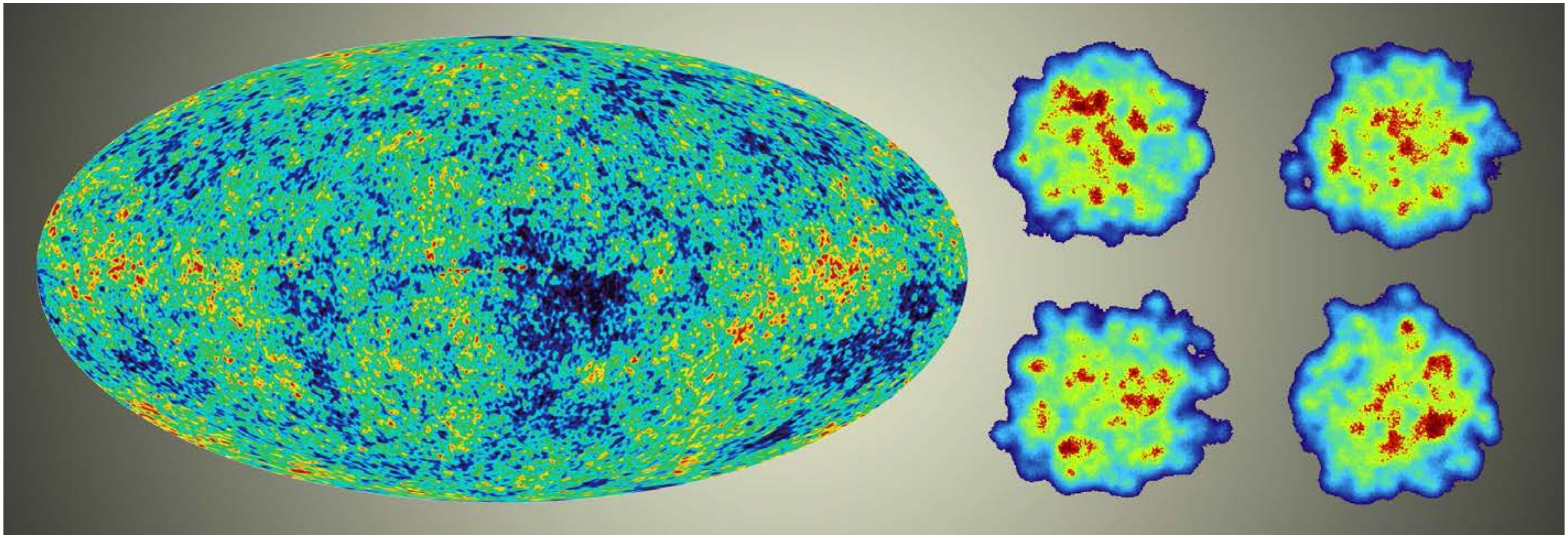
The Little Bang Fluctuation Spectrum[*]

Ulrich Heinz (The Ohio State University)



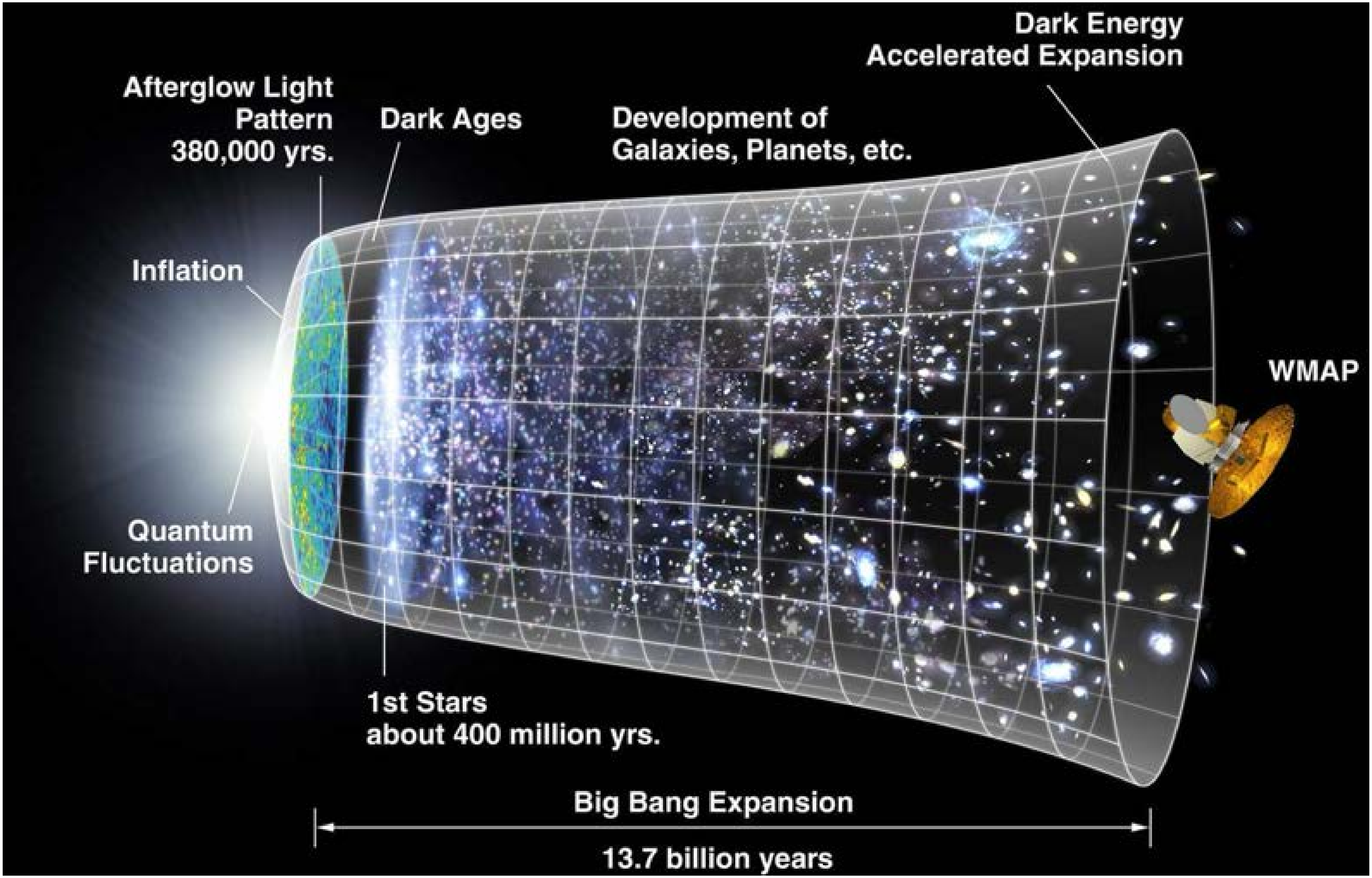
9th International Workshop on Relativistic Aspects of Nuclear Physics (RANP 2013)

(Takeshi Kodama's Fest)

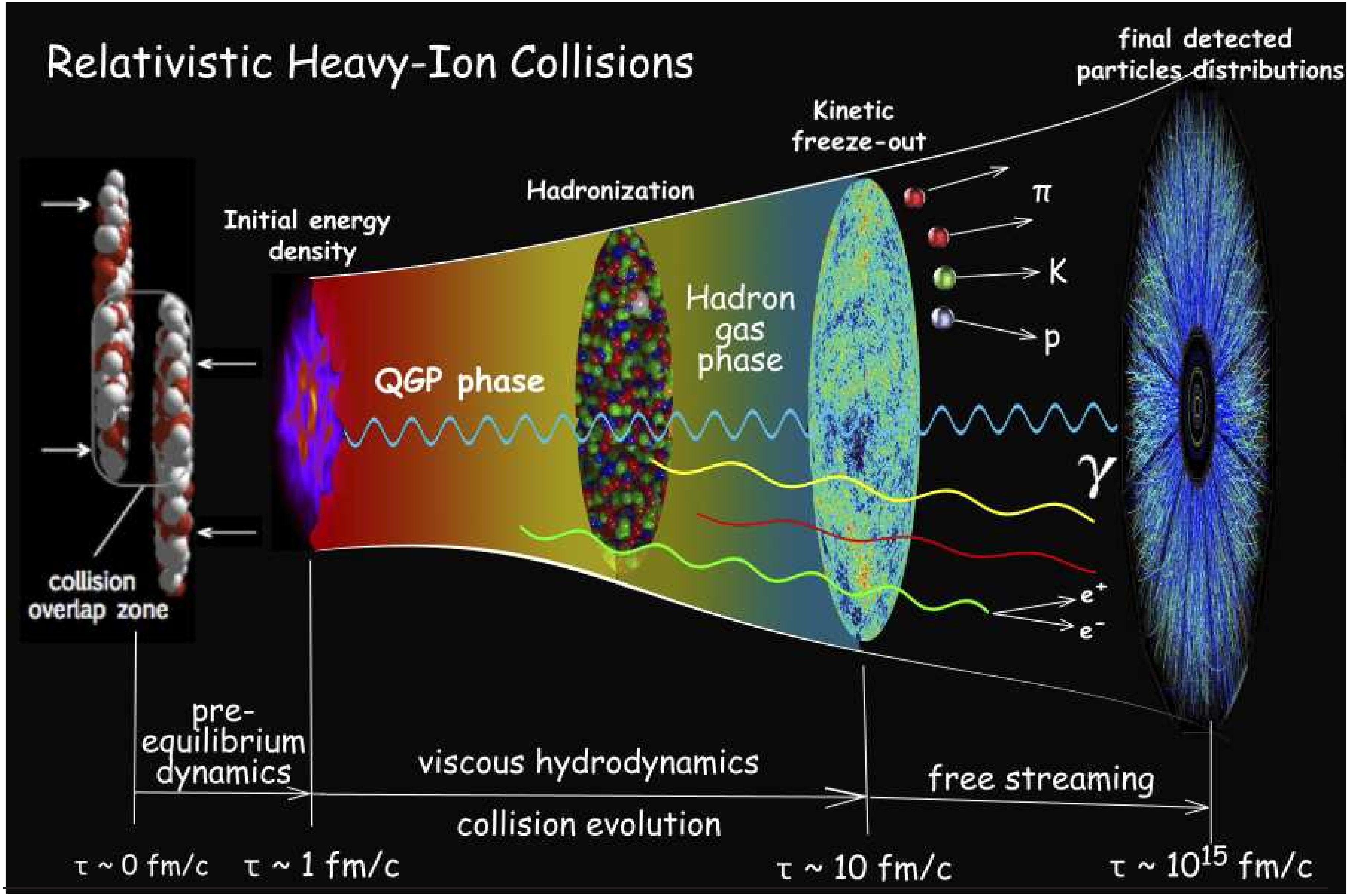
Rio de Janeiro, 23-27 September 2013

[*] Supported by the U.S. Department of Energy

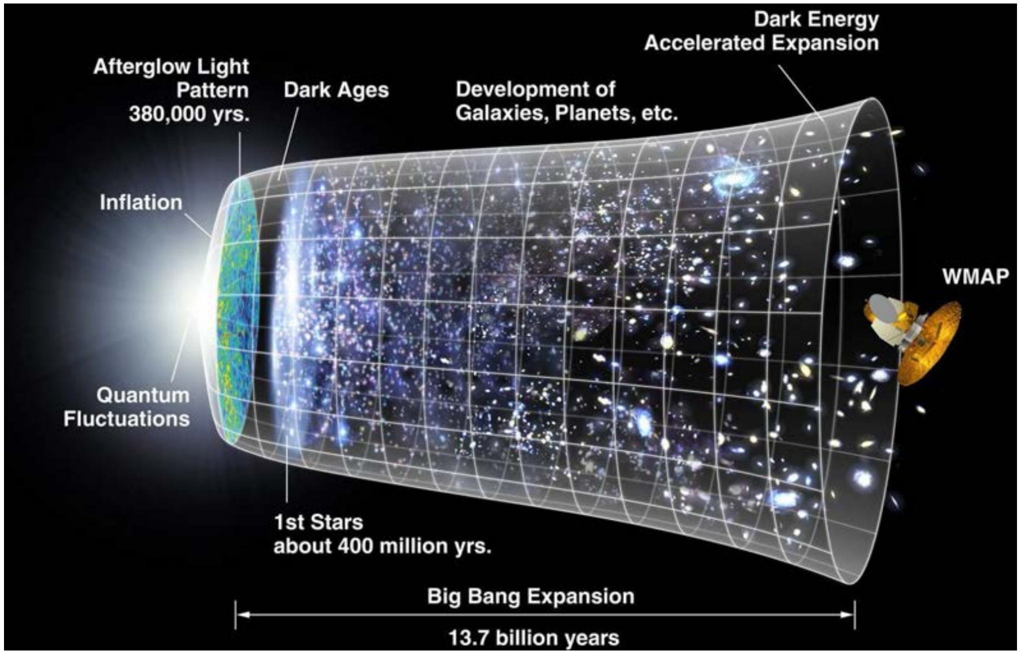
The Big Bang

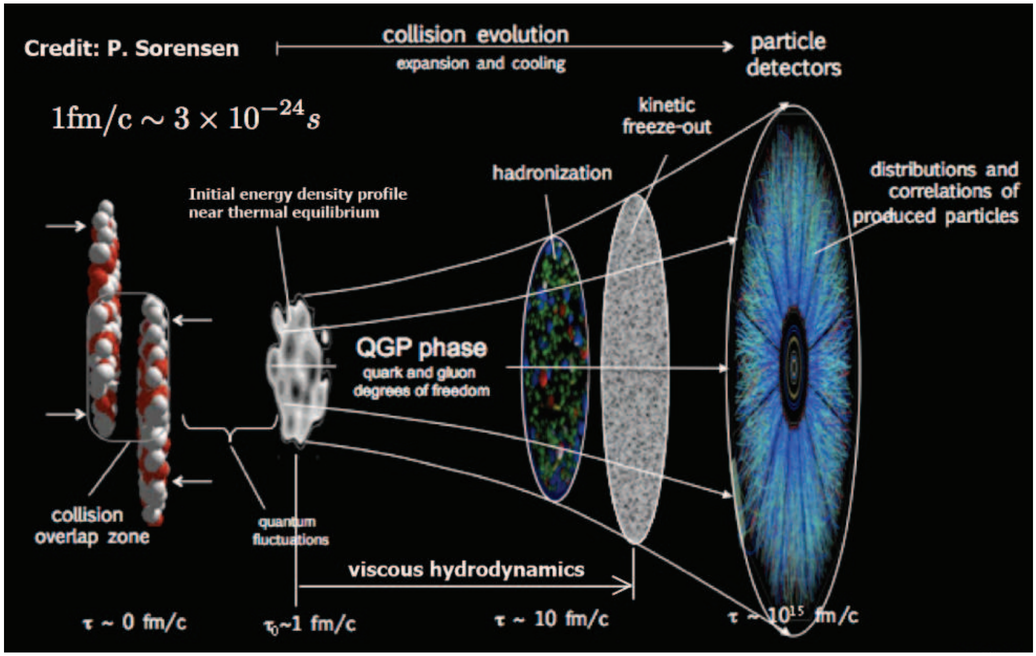

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

The Little Bang



Big Bang vs. Little Bang



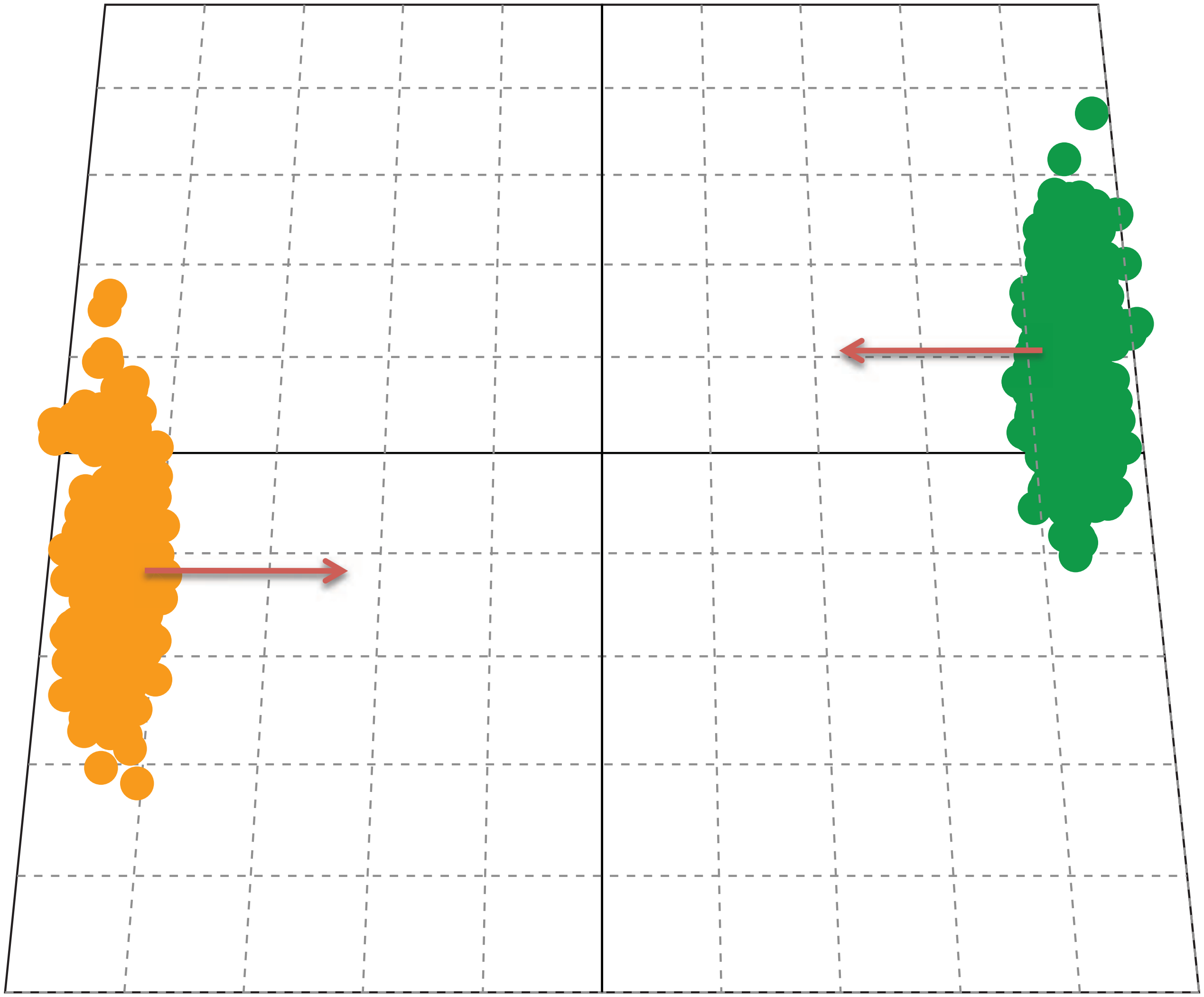


Similarities: Hubble-like expansion, expansion-driven dynamical freeze-out
chemical freeze-out (nucleo-/hadrosynthesis) before thermal freeze-out (CMB, hadron p_T-spectra)
initial-state quantum fluctuations imprinted on final state

Differences: Expansion rates differ by 18 orders of magnitude
Expansion in 3d, not 4d; driven by pressure gradients, not gravity
Time scales measured in fm$/c$ rather than billions of years
Distances measured in fm rather than light years
"Heavy-Ion Standard Model" still under construction $\Longrightarrow$ **this talk**

Relativistic Nucleus-Nucleus Collisions

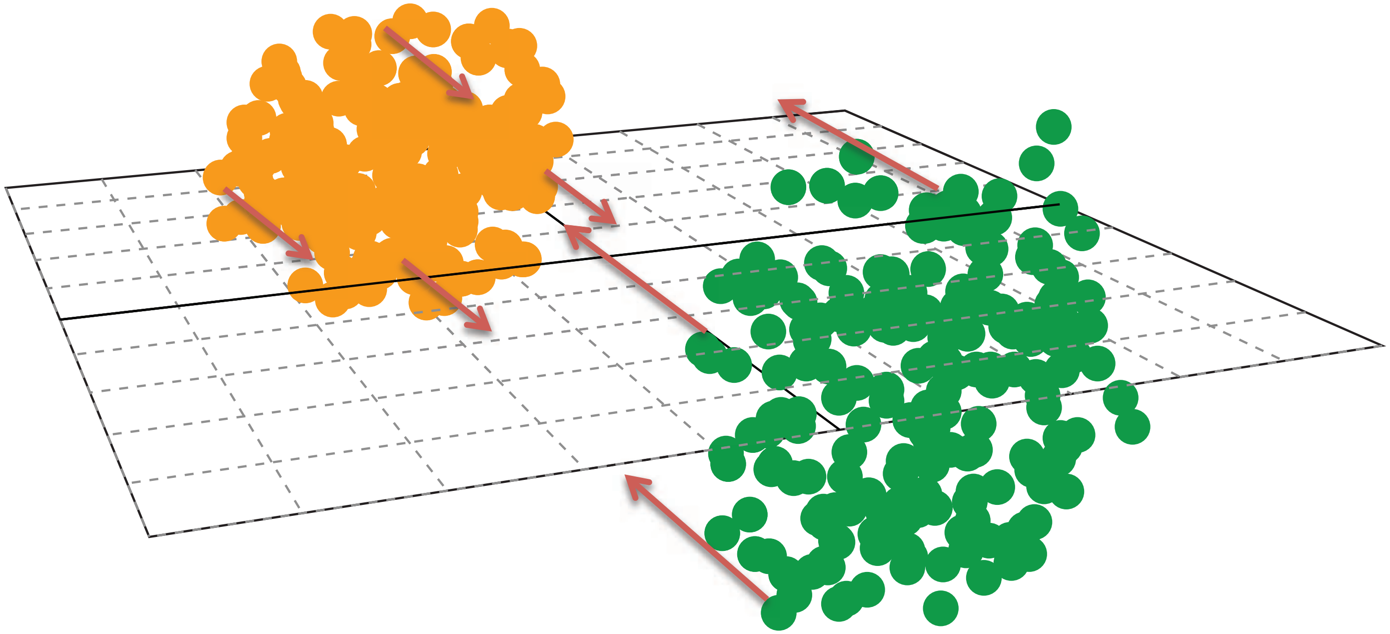
Animation: P. Sorensen



Collision of two Lorentz contracted gold nuclei

Relativistic Nucleus-Nucleus Collisions

Animation: P. Sorensen



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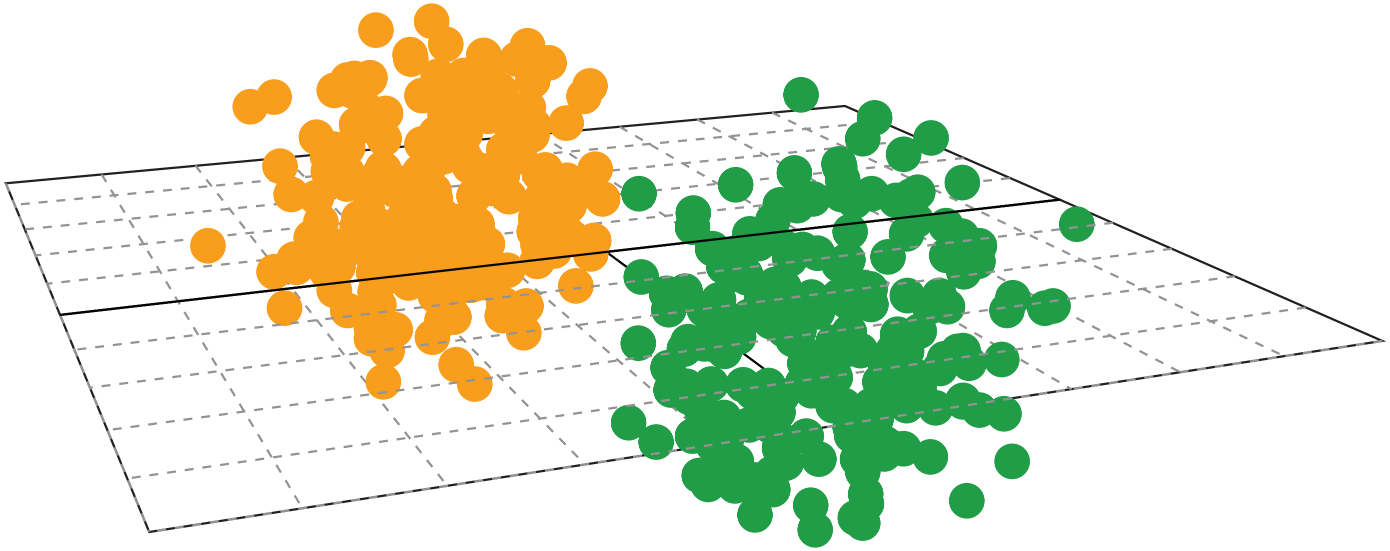

