Borsanyi *et al.* *"The QCD equation of state with dynamical quarks,"* JHEP **1011**, 077 (2010)

Hagedorn resonance gas: hadrons with finite widths

Modified in medium properties

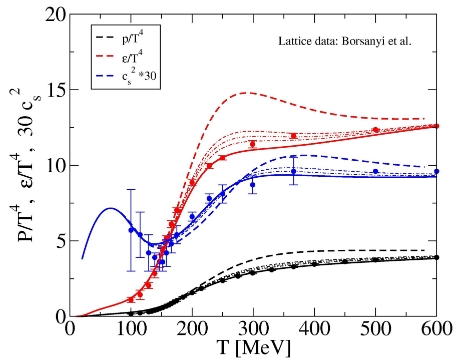
$$A(M,m) \sim \frac{\Gamma \cdot m}{(M^2 - m^2)^2 + \Gamma^2 \cdot m^2} \ ,$$

$$\Gamma(T) = C_\Gamma \ \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)$$

The energy density

$$\begin{aligned} \varepsilon(T,\mu_B,\mu_S) \ &= \sum_{i:\ m_i < m_0} g_i \ \varepsilon_i(T,\mu_i;m_i) \\ &+ \sum_{i:\ m_i \geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2) \ A(M,m_i) \ \varepsilon_i(T,\mu_i;M), \end{aligned}$$

Hagedorn resonance gas: hadrons with finite widths



$$P(T) = T \int_0^T dT' \, \frac{\varepsilon(T')}{T'^2} \, .$$

N_m in the range from $N_m = 2.5$ (dashed line) to $N_m = 3.0$ (solid line).

$C_\Gamma = 10^{-4}$

$N_T = 6.5$

$T_H = 165$ MeV

$$\Gamma(T) = C_\Gamma \, \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)$$

D. Blaschke & K.A. Bugaev, Fizika B **13**, 491 (2004); PPNP **53**, 197 (2004)

Mott-Hagedorn resonance gas

State-dependent hadron resonance width

$$A_i(M, m_i) \sim \frac{\Gamma_i \cdot m_i}{(M^2 - m_i^2)^2 + \Gamma_i^2 \cdot m_i^2} \ ,$$

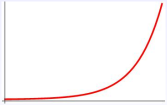
$$\Gamma_i(T) = \tau^{-1}_{\mathrm{coll,i}}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T \ n_j(T)$$

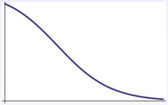
D. B. Blaschke, J. Berdermann, J. Cleymans, K. Redlich: [arXiv:1102.2908]

For mesons

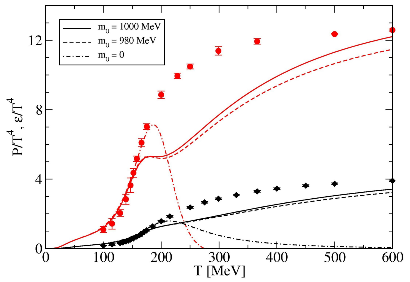
$$r_\pi^2(T) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q} q \rangle_T|^{-1} \ ;$$

$$\langle \bar{q} q \rangle_T \sim [1 - \tanh{(0.002\, T - 1)}]$$





For baryons: $r_N^2(T,\mu) = r_0^2 + r_\pi^2(T,\mu)$ with $r_0 = 0.45\mathrm{fm}$.



Mott-Hagedorn resonance gas:
Pressure and energy density for three values of the mass threshold
$m_0 = 1.0$ GeV (solid lines)
$m_0 = 0.98$ GeV (dashed lines) and
$m_0 = 0$ (dash-dotted lines)

$$\varepsilon(T) = \sum_{i:\ m_i < m_0} g_i\ \varepsilon_i(T; m_i)$$
$$+ \sum_{i:\ m_i \geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2)\ A(M, m_i)\ \varepsilon_i(T; M),$$

Quarks and gluons are missing!

Quarks and gluons in the PNJL model

Systematic expansion of the pressure as the thermodynamical potential in the grand canonical ensemble for a chiral quark model of the PNJL type beyond its mean field description $P_{\mathrm{PNJL,MF}}(T)$ by including perturbative with α_S corrections

$$P(T) = P_{MHRG}(T) + P_{PNJL,MF}(T) + P_2(T) \ .$$

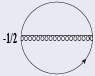
$P_{MHRG}(T)$ stands for the pressure of the MHRG model, accounting for the dissociation of hadrons in hot dense matter.

Quark and gluon contributions

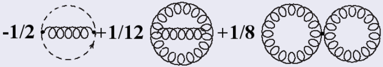
$$P_2(T) = P_2^{\mathrm{quark}}(T) + P_2^{\mathrm{gluon}}(T)$$

Quark and gluon contributions

$P_2^{\text{quark}}(T)$



$P_2^{\text{gluon}}(T)$



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Total perturbative QCD correction

$$P_2 = -\frac{8}{\pi}\alpha_s T^4 (I_\Lambda^+ +$$

$$\frac{3}{\pi^2}((I_\Lambda^+)^2 + (I_\Lambda^-)^2))$$

$$\underset{\Lambda/T \to 0}{\longrightarrow} -\frac{3\pi}{2}\alpha_s T^4$$

where

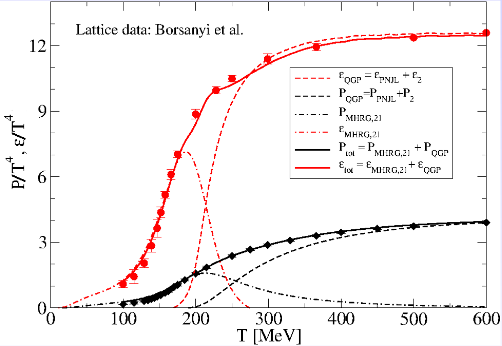
$$I_\Lambda^\pm = \int_{\Lambda/T}^\infty \frac{\mathrm{d}x \; x}{\mathrm{e}^x \pm 1}$$

Energy corrections

$$\varepsilon_2(T) = T\frac{dP_2(T)}{dT} - P_2(T) \, .$$

Quarks, gluons and hadron resonances

$$P_{\mathrm{MHRG}}(T) = \sum_i \delta_i d_i \int \frac{d^3p}{(2\pi)^3} \int dM\, A_i(M, m_i) T \ln\left\{1 + \delta_i \mathrm{e}^{-[\sqrt{p^2+M^2}-\mu_i]/T}\right\} ,$$

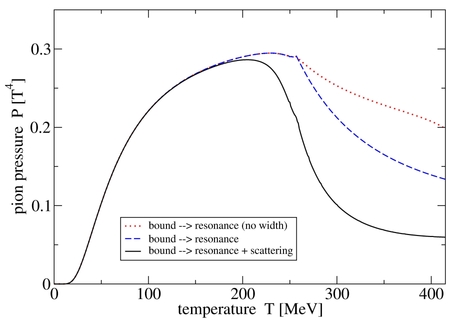


- Quark-gluon plasma contributions are described within the improved PNJL model with α_s corrections .
- Heavy hadrons are described within the resonance gas with finite width exhibiting a Mott effect at the coincident chiral and deconfinement transitions.

Further progress

Taking into account hadrons scattering in the dense medium:

The generalized Beth-Uhlenbeck approach and construction of the scattering phase shift taking into account Levinson theorem (A. Wergieluk, D. Blaschke, Y. .L. Kalinovsky and A. Friesen, arXiv:1212.5245 [nucl-th].)



Conclusions

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances.
- A generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium

Happy 70, Takeshi!

Many happy returns, Takeshi!

Many friends, Takeshi!