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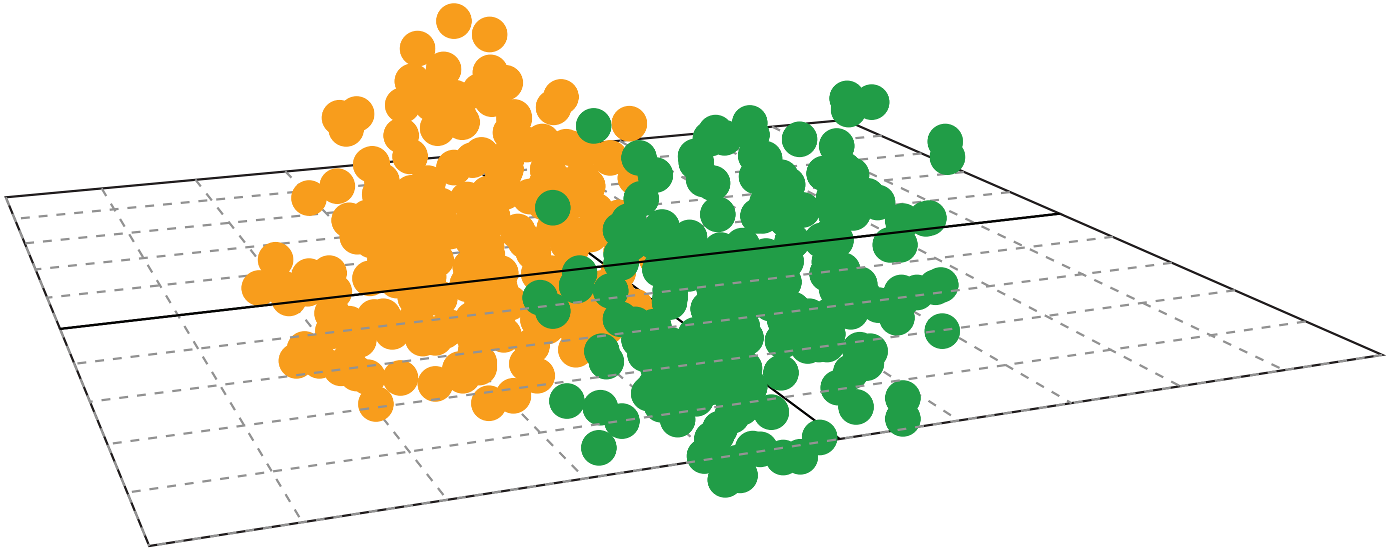

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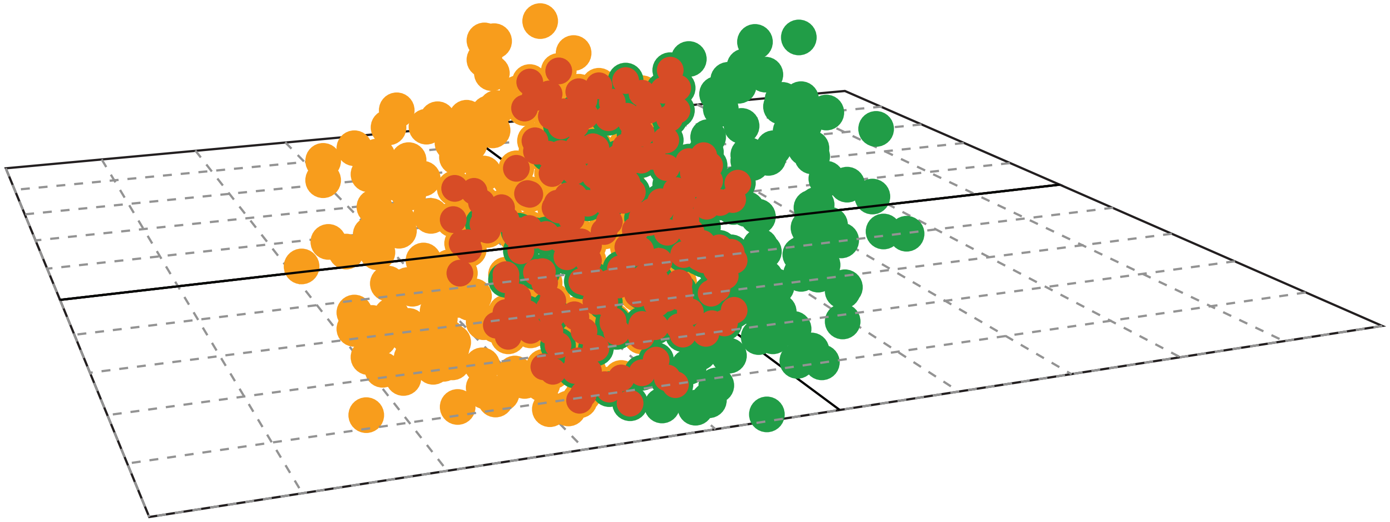

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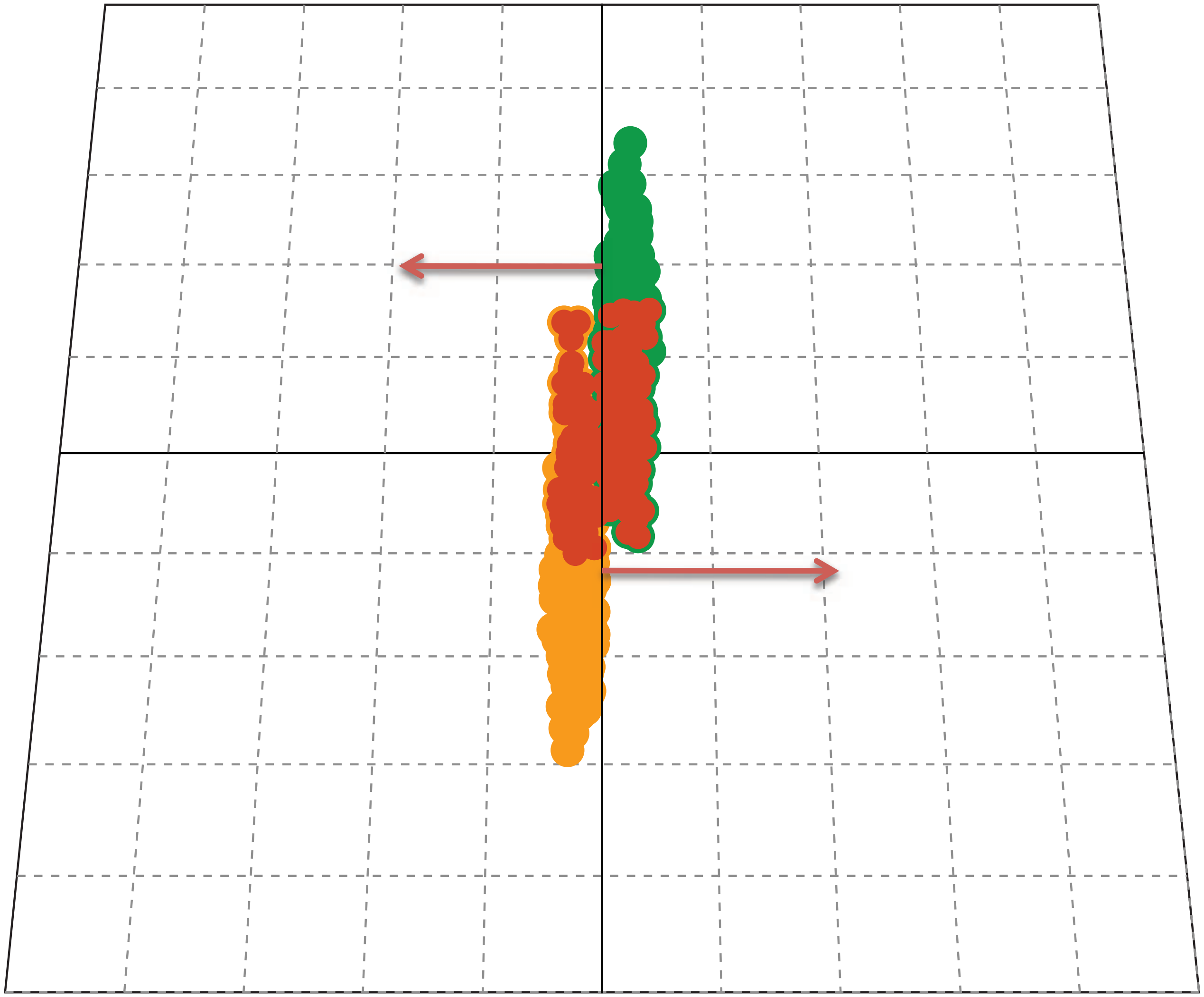

Collision of two Lorentz contracted gold nuclei

Relativistic Nucleus-Nucleus Collisions

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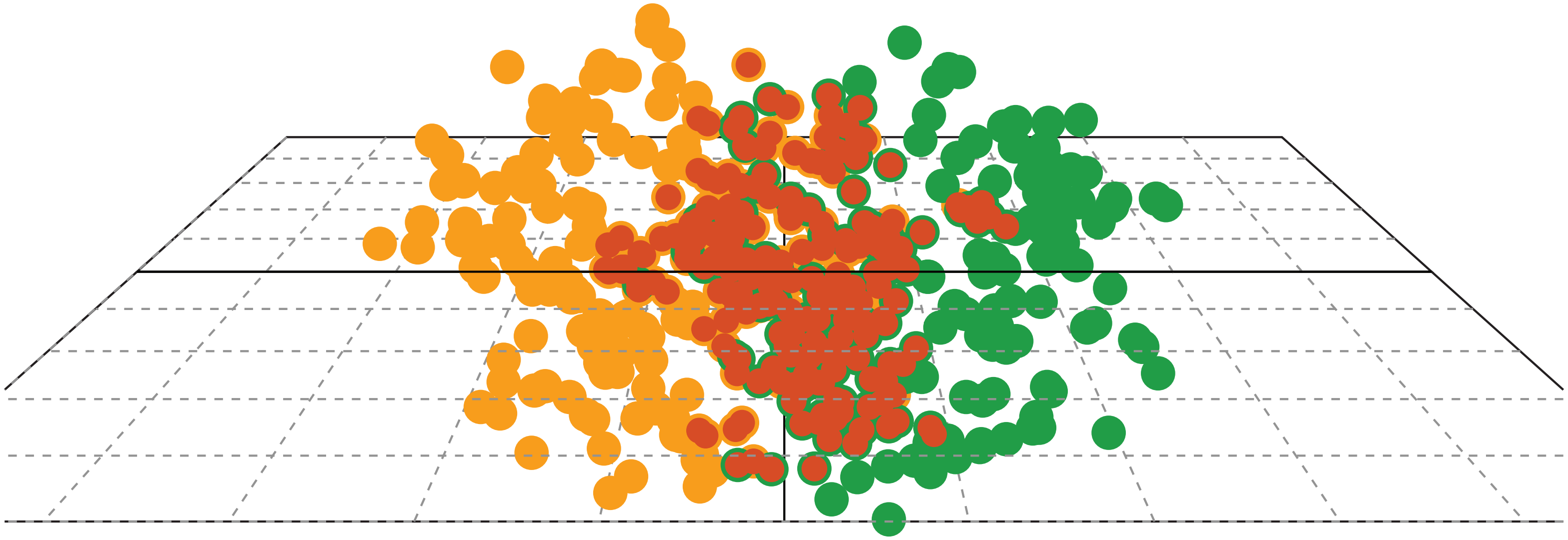
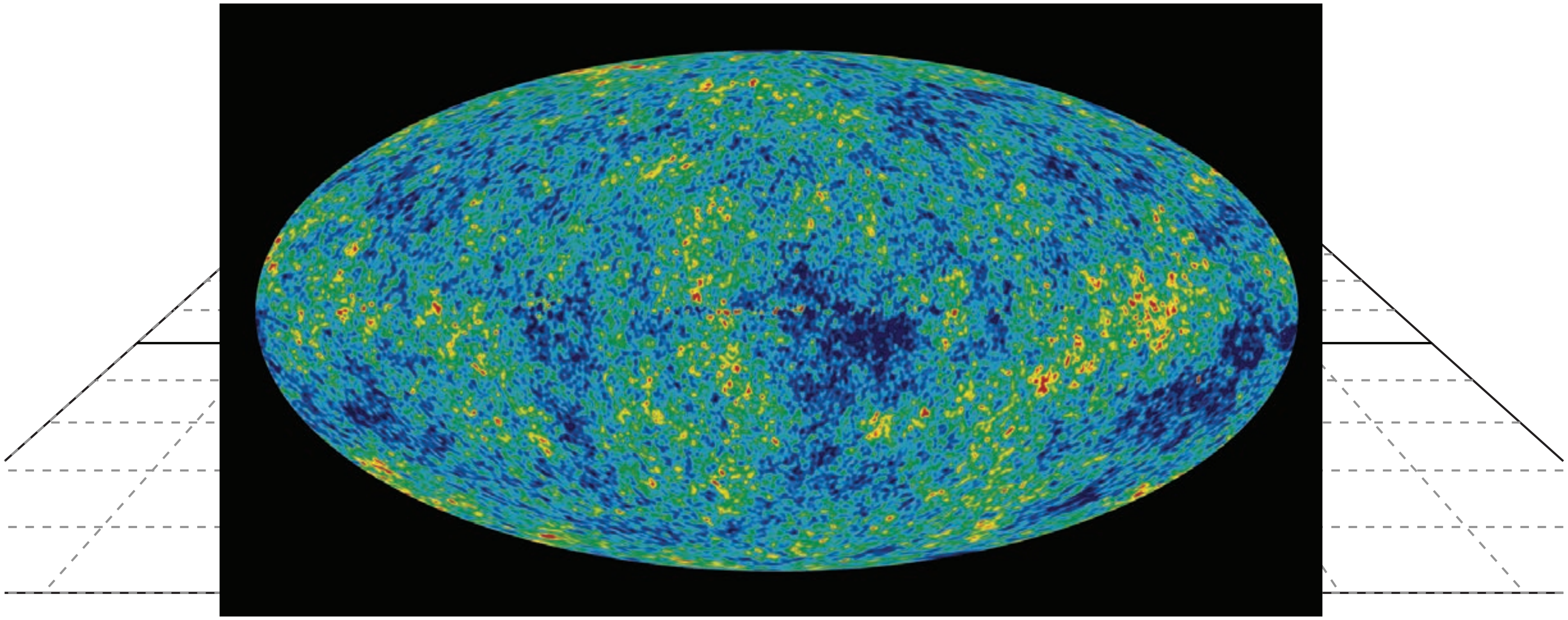


Produced fireball is ~10^{-14} meters across and lives for ~$5\text{x}10^{-23}$ seconds

Collision of two Lorentz contracted gold nuclei

Relativistic Nucleus-Nucleus Collisions

Animation: P. Sorensen

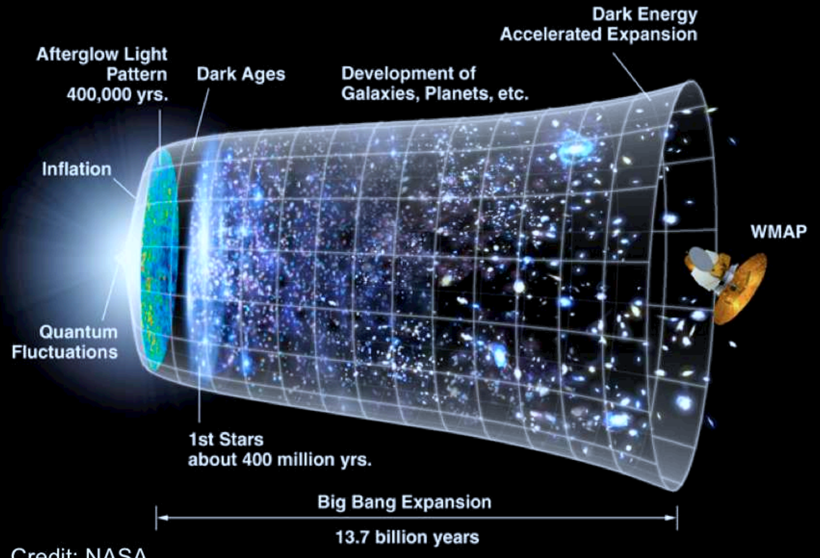


Produced fireball is ~10^{-14} meters across and lives for ~5x10^{-23} seconds

Collision of two Lorentz contracted gold nuclei

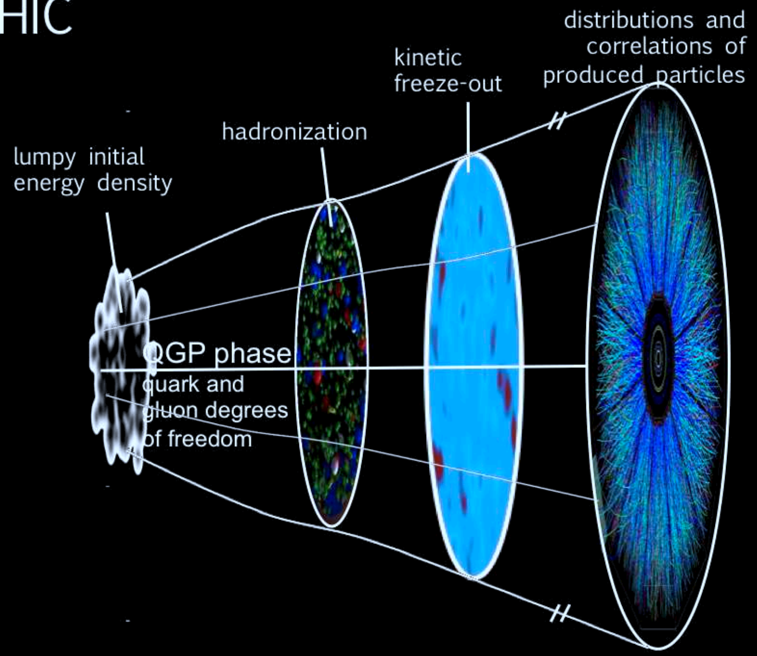
The Big Bang vs the Little Bangs

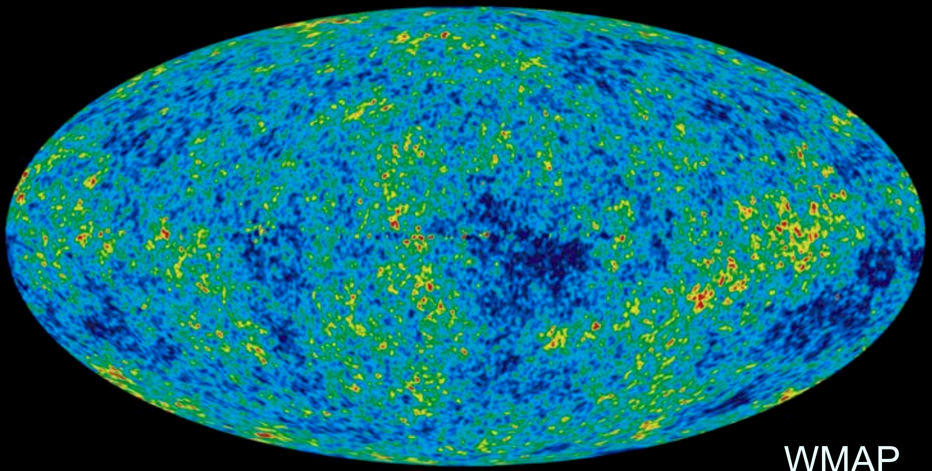
The Universe

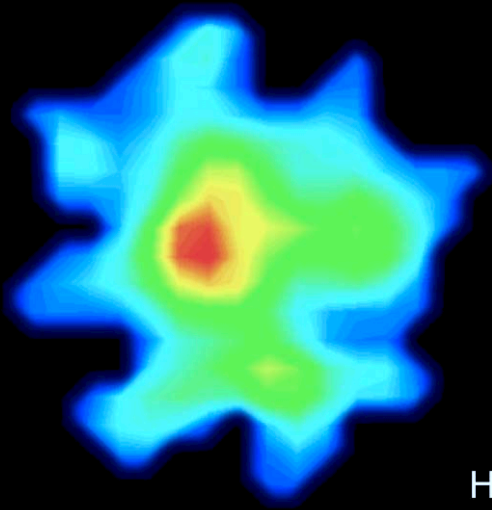


Credit: NASA

HIC





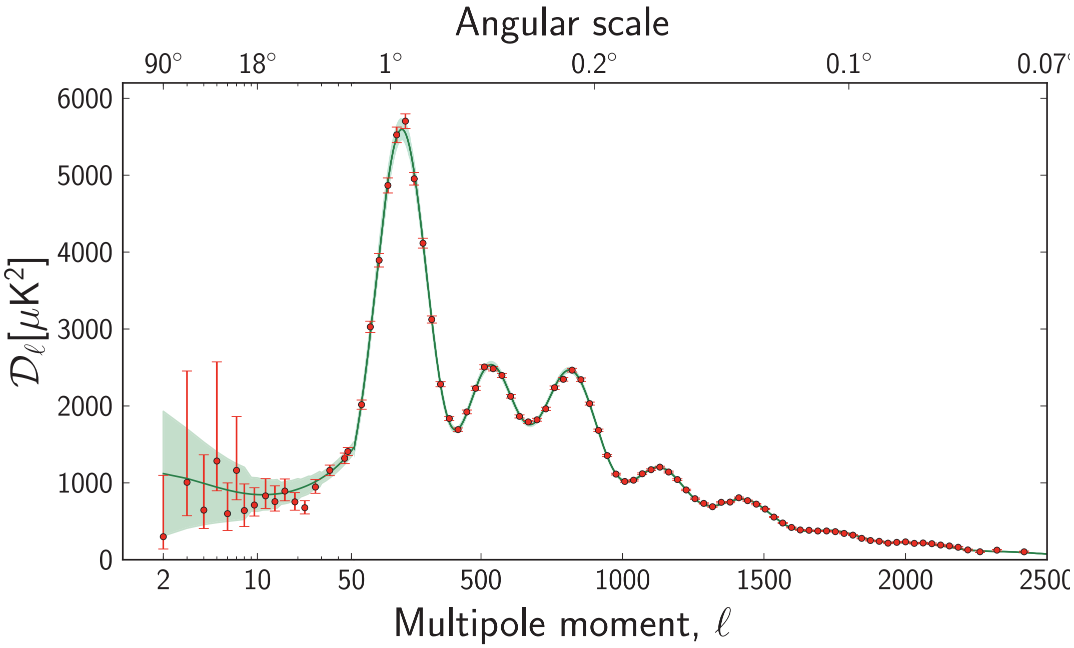


Big vs. Little Bang: The fluctuation power spectrum

Mishra, Mohapatra, Saumia, Srivastava, PRC77 (2008) 064902 and C81 (2010) 034903

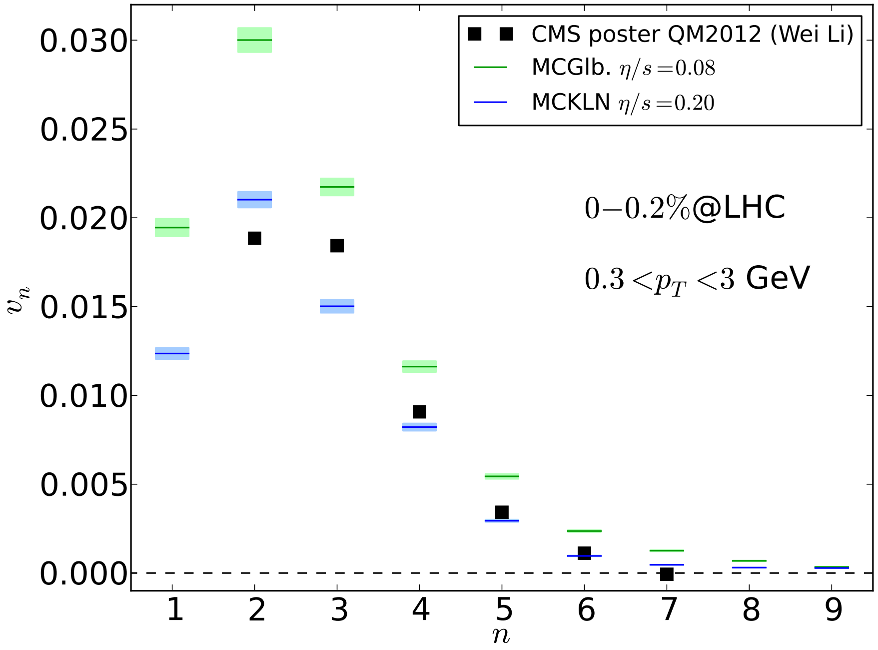
Mocsy & Sorensen, NPA855 (2011) 241, PLB705 (2011) 71

Big Bang temperature power spectrum (Planck 2013)



Flow power spectrum for ultracentral PbPb Little Bangs

(Data: CMS, Quark Matter 2012; Theory: OSU 2013)



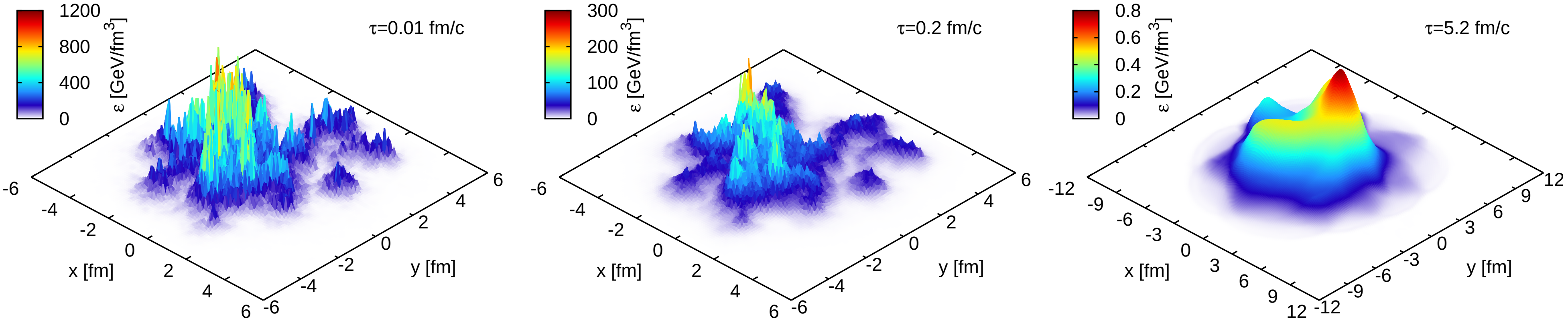
Higher flow harmonics get suppressed by shear viscosity

A detailed study of fluctuations is a powerful discriminator between models!

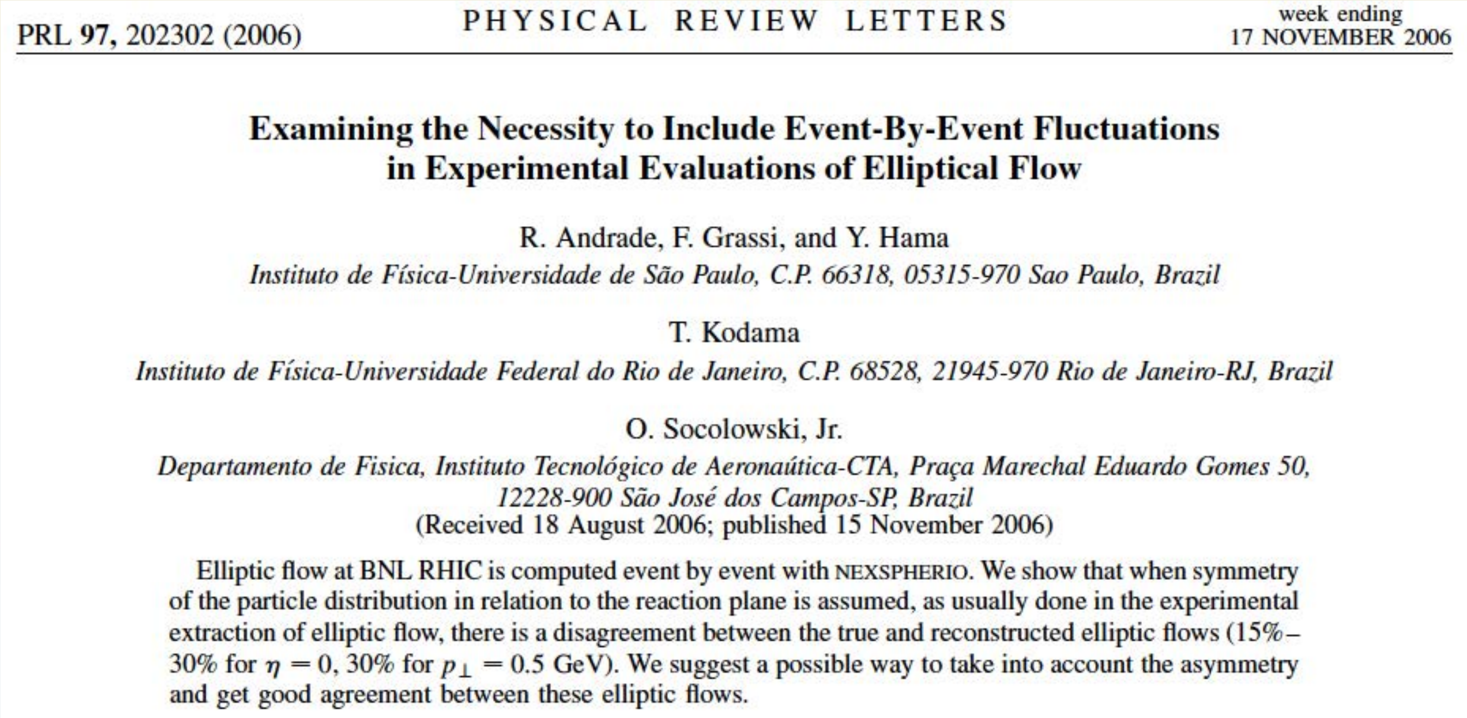
Each Little Bang evolves differently!

Density evolution of a single $b = 8\,\mathrm{fm}$ Au+Au collision at RHIC, with IP-Glasma initial conditions, Glasma evolution to $\tau = 0.2\,\mathrm{fm}/c$ followed by (3+1)-d viscous hydrodynamic evolution with `MUSIC` using $\eta/s = 0.12 = 1.5/(4\pi)$

Schenke, Tribedy, Venugopalan, PRL 108 (2012) 252301:



Takeshi Kodama and his “Brazilians”:
Pioneers of event-by-event hydrodynamics with fluctuating initial conditions



PRL **97,** 202302 (2006) PHYSICAL REVIEW LETTERS week ending 17 NOVEMBER 2006

Examining the Necessity to Include Event-By-Event Fluctuations in Experimental Evaluations of Elliptical Flow

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Instituto de Física-Universidade de São Paulo, C.P. 66318, 05315-970 Sao Paulo, Brazil

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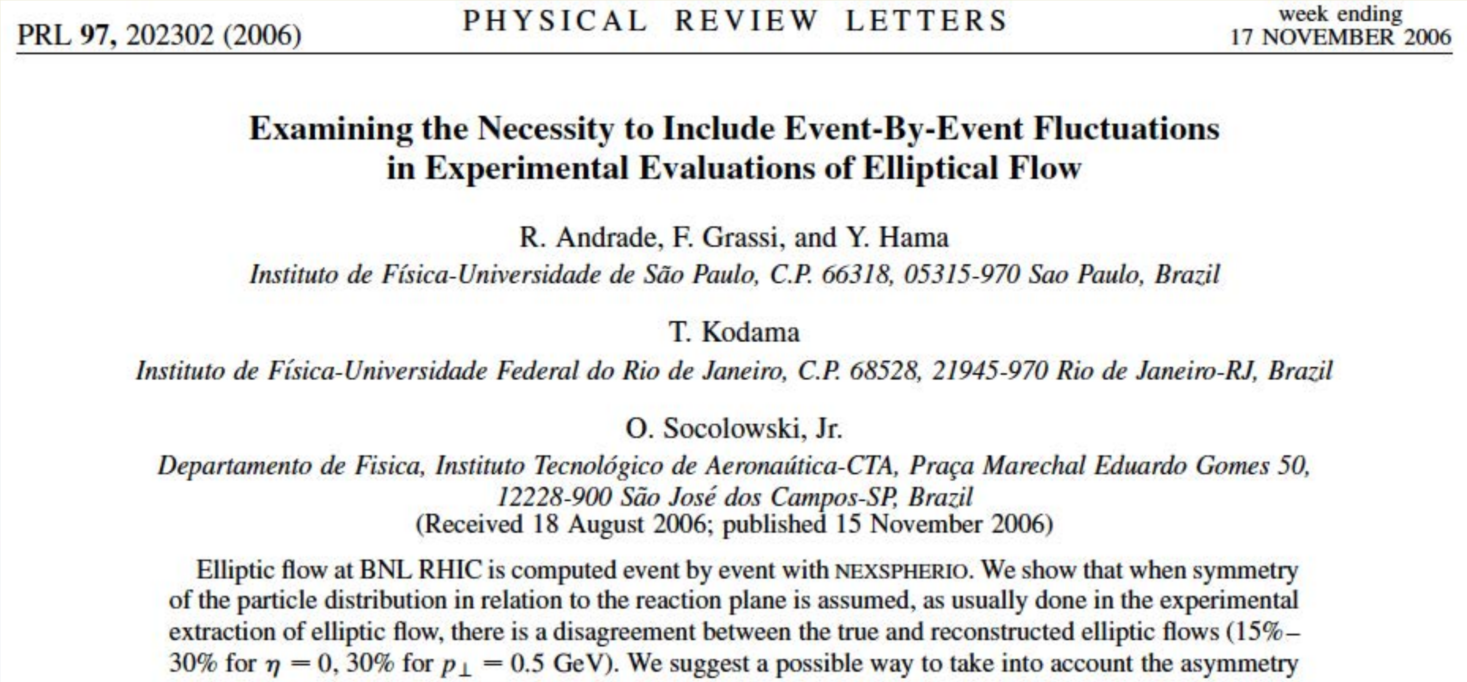
Departamento de Fisica, Instituto Tecnológico de Aeronaútica-CTA, Praça Marechal Eduardo Gomes 50, 12228-900 São José dos Campos-SP, Brazil

(Received 18 August 2006; published 15 November 2006)

Elliptic flow at BNL RHIC is computed event by event with NEXSPHERIO. We show that when symmetry of the particle distribution in relation to the reaction plane is assumed, as usually done in the experimental extraction of elliptic flow, there is a disagreement between the true and reconstructed elliptic flows (15%–30% for $\eta = 0$, 30% for $p_\perp = 0.5$ GeV). We suggest a possible way to take into account the asymmetry and get good agreement between these elliptic flows.

Takeshi Kodama and his "Brazilians": Pioneers of event-by-event hydrodynamics with fluctuating initial conditions



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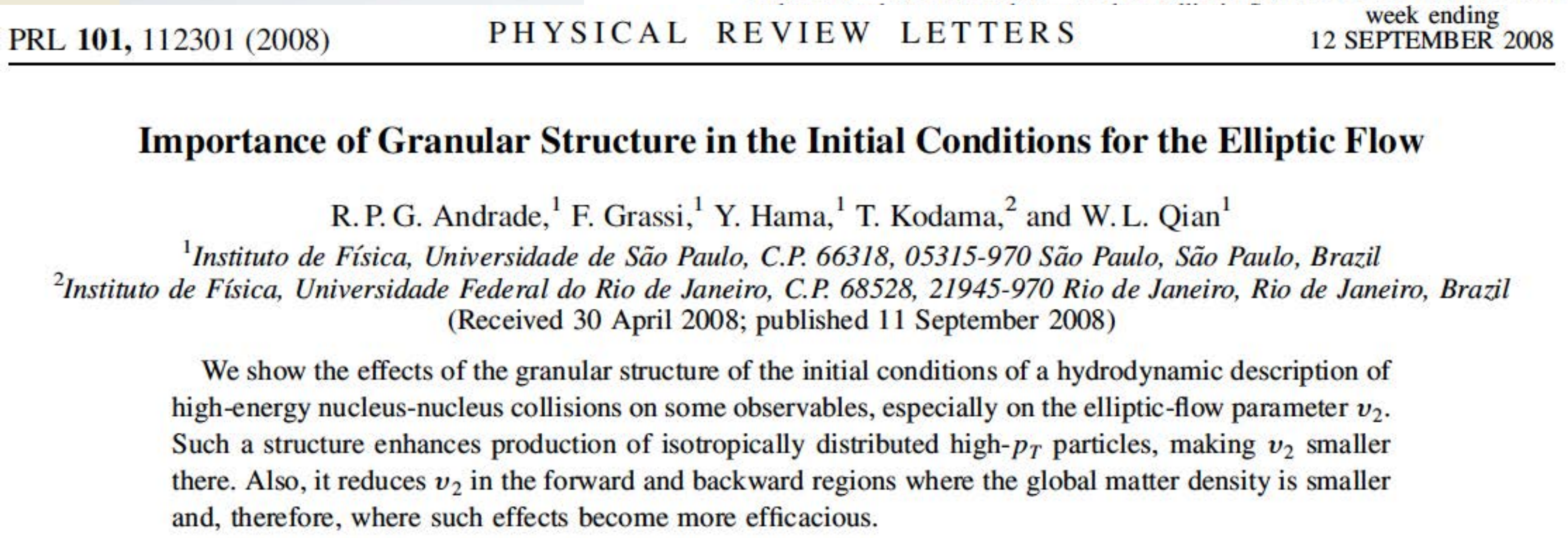


PRL **101,** 112301 (2008) PHYSICAL REVIEW LETTERS week ending 12 SEPTEMBER 2008

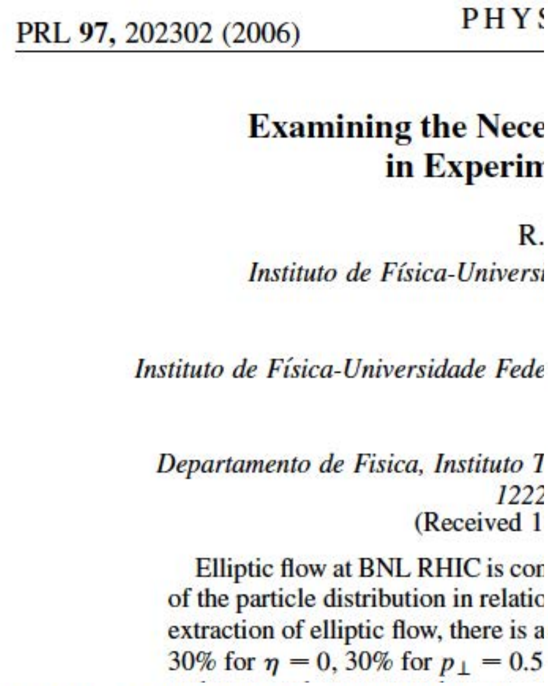
Importance of Granular Structure in the Initial Conditions for the Elliptic Flow

R. P. G. Andrade,[1] F. Grassi,[1] Y. Hama,[1] T. Kodama,[2] and W. L. Qian[1]
[1]*Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 São Paulo, São Paulo, Brazil*
[2]*Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, 21945-970 Rio de Janeiro, Rio de Janeiro, Brazil*

(Received 30 April 2008; published 11 September 2008)

We show the effects of the granular structure of the initial conditions of a hydrodynamic description of high-energy nucleus-nucleus collisions on some observables, especially on the elliptic-flow parameter v_2. Such a structure enhances production of isotropically distributed high-p_T particles, making v_2 smaller there. Also, it reduces v_2 in the forward and backward regions where the global matter density is smaller and, therefore, where such effects become more efficacious.

Takeshi Kodama and his “Brazilians”: Pioneers of event-by-event hydrodynamics with fluctuating initial conditions



PRL **97,** 202302 (2006) PHYS

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extraction of elliptic flow, there is a
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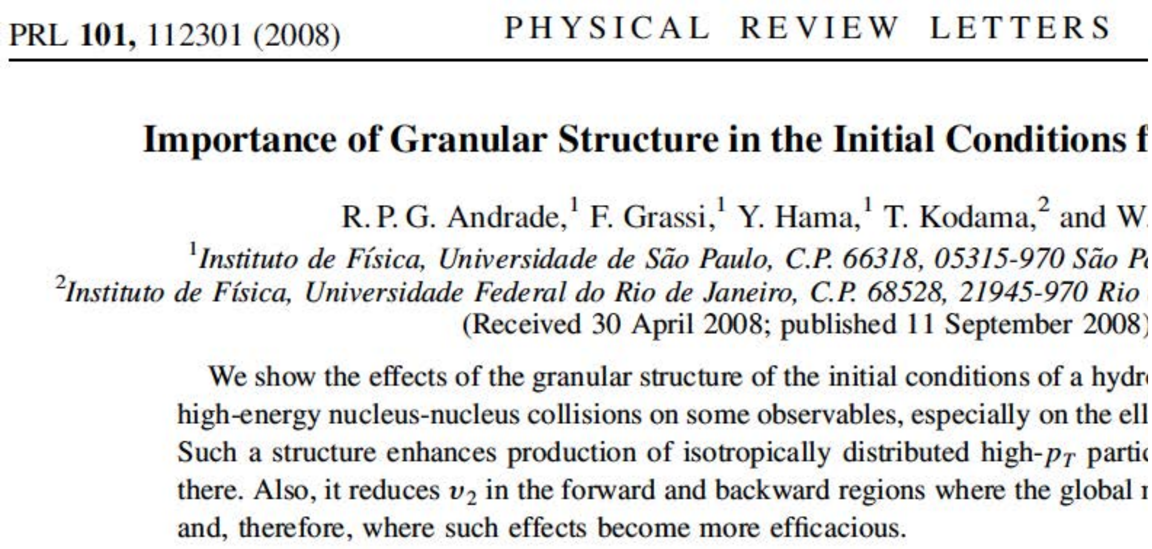

PRL **101,** 112301 (2008) PHYSICAL REVIEW LETTERS

Importance of Granular Structure in the Initial Conditions f

R. P. G. Andrade,[1] F. Grassi,[1] Y. Hama,[1] T. Kodama,[2] and W.
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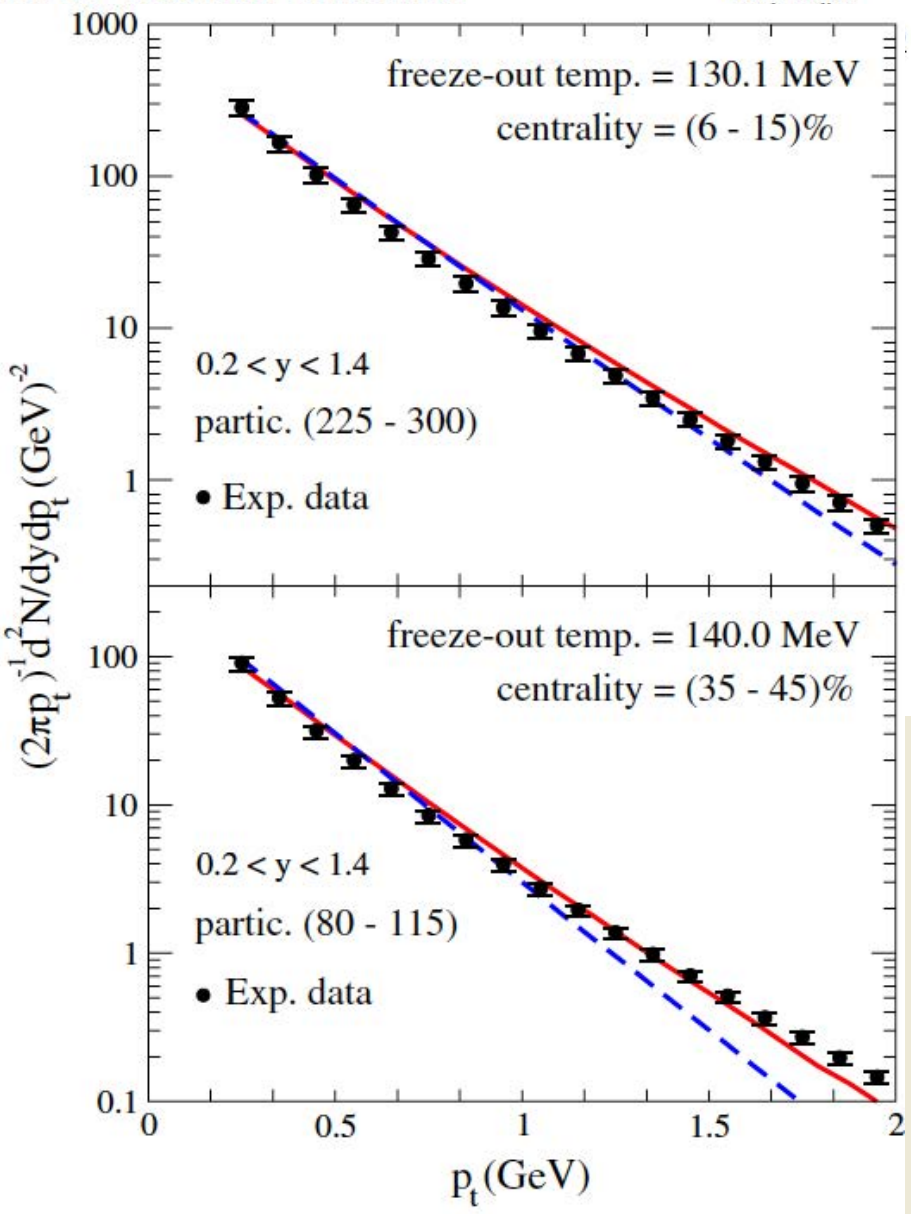
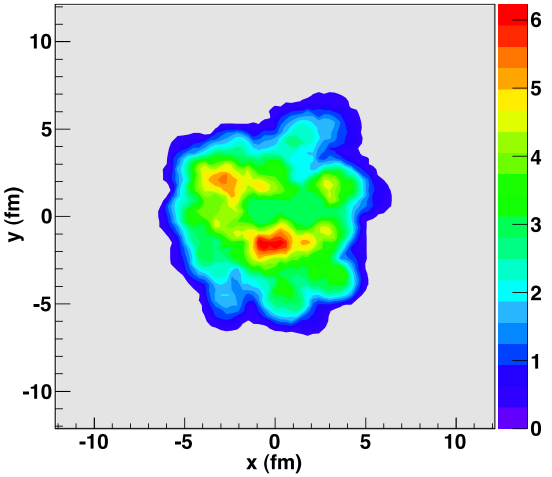
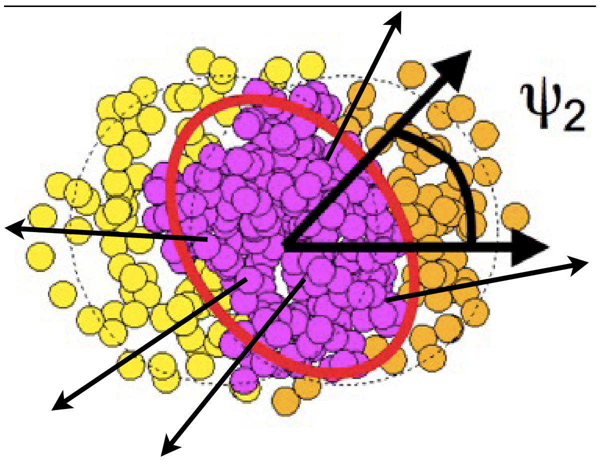
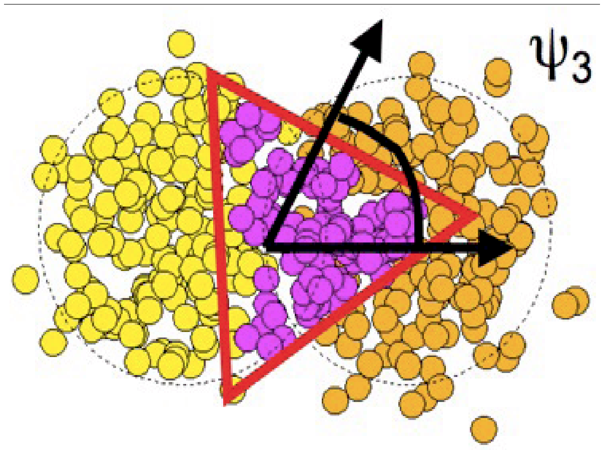


Event-by-event shape and flow fluctuations rule!

(Alver and Roland, PRC81 (2010) 054905)







- Each event has a different initial shape and density distribution, characterized by different set of harmonic eccentricity coefficients ε_n

- Each event develops its individual hydrodynamic flow, characterized by a set of harmonic flow coefficients v_n and flow angles ψ_n

- At small impact parameters fluctuations ("hot spots") dominate over geometric overlap effects
  (Alver & Roland, PRC81 (2010) 054905; Qin, Petersen, Bass, Müller, PRC82 (2010) 064903)

How anisotropic flow is measured:

Definition of flow coefficients:

$$\frac{dN^{(i)}}{dy\,p_T dp_T\,d\phi_p}(b) = \frac{dN^{(i)}}{dy\,p_T dp_T}(b)\left(1 + 2\sum_{n=1}^{\infty} \boldsymbol{v_n^{(i)}(y, p_T; b)} \cos\left(n(\phi_p - \Psi_n^{(i)})\right)\right).$$

Define event average $\{\dots\}$, ensemble average $\langle\dots\rangle$

Flow coefficients v_n typically extracted from azimuthal correlations (k-particle cumulants). E.g. $k = 2, 4$:

$$c_n\{2\} = \langle\{e^{ni(\phi_1-\phi_2)}\}\rangle = \langle\{e^{ni(\phi_1-\psi_n)}\}\{e^{-ni(\phi_2-\psi_n)}\} + \delta_2\rangle = \langle v_n^2 + \delta_2\rangle$$
$$c_n\{4\} = \langle\{e^{ni(\phi_1+\phi_2-\phi_3-\phi_4)}\}\rangle - 2\langle\{e^{ni(\phi_1-\phi_2)}\}\rangle = \langle -v_n^4 + \delta_4\rangle$$

v_n is correlated with the event plane while δ_n is not ("non-flow"). $\delta_2 \sim 1/M$, $\delta_4 \sim 1/M^3$. 4th-order cumulant is free of 2-particle non-flow correlations.

These measures are affected by event-by-event flow fluctuations:

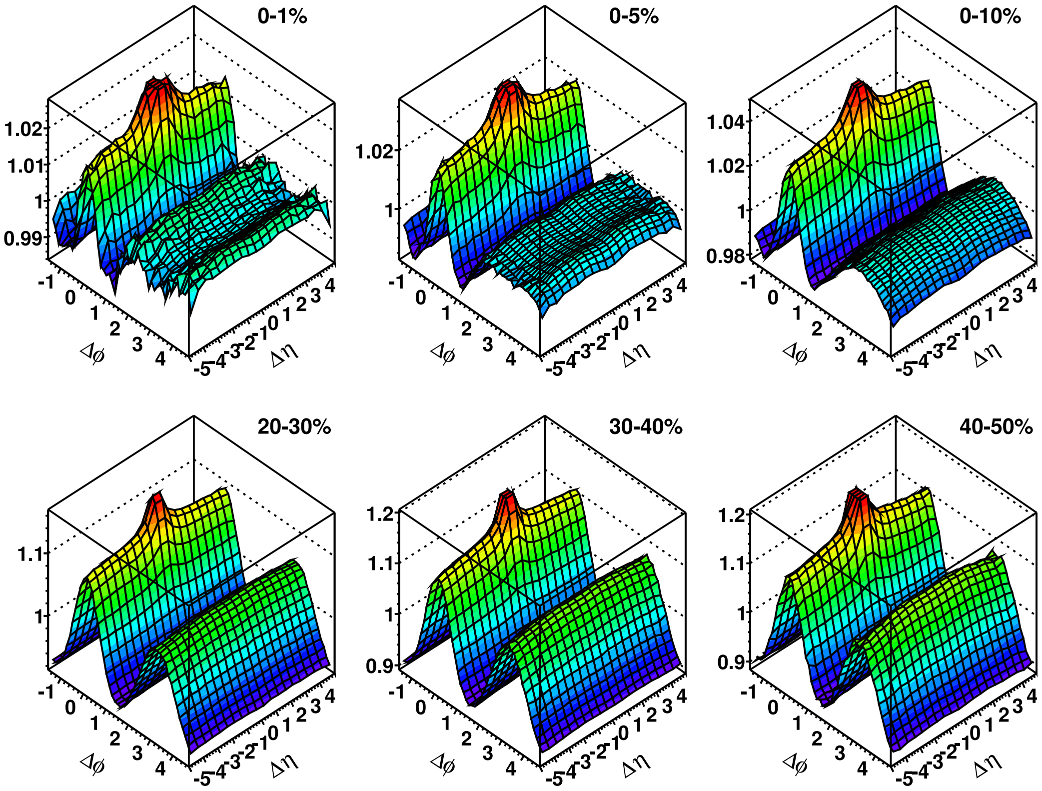
$$\langle v_2^2\rangle = \langle v_2\rangle^2 + \sigma^2, \qquad \langle v_2^4\rangle = \langle v_2\rangle^4 + 6\sigma^2\langle v_2\rangle^2$$

$\boldsymbol{v_n\{k\}}$ denotes the value of v_n extracted from the k^{th}-order cumulant:

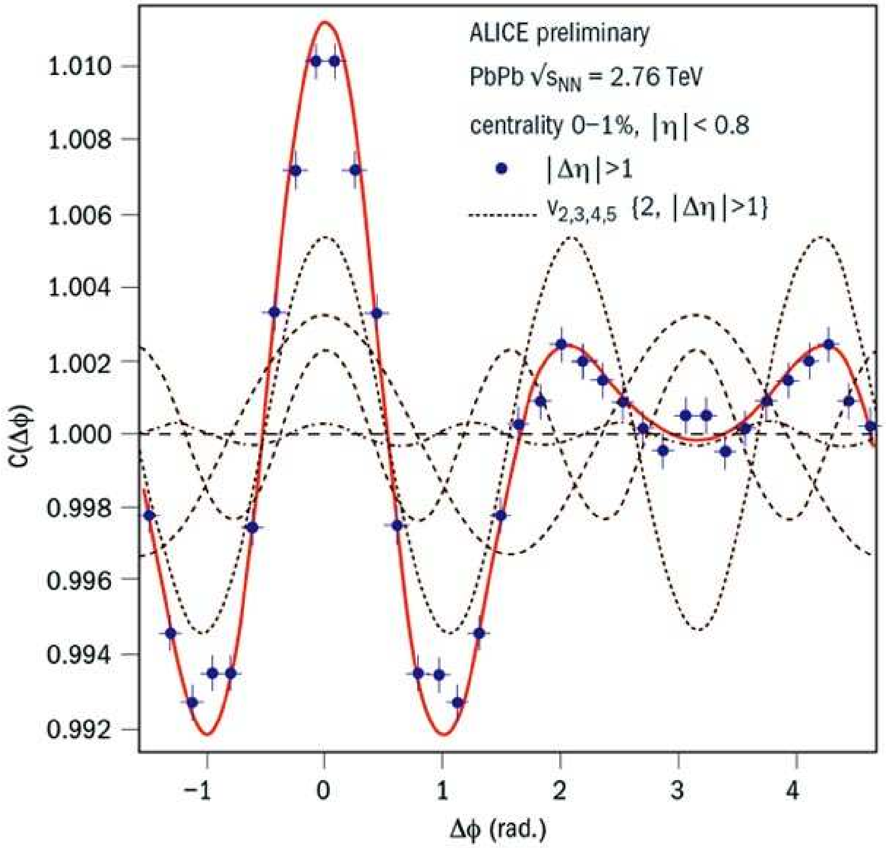
$$v_2\{2\} = \sqrt{\langle v_2^2\rangle}, \qquad v_2\{4\} = \sqrt[4]{2\langle v_2^2\rangle^2 - \langle v_2^4\rangle}$$

Panta rhei: “soft ridge”=“Mach cone”=flow!

ATLAS (J. Jia), Quark Matter 2011



ALICE (J. Grosse-Oetringhaus), QM11



- anisotropic flow coefficients v_n and flow angles ψ_n correlated over large rapidity range!

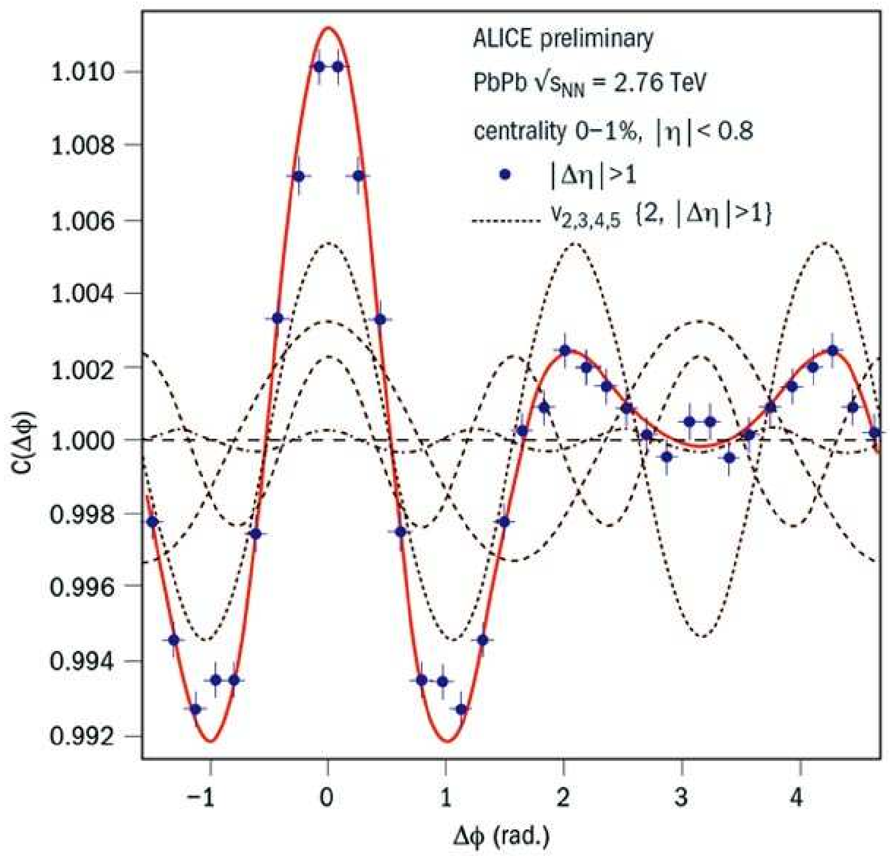
  M. Luzum, PLB 696 (2011) 499: All long-range rapidity correlations seen at RHIC are consistent with being entirely generated by hydrodynamic flow.

- in the 1% most central collisions $v_3 > v_2$

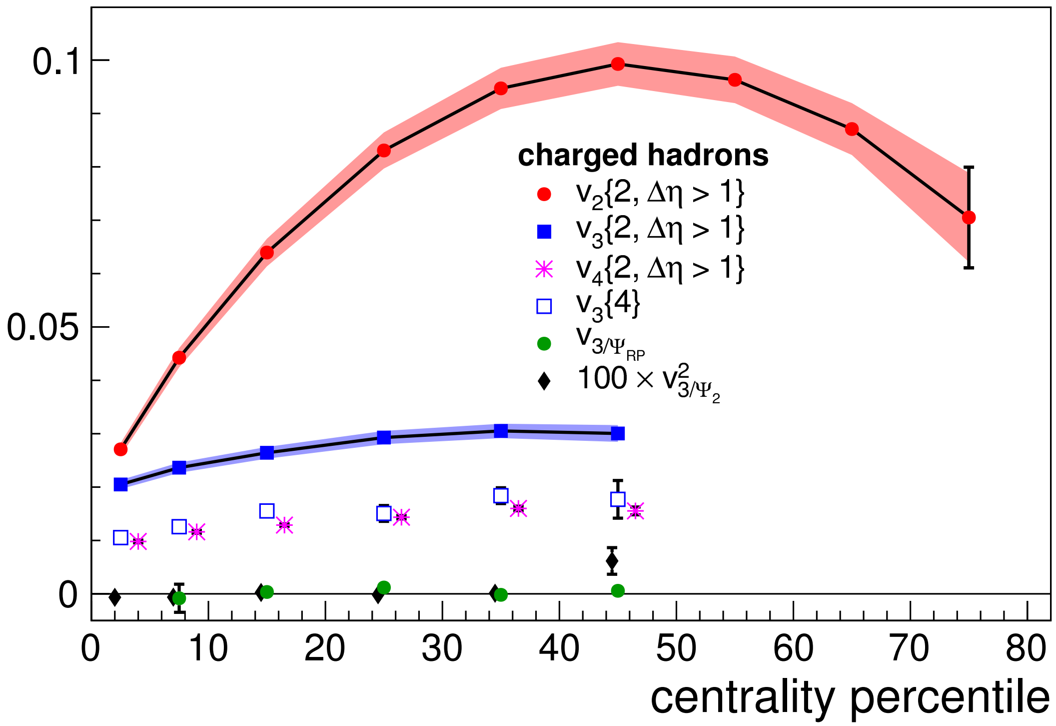
  $\Longrightarrow$ prominent “Mach cone”-like structure!

  $\Longrightarrow$ event-by-event eccentricity fluctuations dominate!

Event-by-event shape and flow fluctuations rule!



ALICE (A. Bilandzic) Quark Matter 2011



- in the 1% most central collisions $v_3 > v_2 \Longrightarrow$ prominent "Mach cone"-like structure!

- triangular flow angle uncorrelated with reaction plane and elliptic flow angles
  $\Longrightarrow$ due to event-by-event eccentricity fluctuations which dominate the anisotropic flows in the most central collisions

Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity η, neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = \big(e(x)+p(x)\big)u^\mu(x)u^\nu(x) - g^{\mu\nu}p(x) + \pi^{\mu\nu}.$$

$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to 2η times the shear tensor $\nabla^{\langle\mu}u^{\nu\rangle}$ on a microscopic kinetic time scale τ_π:

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) + \dots$$

where $D \equiv u^\mu\partial_\mu$ is the time derivative in the local rest frame.

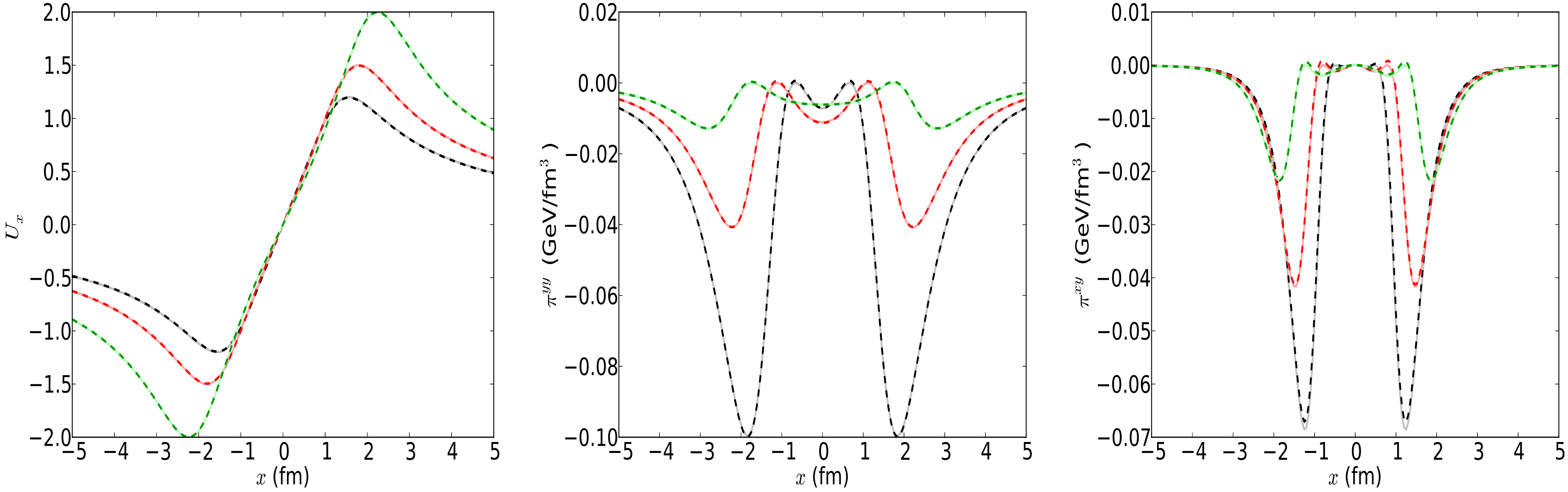
Kinetic theory relates η and τ_π, but for a strongly coupled QGP neither η nor this relation are known $\Longrightarrow$ treat η and τ_π as independent phenomenological parameters. For consistency: $\tau_\pi\theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

Numerical precision: “Gubser-Test”

Gubser (PRD82 (2010) 085027) found analytical solution for relativistic Navier-Stokes equation with conformal EOS, boost-invariant longitudinal and non-zero transverse flow, corresponding to a specific transverse temperature profile.

Marrochio, Noronha *et al.* (arXiv:1307.6130) found semianalytical generalization of this solution for Israel-Stewart theory. This solution provides a stringent test for numerical Irael-Stewart codes (very rapid and non-trivial transverse expansion!)

VISH2+1 (C. Shen, 2013)

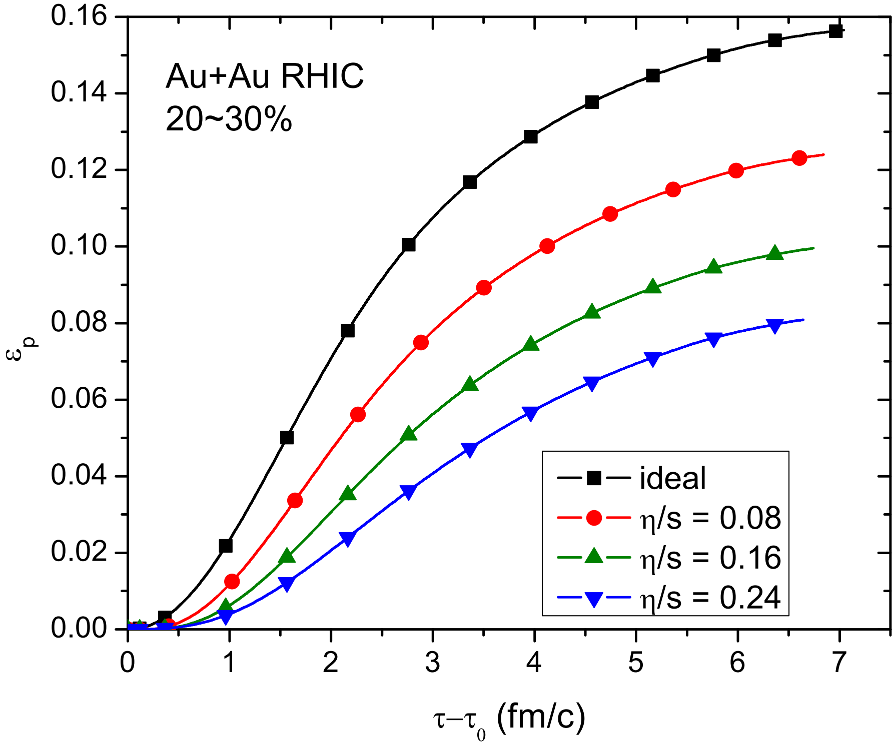


Converting initial shape fluctuations into final flow anisotropies – the QGP shear viscosity

$$(\eta/\boldsymbol{s})_{\rm QGP}$$

How to use elliptic flow for measuring $(\eta/s)_{\mathrm{QGP}}$

Hydrodynamics converts
spatial deformation of initial state $\Longrightarrow$
momentum anisotropy of final state,
through anisotropic pressure gradients



Shear viscosity degrades conversion efficiency

$$\varepsilon_x = \frac{\langle\langle y^2-x^2\rangle\rangle}{\langle\langle y^2+x^2\rangle\rangle} \Longrightarrow \varepsilon_p = \frac{\langle T^{xx}-T^{yy}\rangle}{\langle T^{xx}+T^{yy}\rangle}$$

of the fluid; the suppression of ε_p is monotonically related to η/s.

The observable that is most directly related to the total hydrodynamic momentum anisotropy ε_p is the **total (p_T-integrated) charged hadron elliptic flow v_2^{ch}**:

$$\varepsilon_p = \frac{\langle T^{xx}-T^{yy}\rangle}{\langle T^{xx}+T^{yy}\rangle} \Longleftrightarrow \frac{\sum_i \int p_T dp_T \int d\phi_p\, p_T^2 \cos(2\phi_p)\, \frac{dN_i}{dy p_T dp_T d\phi_p}}{\sum_i \int p_T dp_T \int d\phi_p\, p_T^2\, \frac{dN_i}{dy p_T dp_T d\phi_p}} \Longleftrightarrow v_2^{\mathrm{ch}}$$

How to use elliptic flow for measuring $(\eta/s)_{\rm QGP}$ (ctd.)

- **If** ε_p **saturates** before hadronization (e.g. in PbPb@LHC (?))

$\Rightarrow$ $v_2^{\rm ch} \approx$ not affected by details of hadronic rescattering below $T_{\rm c}$

but: $v_2^{(i)}(p_T)$, $\frac{dN_i}{dyd^2p_T}$ change during hadronic phase (addl. radial flow!), and these changes depend on details of the hadronic dynamics (chemical composition etc.)

$\Rightarrow$ $v_2(p_T)$ of a single particle species **not** a good starting point for extracting η/s

- **If** ε_p **does not saturate** before hadronization (e.g. AuAu@RHIC), dissipative hadronic dynamics affects not only the distribution of ε_p over hadronic species and in p_T, but even the final value of ε_p itself (from which we want to get η/s)

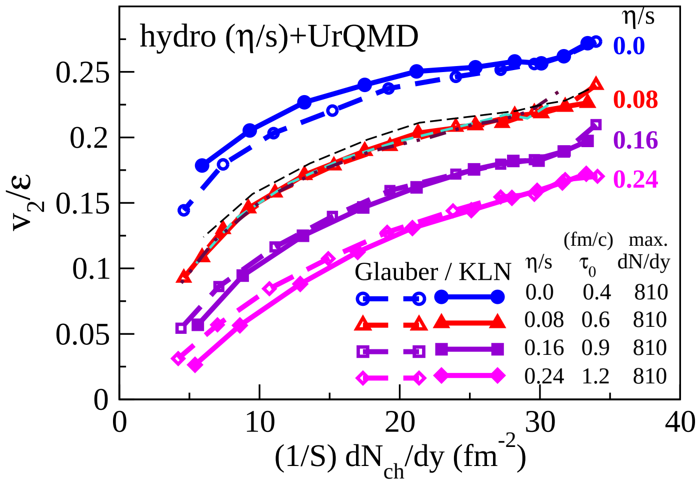
$\Rightarrow$ need hybrid code that couples viscous hydrodynamic evolution of QGP to **realistic microscopic dynamics** of late-stage hadron gas phase

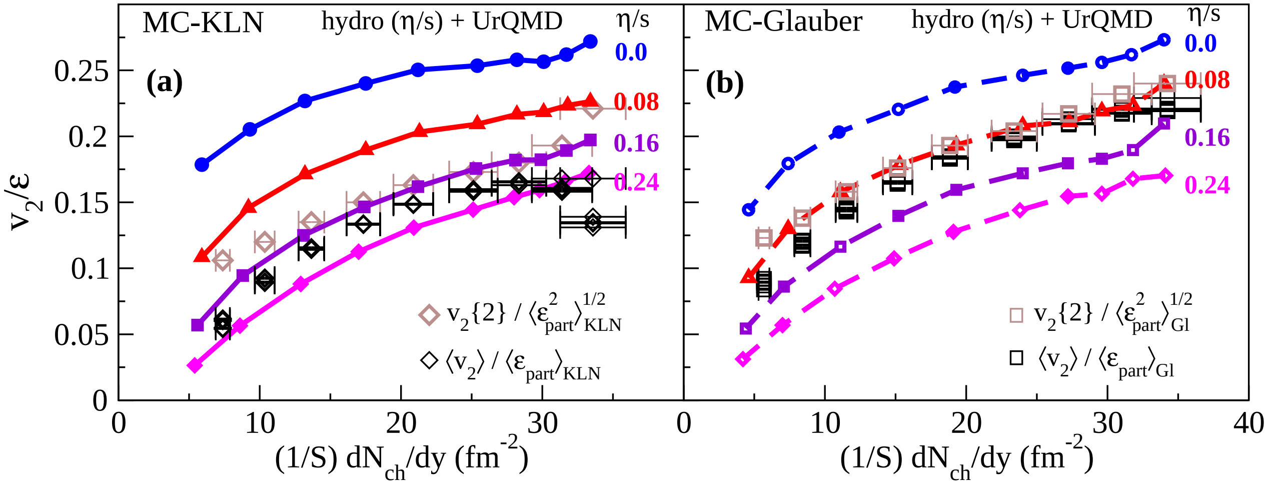
$\Rightarrow$ **VISHNU** ("Viscous Israel-Stewart Hydrodynamics 'n' UrQMD")

(Song, Bass, UH, PRC83 (2011) 024912) Note: this paper shows that UrQMD $\neq$ viscous hydro!

Extraction of $(\eta/s)_{\rm QGP}$ from AuAu@RHIC

H. Song, S.A. Bass, UH, T. Hirano, C. Shen, PRL106 (2011) 192301



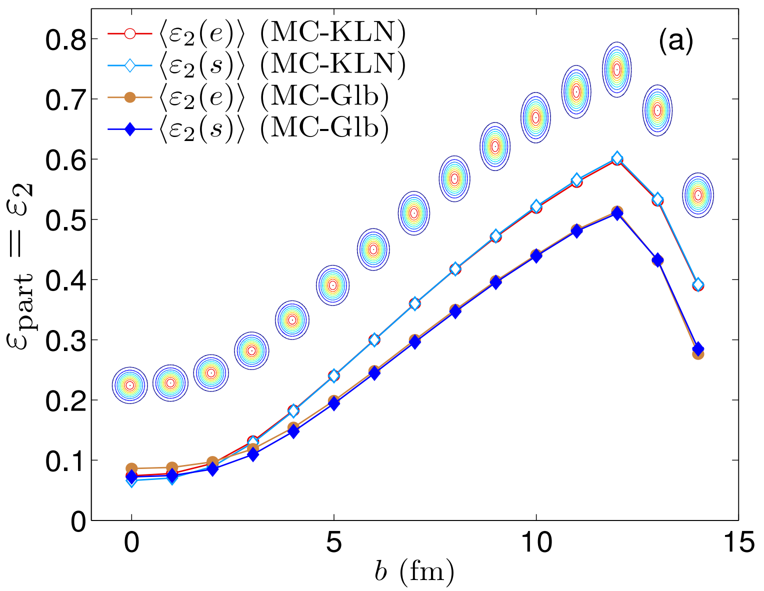


$$1 < 4\pi(\eta/s)_{\rm QGP} < 2.5$$

- All shown theoretical curves correspond to parameter sets that correctly describe centrality dependence of charged hadron production as well as p_T-spectra of charged hadrons, pions and protons at all centralities
- $v_2^{\rm ch}/\varepsilon_x$ vs. $(1/S)(dN_{\rm ch}/dy)$ is "universal", i.e. depends **only on** $\boldsymbol{\eta/s}$ but (in good approximation) not on initial-state model (Glauber vs. KLN, optical vs. MC, RP vs. PP average, etc.)
- dominant source of uncertainty: $\varepsilon_x^{\rm Gl}$ vs. $\varepsilon_x^{\rm KLN}$ ⟶
- smaller effects: *early flow* → increases $\frac{v2}{\varepsilon}$ by $\sim$ few % → larger η/s

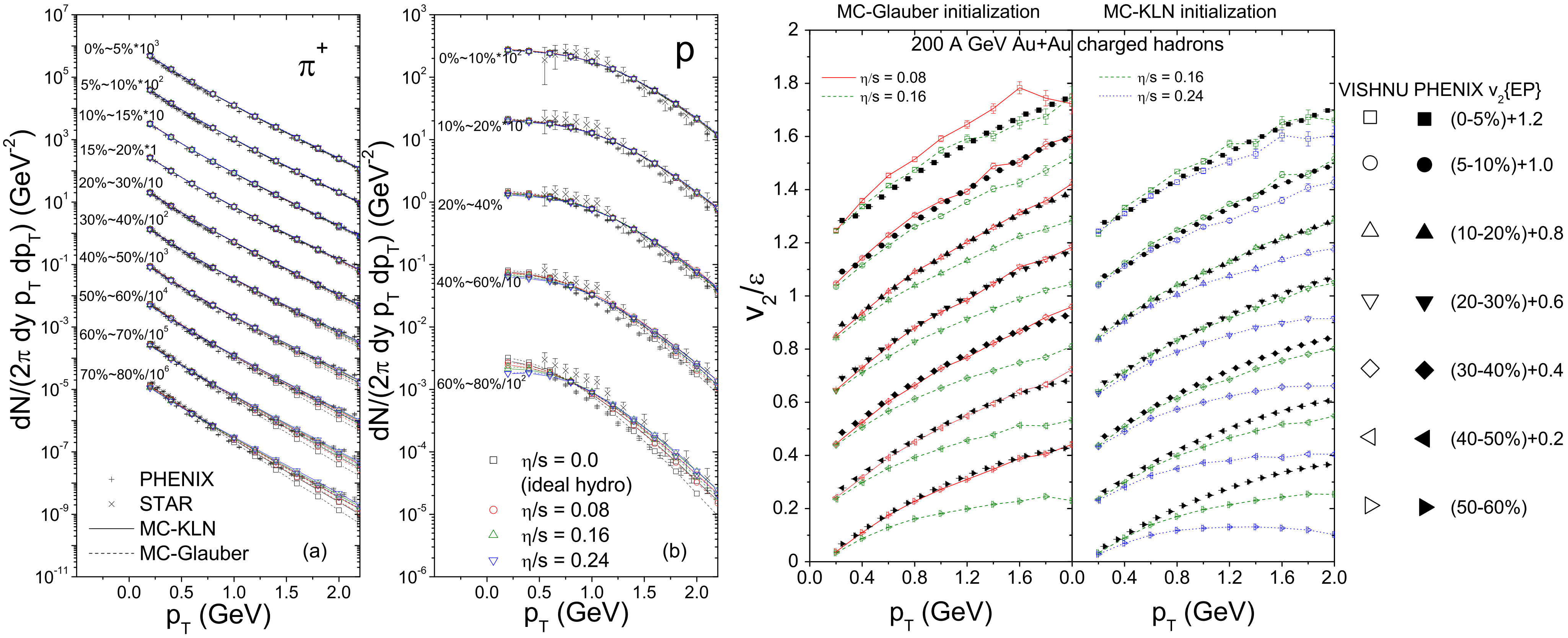
  bulk viscosity → affects $v_2^{\rm ch}(p_T)$, but $\approx$ not $v_2^{\rm ch}$

Zhi Qiu, UH, PRC84 (2011) 024911



Global description of AuAu@RHIC spectra and v_2

VISHNU (H. Song, S.A. Bass, UH, T. Hirano, C. Shen, PRC83 (2011) 054910)

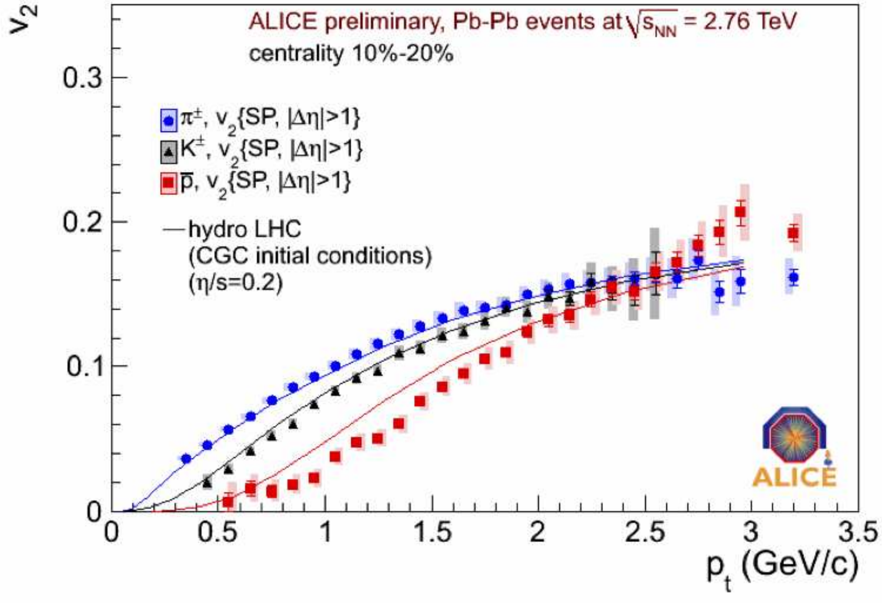


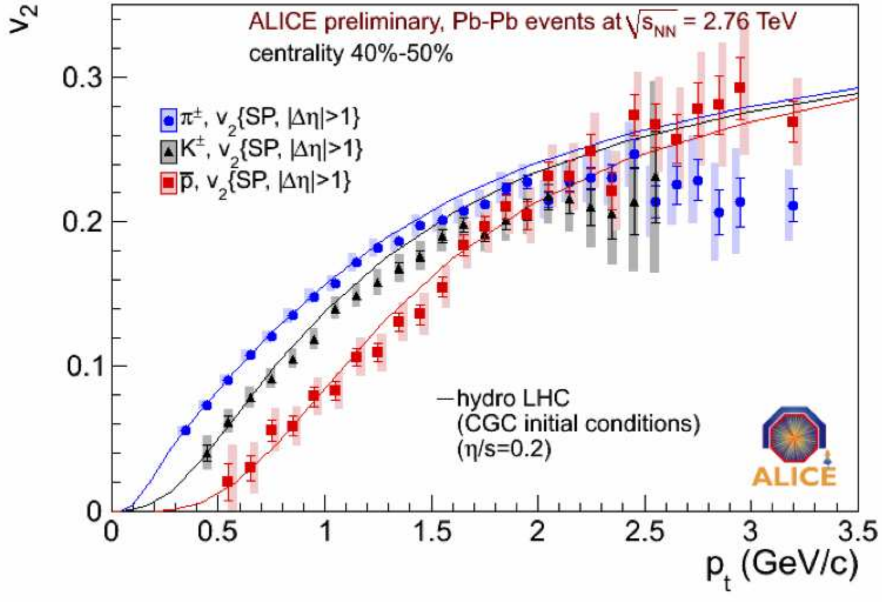
$(\eta/s)_{\rm QGP} = 0.08$ for MC-Glauber and $(\eta/s)_{\rm QGP} = 0.16$ for MC-KLN work well for charged hadron, pion and proton spectra and $v_2(p_T)$ at all collision centralities

Successful prediction of $v_2(p_T)$ for identified hadrons in PbPb@LHC

Data: ALICE, Quark Matter 2011

Prediction: Shen et al., PRC84 (2011) 044903 (VISH2+1)





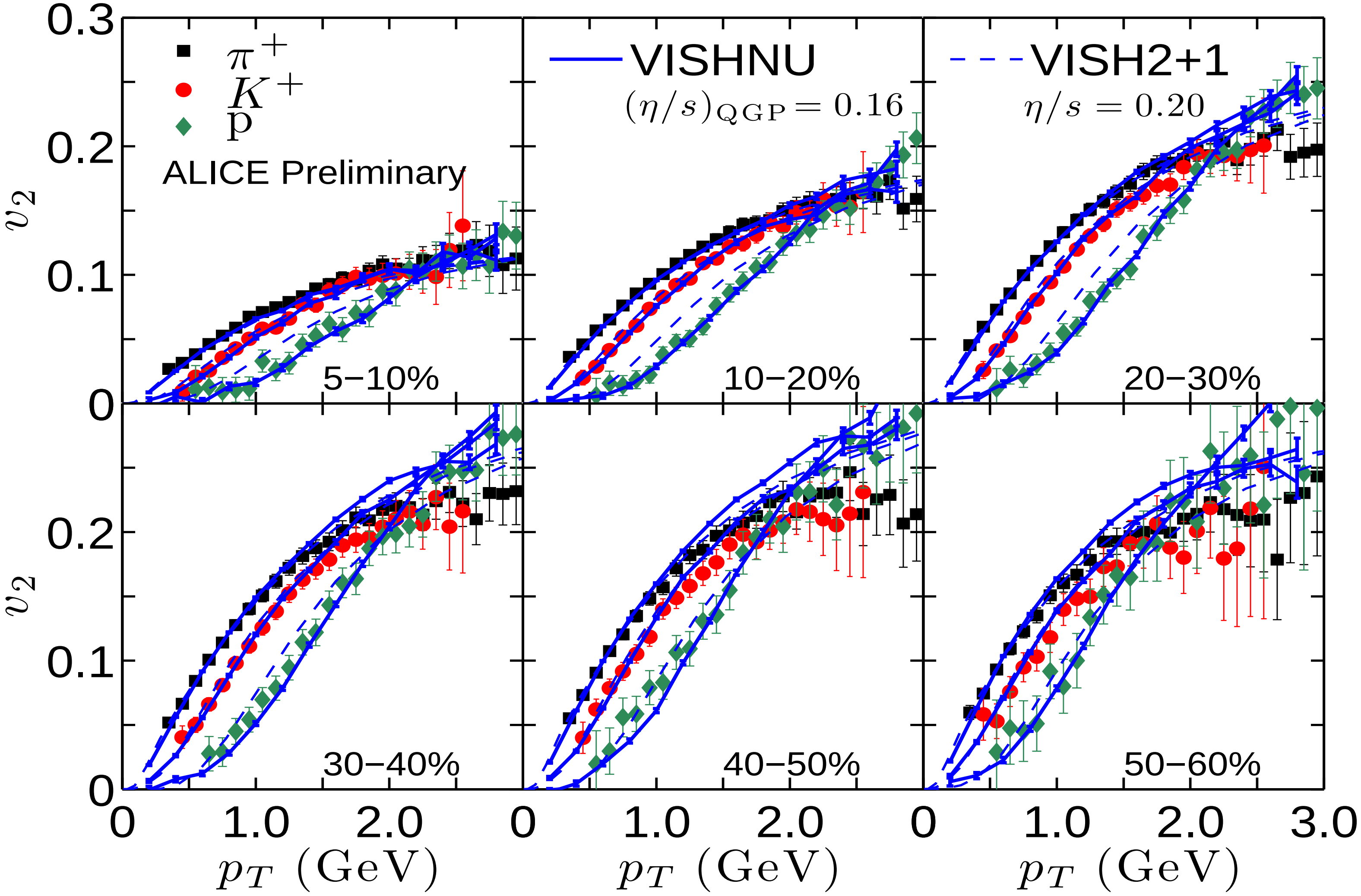
Perfect fit in semi-peripheral collisions!

The problem with insufficient proton radial flow exists only in more central collisions

Adding the hadronic cascade (VISHNU) helps:

$v_2(p_T)$ in PbPb@LHC: ALICE vs. VISHNU

Data: ALICE, preliminary (Snellings, Krzewicki, Quark Matter 2011)
Dashed lines: Shen et al., PRC84 (2011) 044903 (VISH2+1, MC-KLN, $(\eta/s)_{\rm QGP}$=0.2)
Solid lines: Song, Shen, UH 2011 (VISHNU, MC-KLN, $(\eta/s)_{\rm QGP}$=0.16)

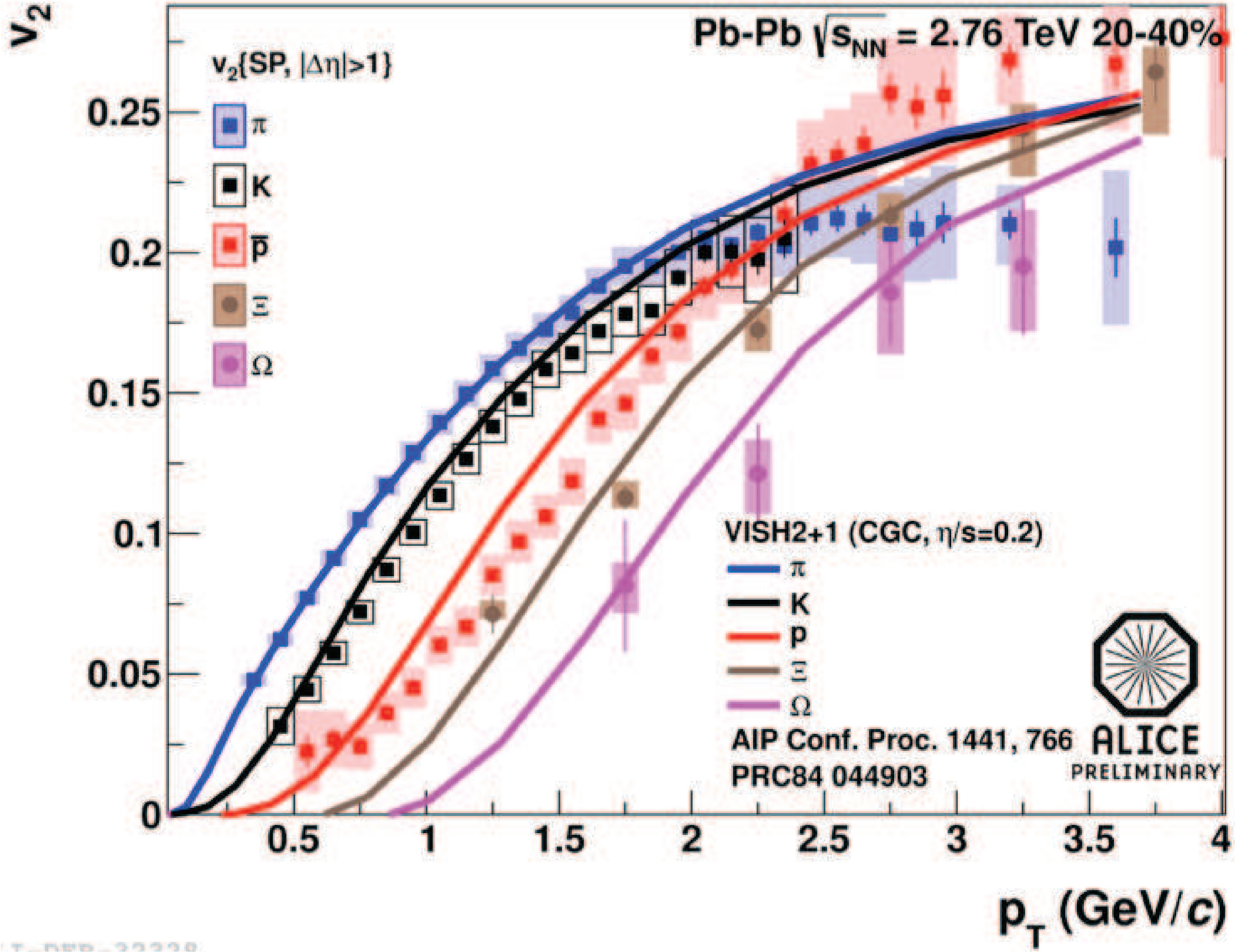


VISHNU yields correct magnitude and centrality dependence of $v_2(p_T)$ for pions, kaons **and protons!**

Same $(\eta/s)_{\rm QGP} = 0.16$ (for MC-KLN) at RHIC and LHC!

Successful prediction of $v_2(p_T)$ for identified hadrons in PbPb@LHC (II)

Data: ALICE, Quark Matter 2012 Prediction: Shen et al., PRC84 (2011) 044903 (VISH2+1)



Radial flow pushes v_2 for heavier hadrons to larger p_T

Theory curves are true predictions, without any parameter adjustment

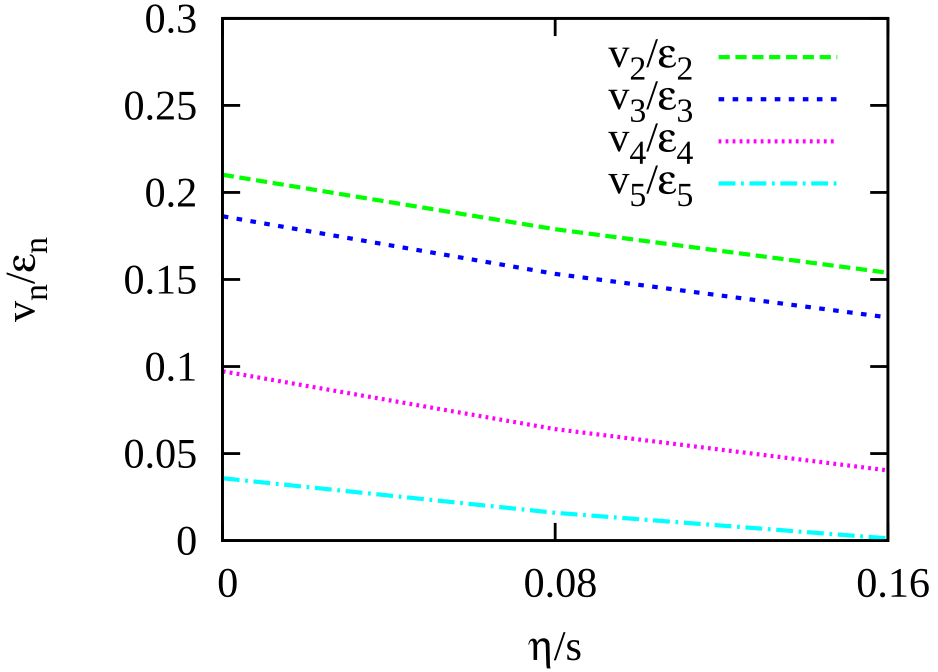
Back to the “elephant in the room”: How to eliminate the large model uncertainty in the initial eccentricity?

Two observations:

I. Shear viscosity suppresses higher flow harmonics more strongly

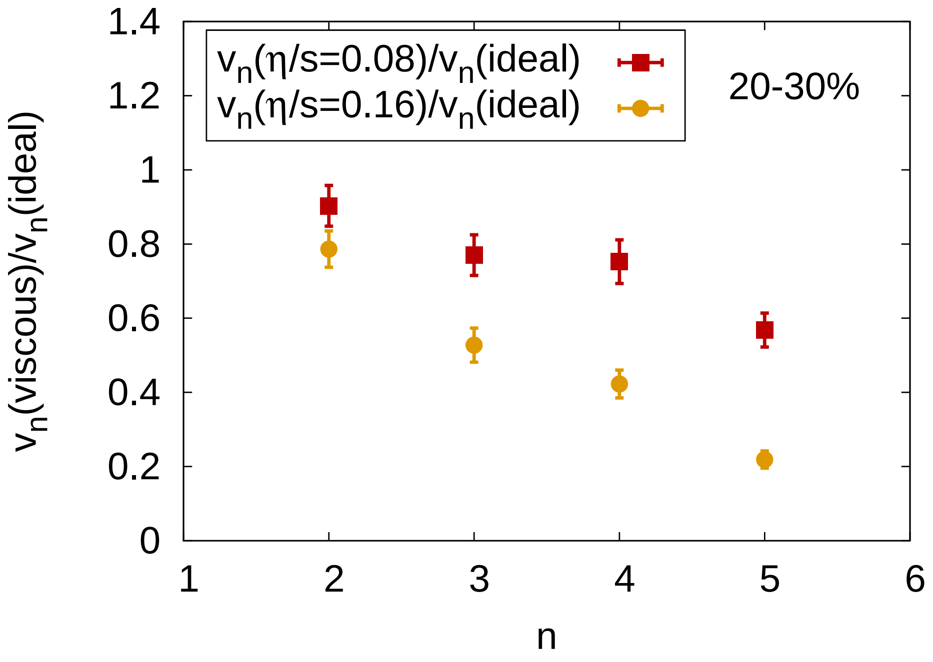
Alver et al., PRC82 (2010) 034913

(averaged initial conditions)



Schenke et al., arXiv:1109.6289

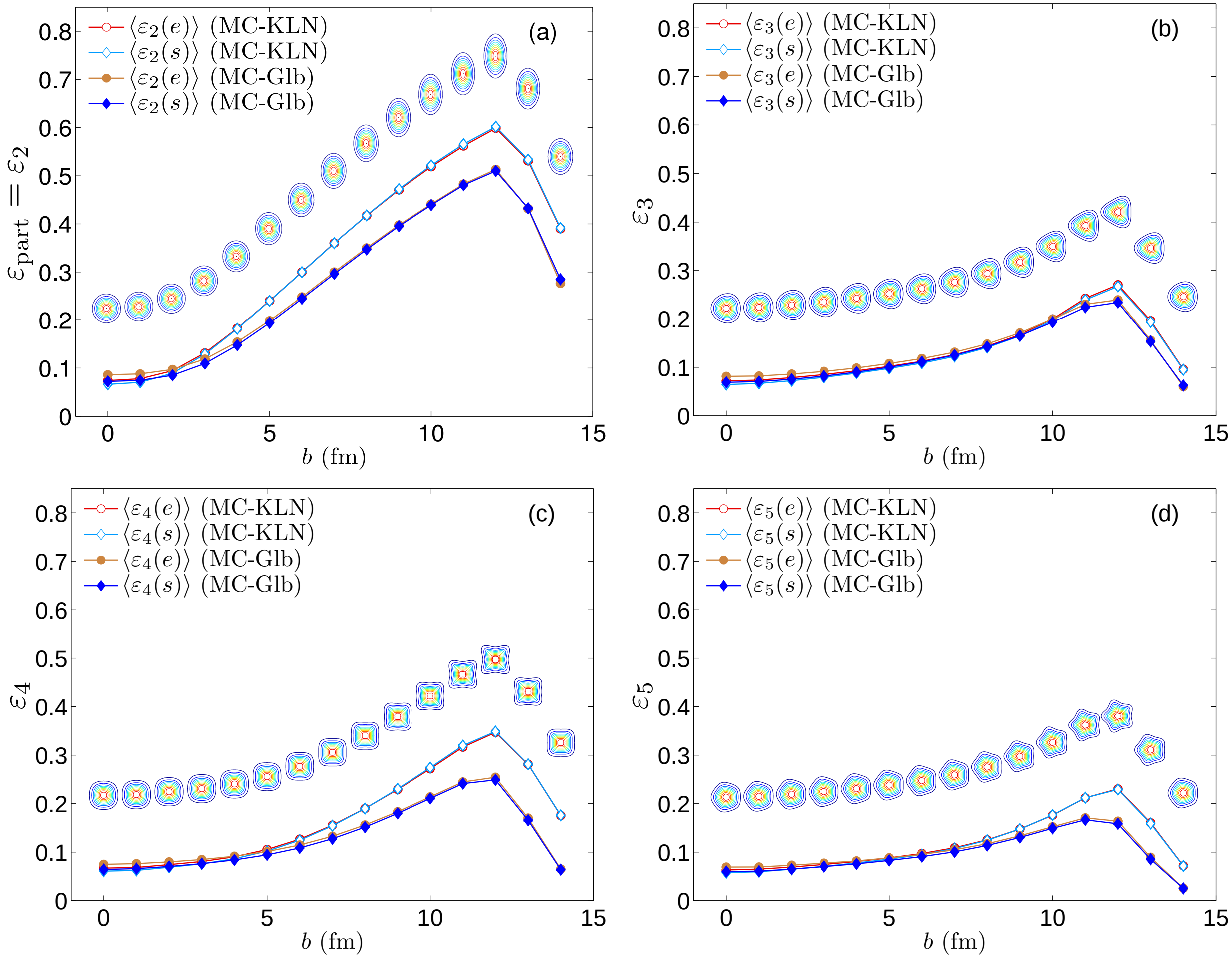
(event-by-event hydro)



$\Longrightarrow$ **Idea:** Use simultaneous analysis of elliptic and triangular flow to constrain initial state models (see also Bhalerao, Luzum Ollitrault, PRC 84 (2011) 034910)

Two observations:
II. ε_3 is $\approx$ model independent

Zhi Qiu, UH, PRC84 (2011) 024911



Initial eccentricities ε_n and angles ψ_n:

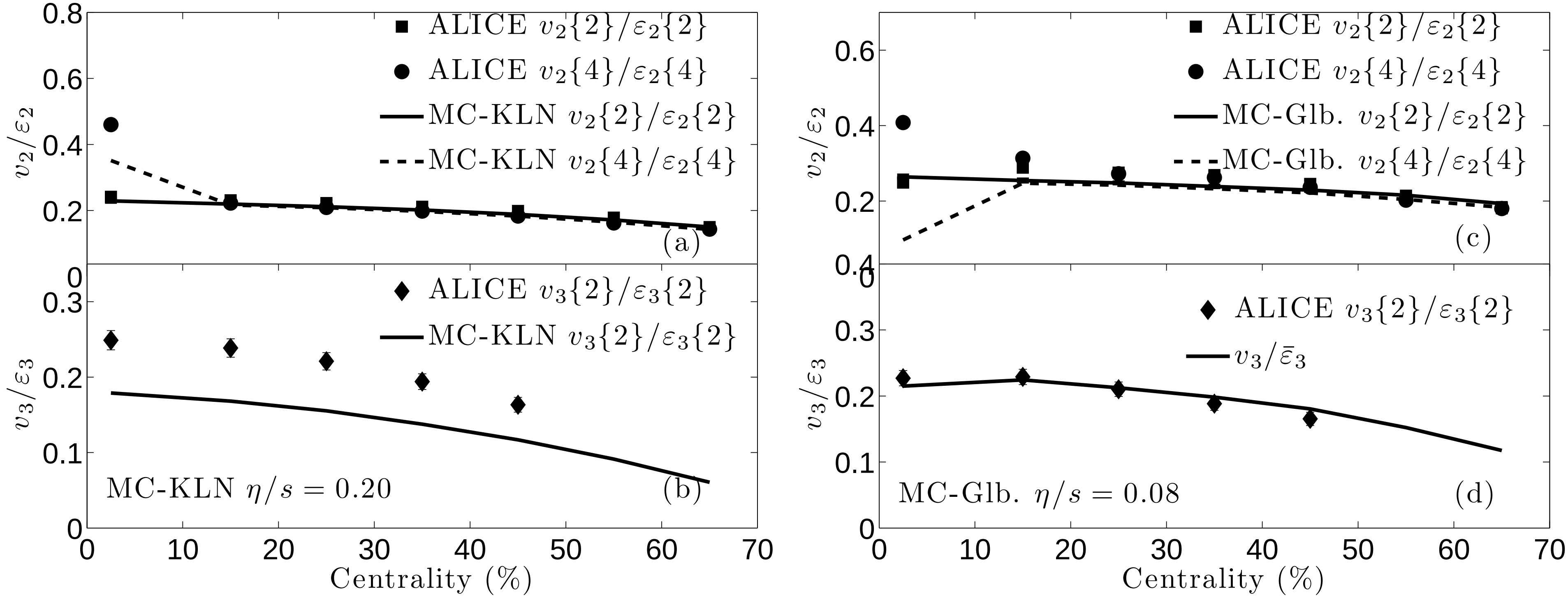
$$\varepsilon_n e^{in\psi_n} = -\frac{\int r dr d\phi\, r^2 e^{in\phi}\, e(r,\phi)}{\int r dr d\phi\, r^2\, e(r,\phi)}$$

- MC-KLN has larger ε_2 and ε_4, but similar ε_5 and almost identical ε_3 as MC-Glauber
- Angles of ε_2 and ε_4 are correlated with reaction plane by geometry, whereas those of ε_3 and ε_5 are random (purely fluctuation-driven)
- While v_4 and v_5 have mode-coupling contributions from ε_2, v_3 is almost pure response to ε_3 and $v_3/\varepsilon_3 \approx$const. over a wide range of centralities

$\Longrightarrow$ **Idea:** Use total charged hadron v_3^{ch} to determine $(\eta/s)_{\text{QGP}}$,
then check v_2^{ch} to distinguish between MC-KLN and MC-Glauber!

Combined v_2 & v_3 analysis: η/s is small!

Zhi Qiu, C. Shen, UH, PLB707 (2012) 151 and QM2012 (e-by-e VISH2+1)



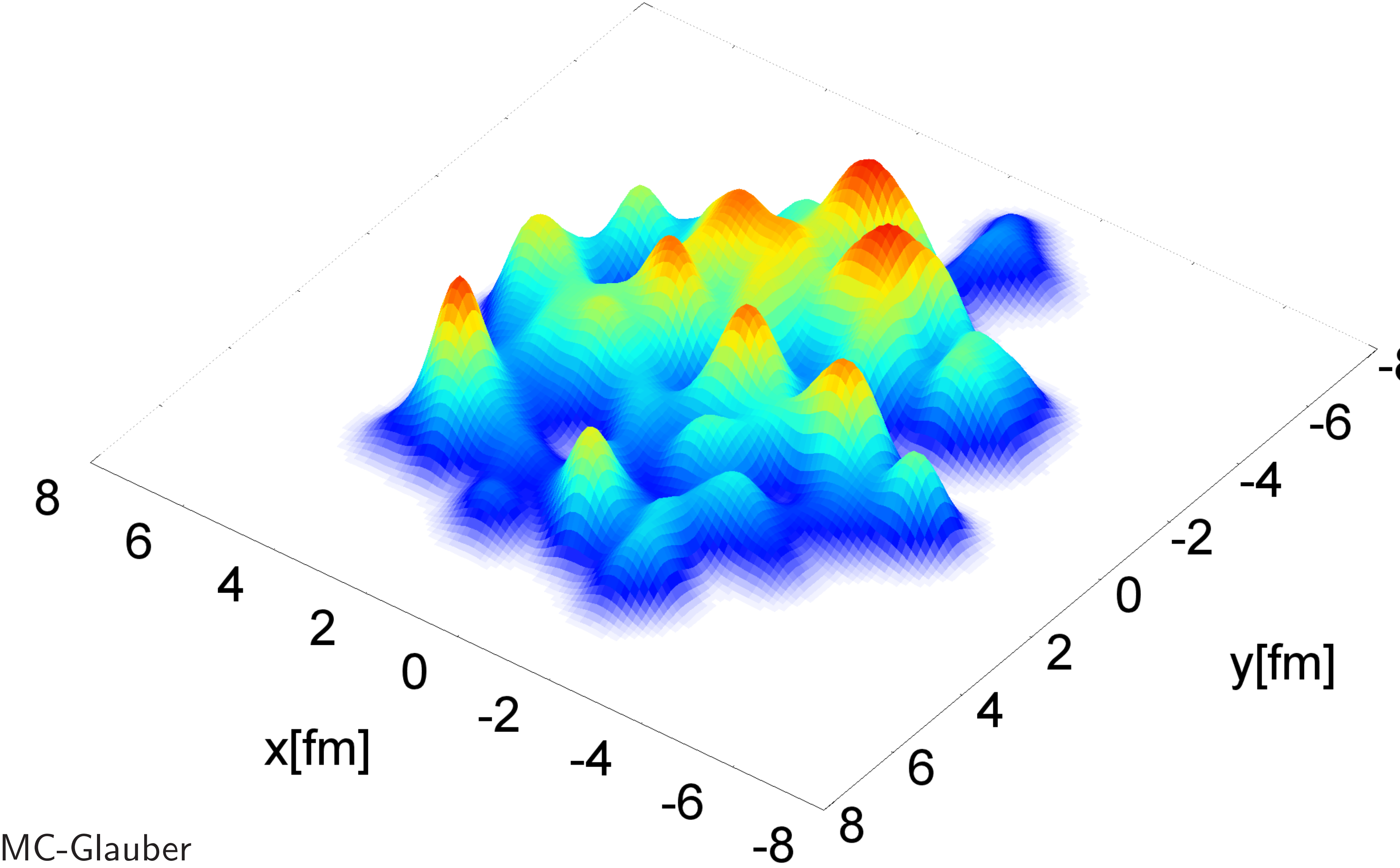
- Both MC-KLN with $\eta/s = 0.2$ and MC-Glauber with $\eta/s = 0.08$ give very good description of v_2/ε_2 at all centralities.
- **Only $\eta/s = 0.08$ (with MC-Glauber initial conditions) describes v_3/ε_3!**

  PHENIX, comparing to calculations by Alver et al. (PRC82 (2010) 034913), come to similar conclusions at RHIC energies (Adare et al., arXiv:1105.3928, and Lacey et al., arXiv:1108.0457)
- **Large v_3 measured at RHIC and LHC requires small $(\eta/s)_{\mathrm{QGP}} \simeq 1/(4\pi)$** unless the fluctuations in these models are completely wrong and ε_3 is really 50% larger than these models predict!

Sub-nucleonic fluctuations

Adding sub-nucleonic quantum fluctuations

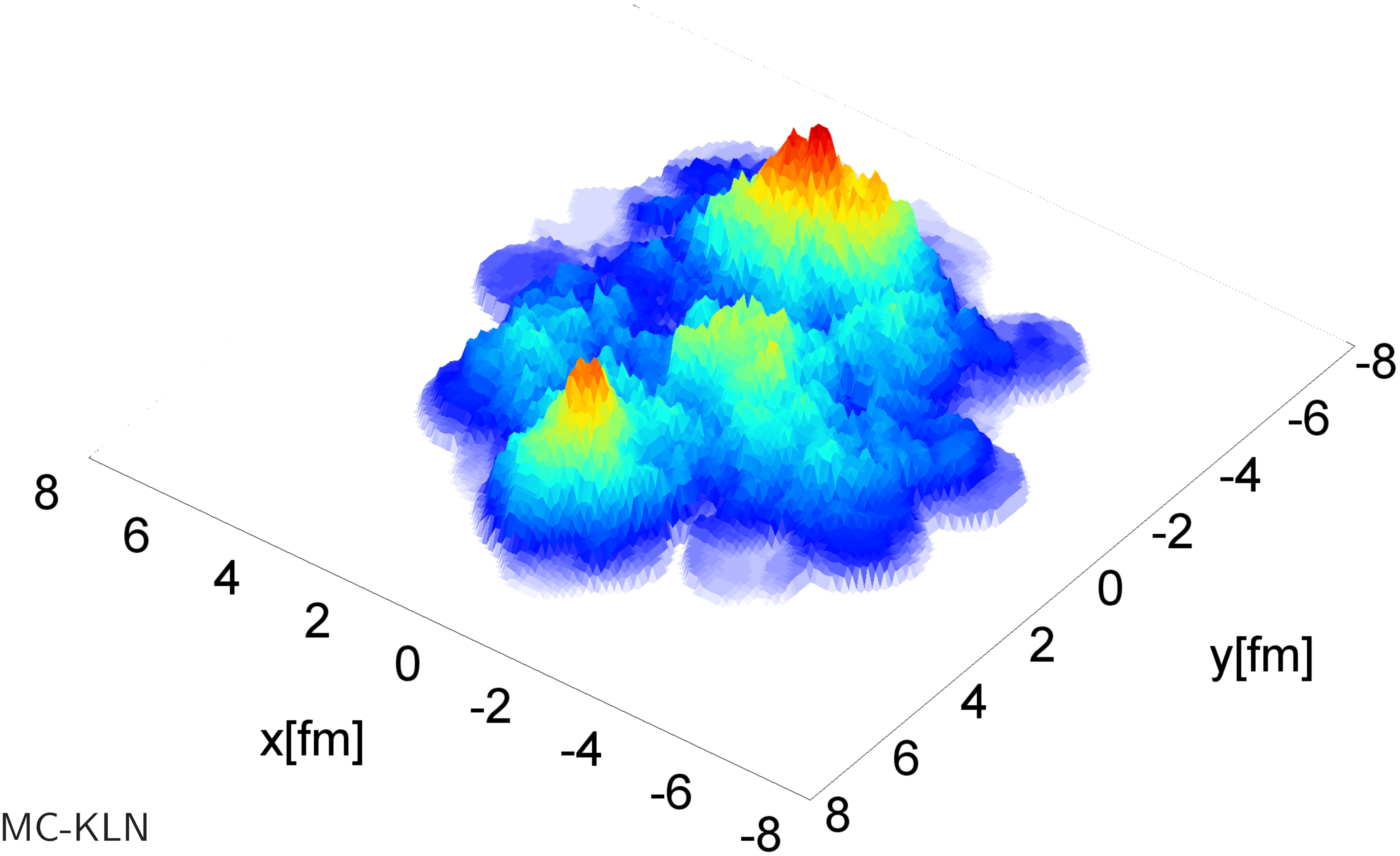
Schenke, Tribedy, Venugopalan, PRL108, 252301 (2012)



MC-Glauber

Adding sub-nucleonic quantum fluctuations

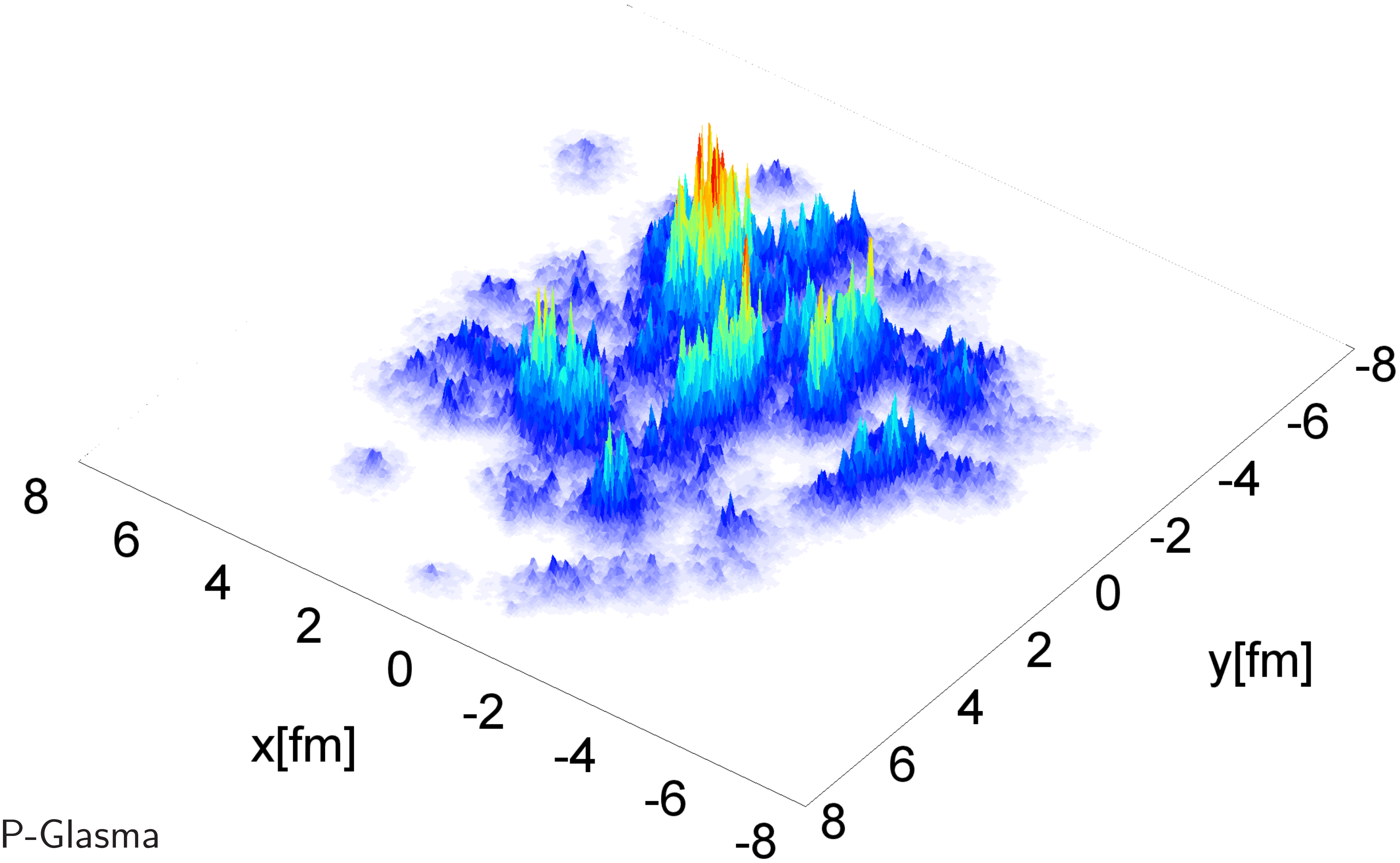
Schenke, Tribedy, Venugopalan, PRL108, 252301 (2012)



MC-KLN

Adding sub-nucleonic quantum fluctuations

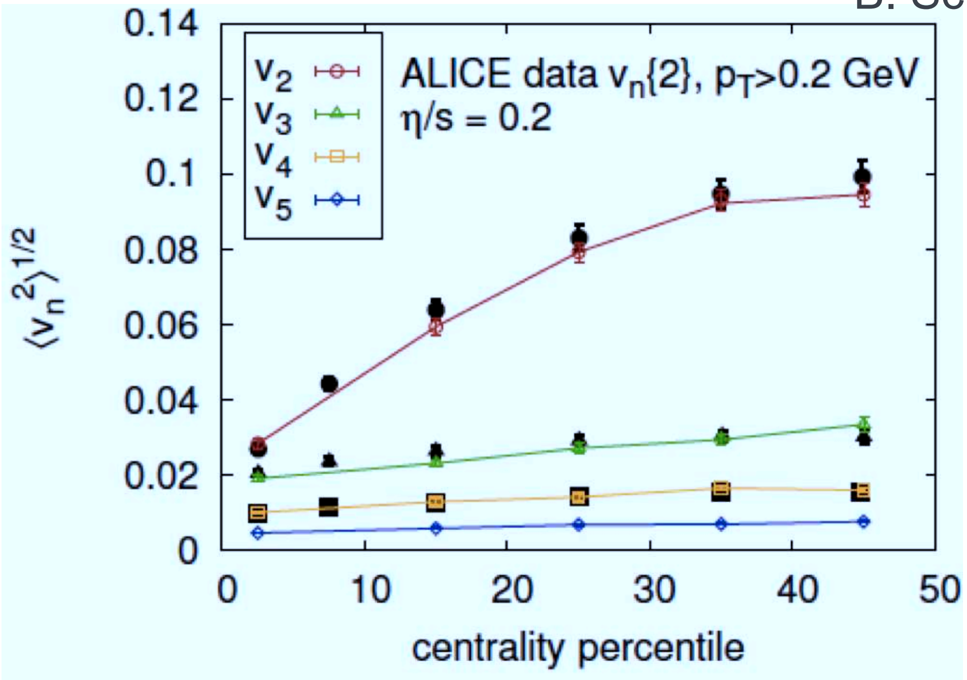
Schenke, Tribedy, Venugopalan, PRL108, 252301 (2012)

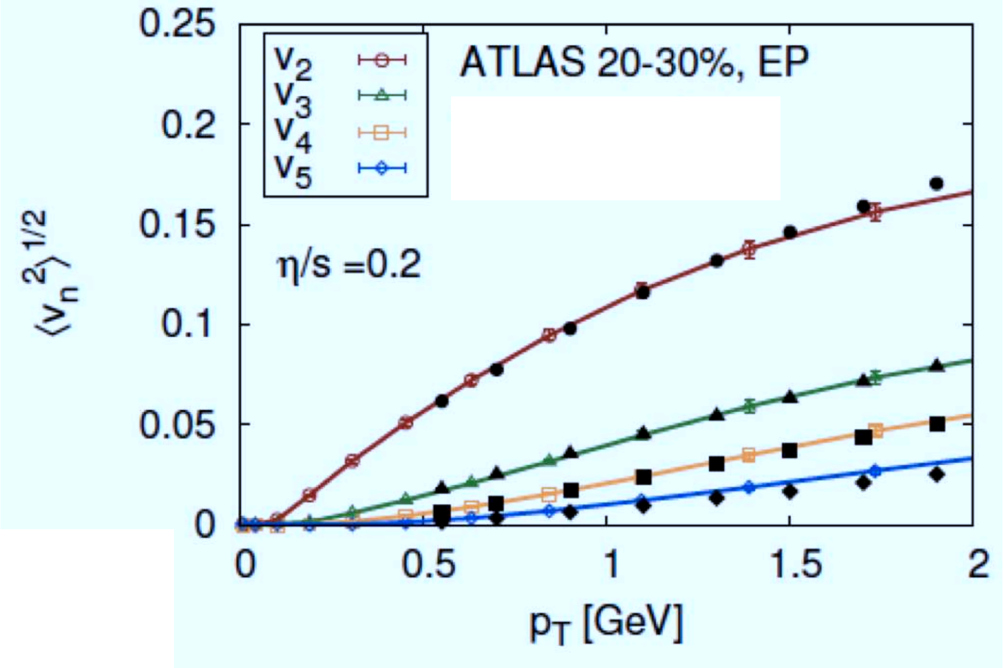


IP-Glasma

Towards a Standard Model of the Little Bang

B. Schenke: QM2012





Schenke, Tribedy, Venugopalan,
Phys.Rev.Lett. 108:25231 (2012)

With inclusion of sub-nucleonic quantum fluctuations and pre-equilbrium dynamics of gluon fields:

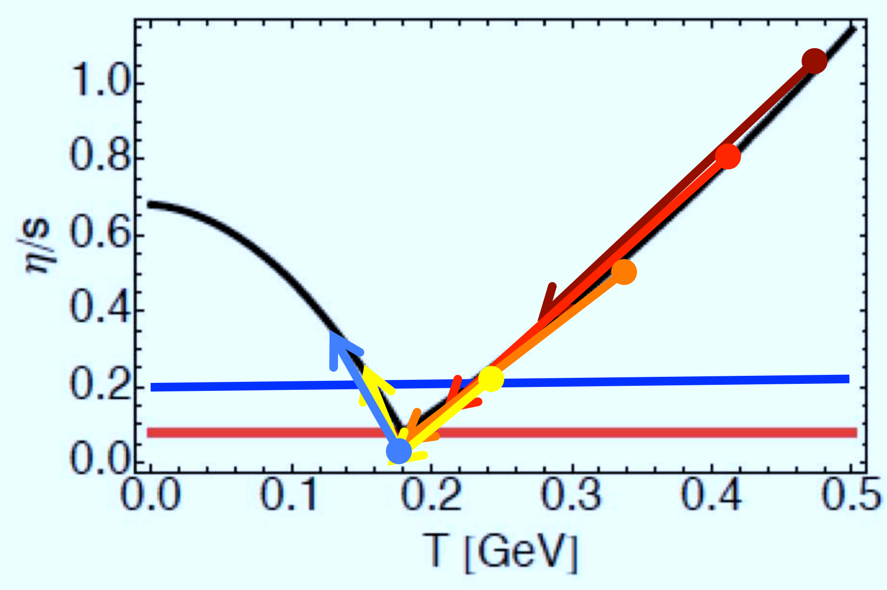
→ outstanding agreement between data and model

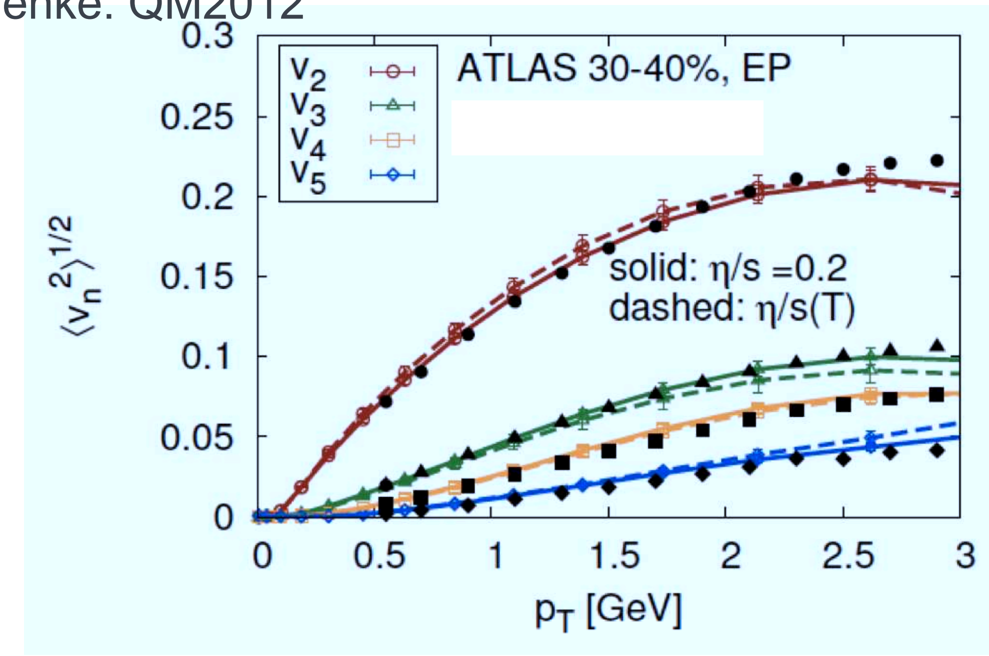
Rapid convergence on a standard model of the Little Bang!

Perfect liquidity reveals in the final state initial-state gluon field correlations of size $1/Q_s$ (sub-hadronic)!

What We Don’t Know

B. Schenke: QM2012



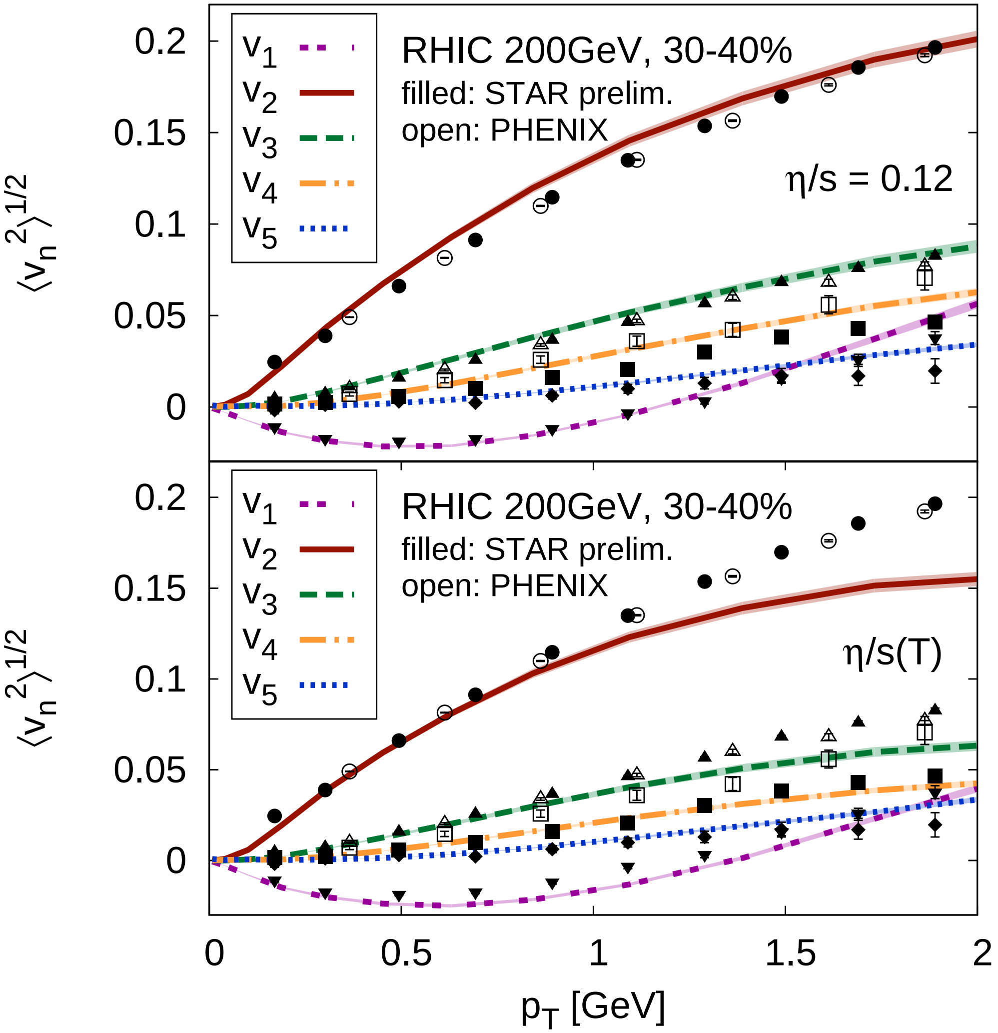


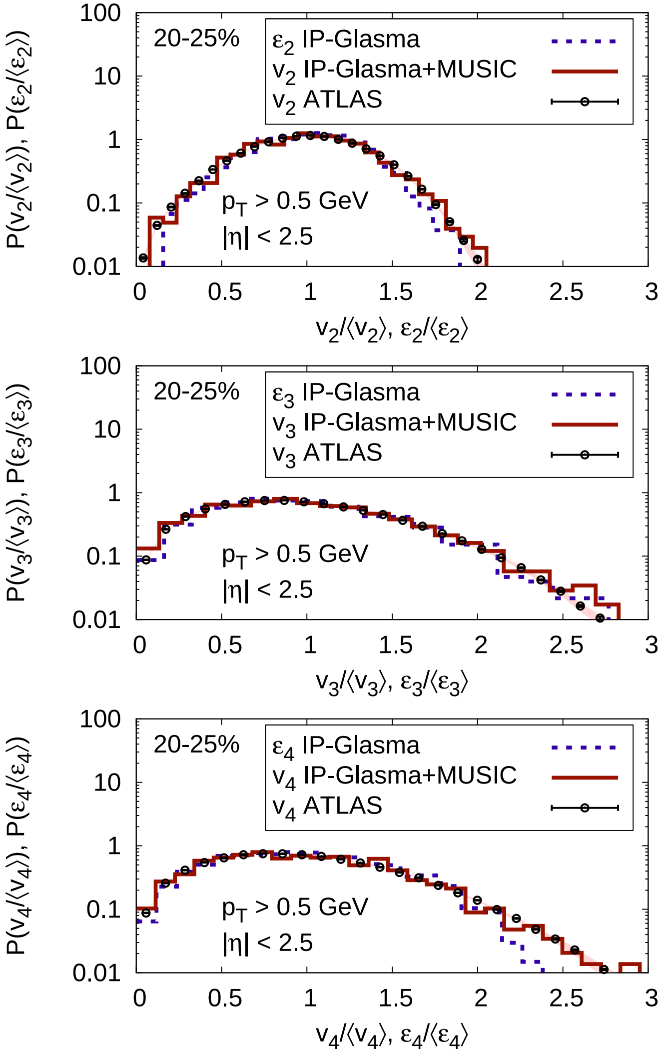
Model doesn’t distinguish between a constant η/s of 0.2 or a temperature dependent η/s with a minimum of 1/4π

Need both RHIC and LHC to sort this out!

Other successes of the Little Bang Standard Model

Gale, Jeon, Schenke, Tribedy, Venugopalan, arXiv:1209.6330 (PRL 2012)

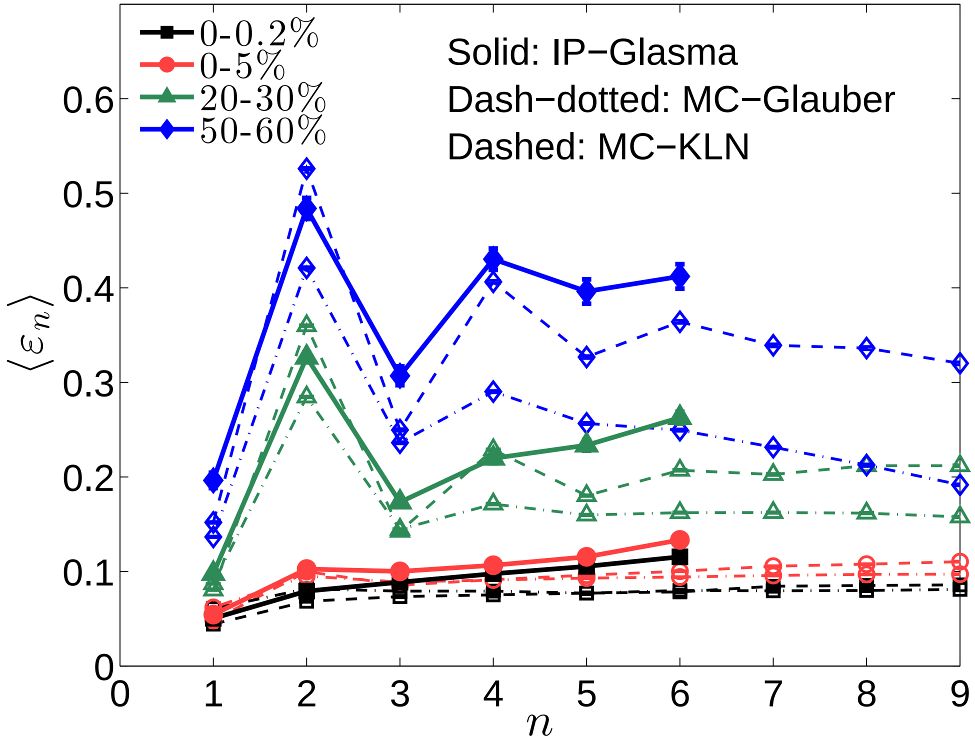




- Model describes RHIC data with lower effective specific shear viscosity $\eta/s = 0.12$
- In contrast to MC-Glauber and MC-KLN, IP-Sat initial conditions correctly reproduce the final flow fluctuation spectrum, generated from initial shape fluctuations by viscous hydrodynamics

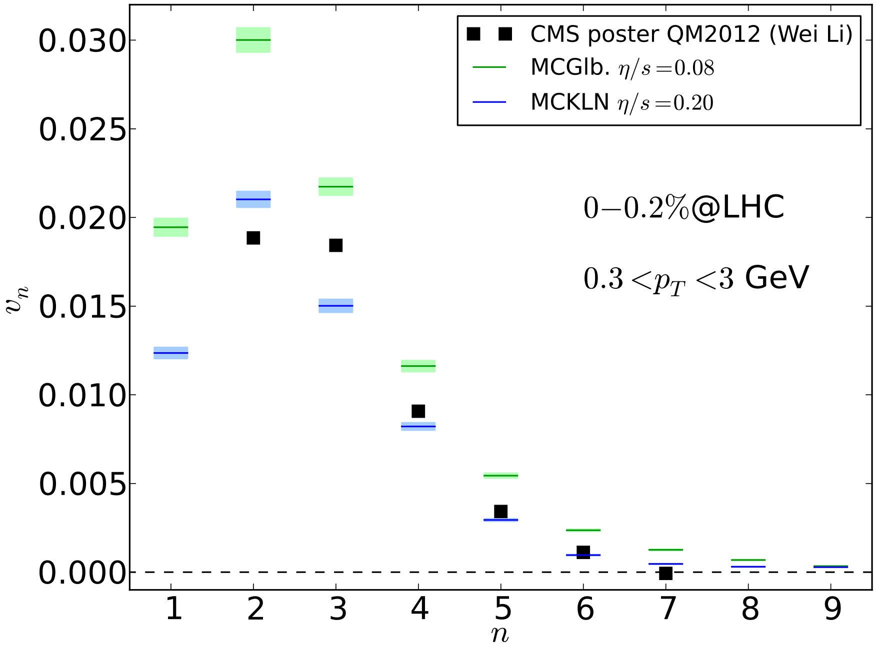
The Little Bang fluctuation power spectrum: initial vs. final

Little Bang density power spectra



Flow power spectrum for ultracentral PbPb Little Bangs

(Data: CMS, Quark Matter 2012; Theory: OSU 2013)

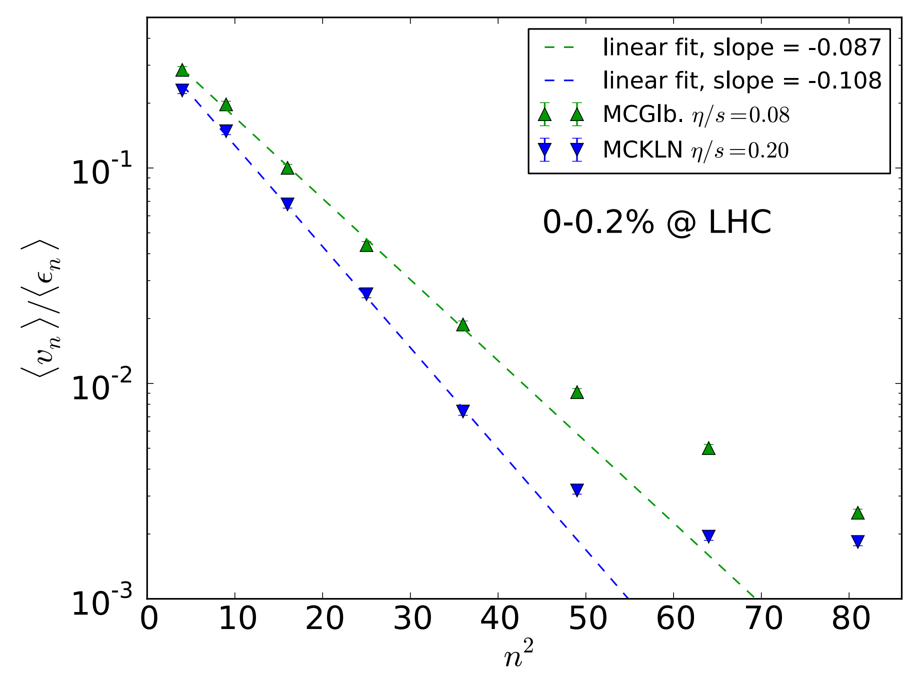


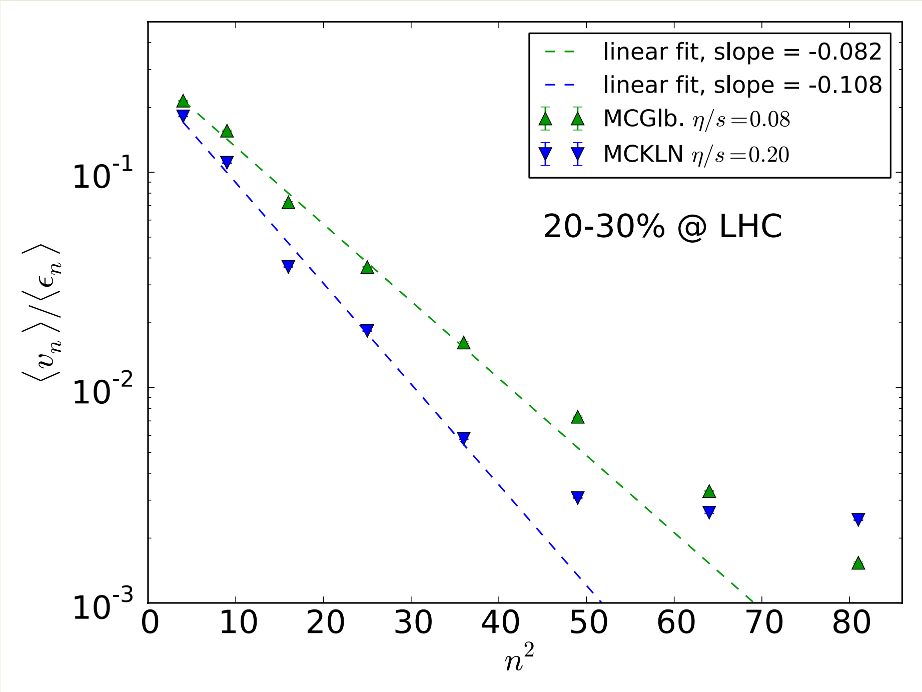
Higher flow harmonics get suppressed by shear viscosity

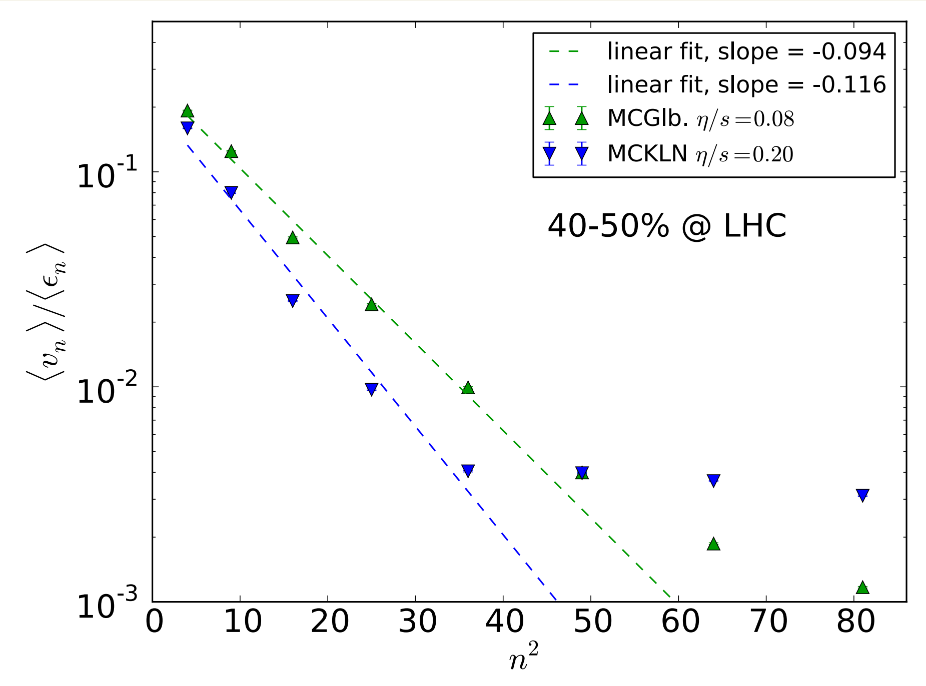
Neither MC-Glb nor MC-KLN gives the correct initial power spectrum! † R.I.P.

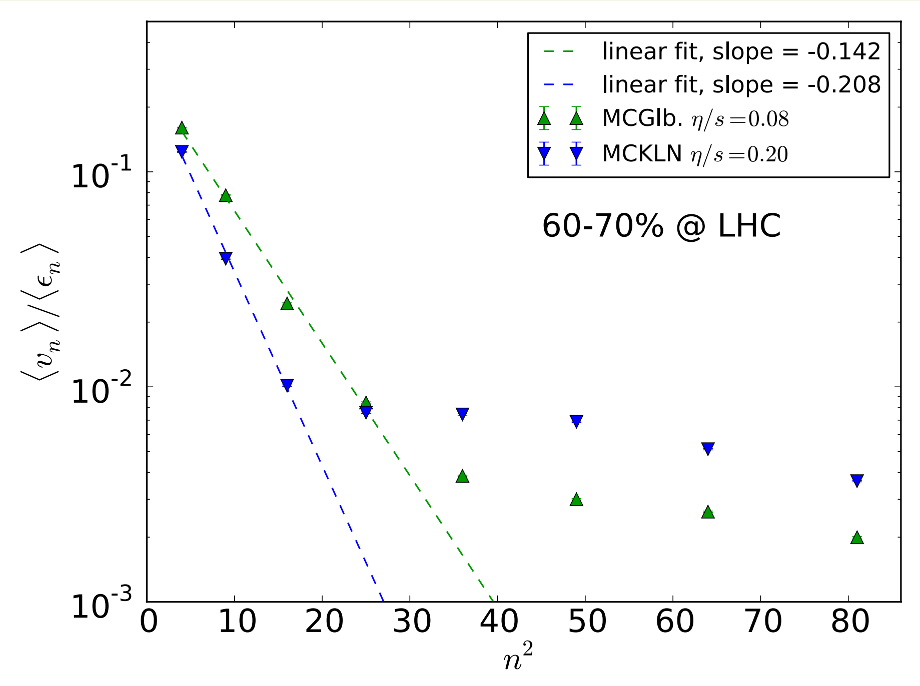
A detailed study of fluctuations is a powerful discriminator between models!

“Acoustic scaling” in Pb+Pb @ LHC (Chun Shen)









Conclusions

- Quark-Gluon Plasma is by far the hottest and densest form of matter ever observed in the laboratory. Its properties and interactions are controlled by QCD, not QED.
- It is a liquid with almost perfect fluidity. Its specific shear viscosity at RHIC and LHC energies is

  $$(\eta/s)_{\mathrm{QGP}}(T_{\mathrm{c}}<T<2T_{\mathrm{c}}) = \frac{2}{4\pi} \pm 50\%$$

  This is significantly below that of any other known real fluid.
  Precision comparison of harmonic flow coefficients at RHIC and LHC provides first serious indications for a moderate increase of the specific QGP shear viscosity between $2T_{\mathrm{c}}$ and $3T_{\mathrm{c}}$.
- Viscous relativistic hydrodynamics provides a quantitative description of QGP evolution.
- By coupling viscous fluid dynamics for the QGP stage to microscopic evolution models of the dense early pre-equilibrium and dilute late hadronic freeze-out stages, a complete dynamical description of the strongly interacting matter created in ultra-relativistic heavy-ion collisions has been achieved. This dynamical theory has made successful predictions for the first Pb+Pb collisions at the LHC that were quantitatively precise and non-trivial (in the sense that they disagreed with other predictions that were falsified by the data).
- The Color Glass Condensate theory (IP-Sat model) appears to give the correct spectrum of initial-state gluon field fluctuations.
- A large set of flow fluctuation observables, so far only partially explored, (over)constrains this initial fluctuation spectrum.

$\Longrightarrow$ We are rapidly converging on the Standard Model for the Little Bang

Supplements

Single event anisotropic flow coefficients

In a single event, the specific initial density profile results in a set of complex, y- and p_T-dependent flow coefficients (we'll suppress the y-dependence):

$$V_n = v_n e^{in\Psi_n} := \frac{\int p_T dp_T d\phi \, e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\int p_T dp_T d\phi \, \frac{dN}{dy p_T dp_T d\phi}} \equiv \{e^{in\phi}\},$$

$$V_n(p_T) = v_n(p_T) e^{in\Psi_n(p_T)} := \frac{\int d\phi \, e^{in\phi} \frac{dN}{dy p_T dp_T d\phi}}{\int d\phi \, \frac{dN}{dy p_T dp_T d\phi}} \equiv \{e^{in\phi}\}_{p_T}.$$

Together with the azimuthally averaged spectrum, these completely characterize the measurable single-particle information for that event:

$$\frac{dN}{dy\,d\phi} = \frac{1}{2\pi}\frac{dN}{dy}\left(1 + 2\sum_{n=1}^{\infty} v_n \,\cos[n(\phi - \Psi_n)]\right),$$

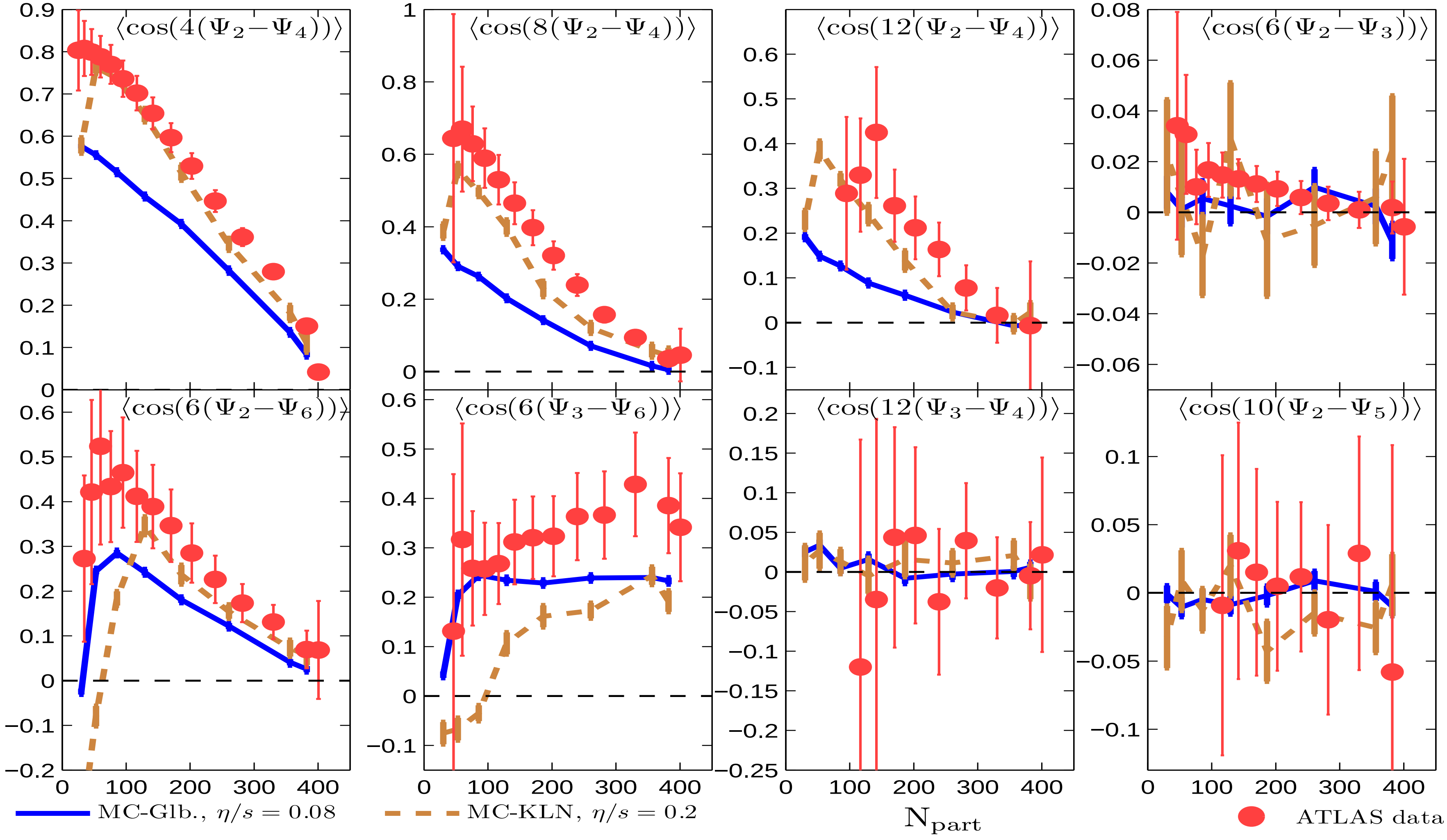
$$\frac{dN}{dy\,p_T\,dp_T\,d\phi} = \frac{1}{2\pi}\frac{dN}{dy\,p_T\,dp_T}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T) \,\cos[n(\phi - \Psi_n(p_T))]\right).$$

- Both the magnitude v_n and the direction Ψ_n ("flow angle") depend on p_T.
- v_n, Ψ_n, $v_n(p_T)$, $\Psi_n(p_T)$ **all fluctuate from event to event.**
- $\Psi_n(p_T) - \Psi_n$ fluctuates from event to event.

Higher order event plane correlations in PbPb@LHC

Data: ATLAS Coll., J. Jia et al., Hard Probes 2012

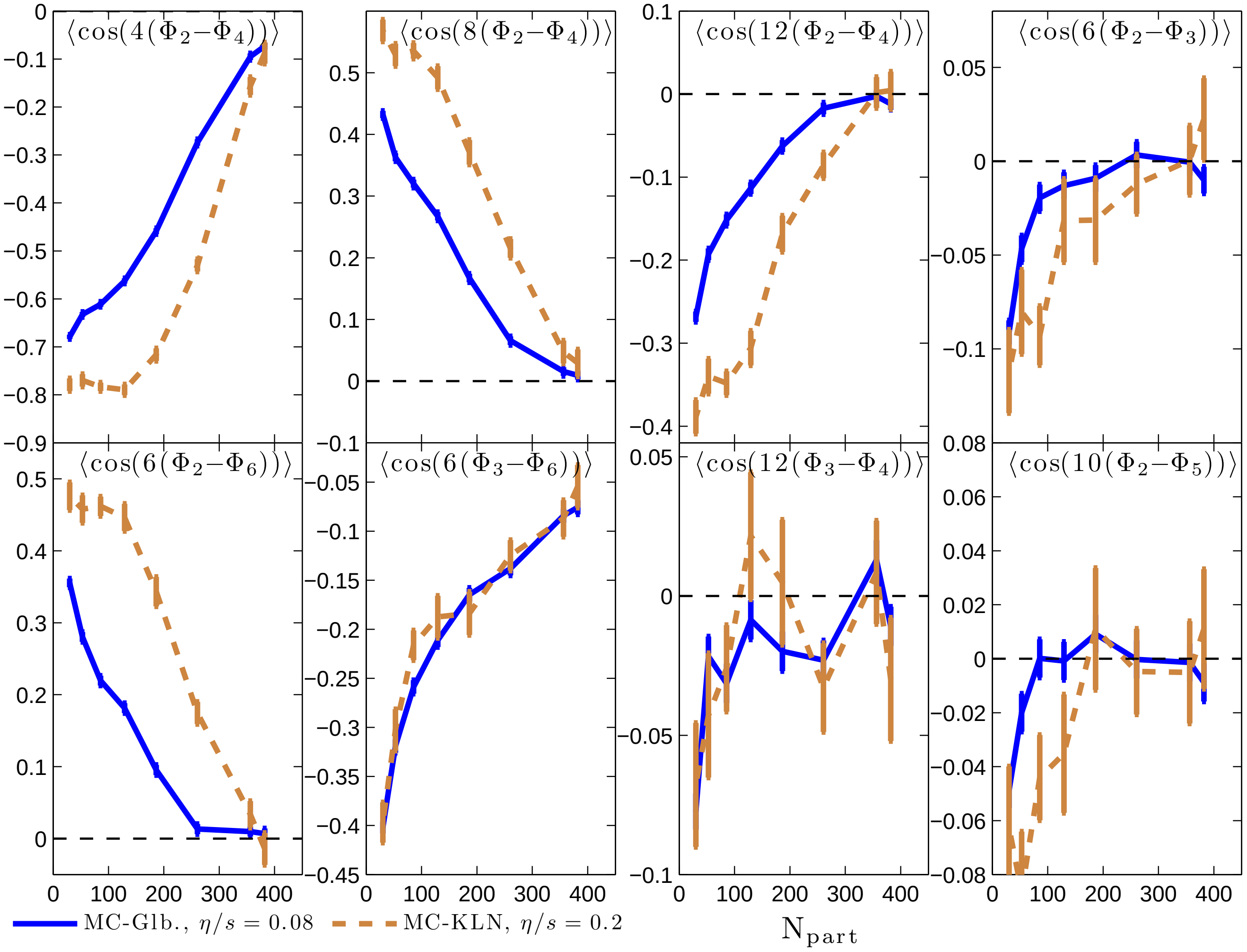
Event-by-event hydrodynamics: Zhi Qiu, UH, PLB 717 (2012) 261 (VISH2+1)



VISH2+1 reproduces qualitatively the centrality dependence of all measured event-plane correlations

Higher order event plane correlations in PbPb@LHC

Zhi Qiu, UH, PLB 717 (2012) 261



Initial-state participant plane correlations disagree with final-state flow-plane correlations

$\Longrightarrow$ Nonlinear mode coupling through hydrodynamic evolution essential to describe the data!

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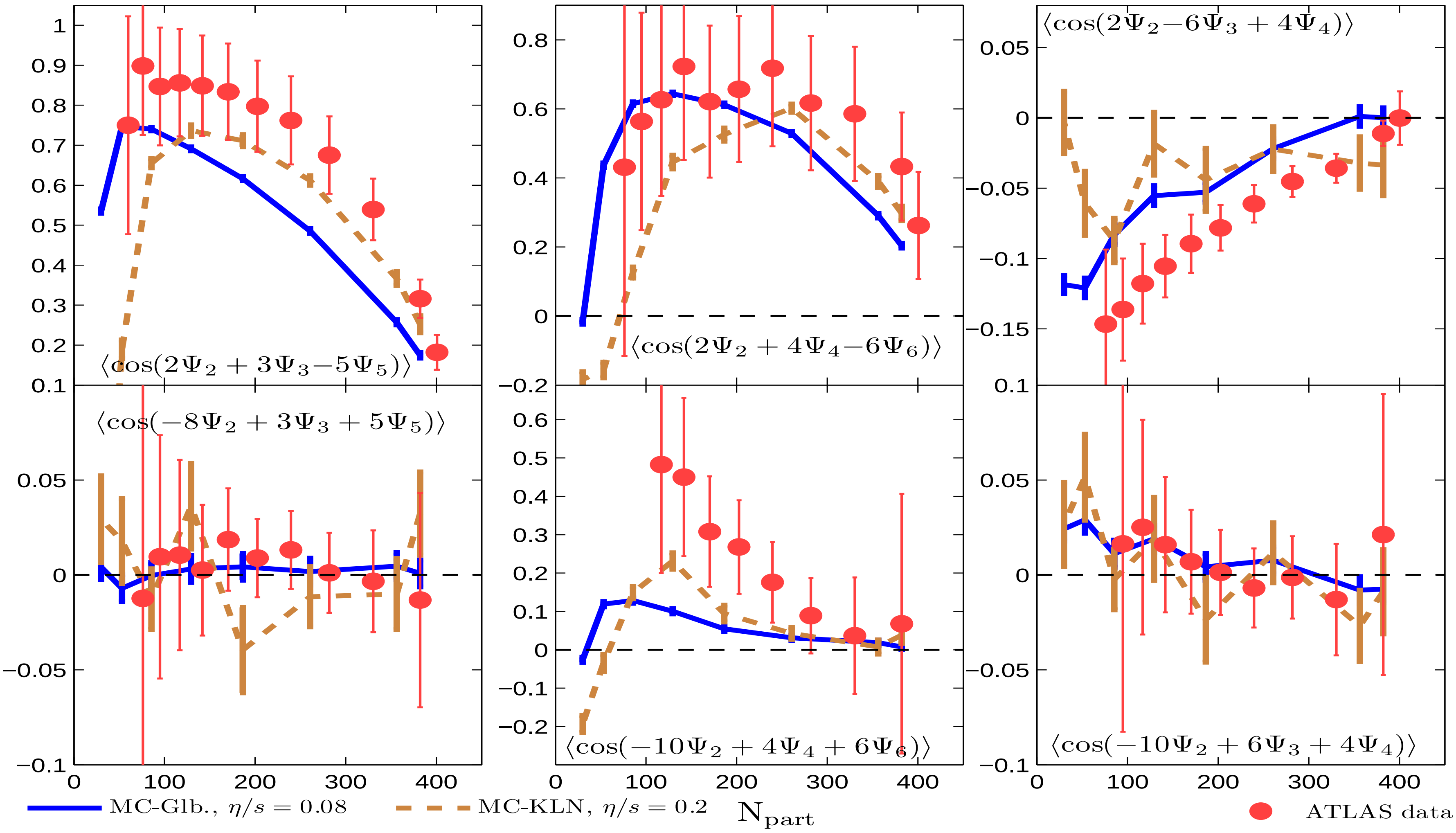

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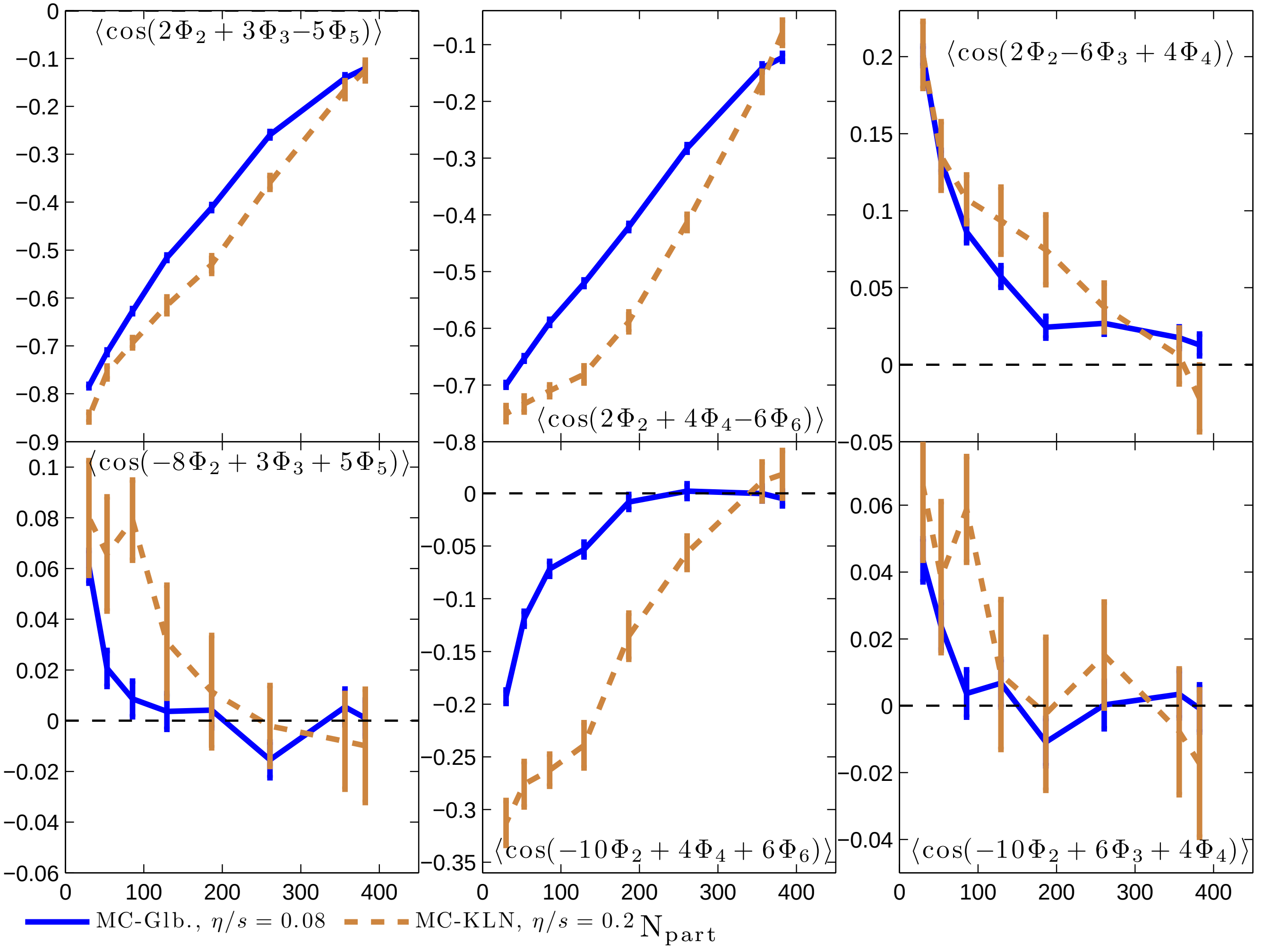
Zhi Qiu, UH, PLB 717 (2012) 261



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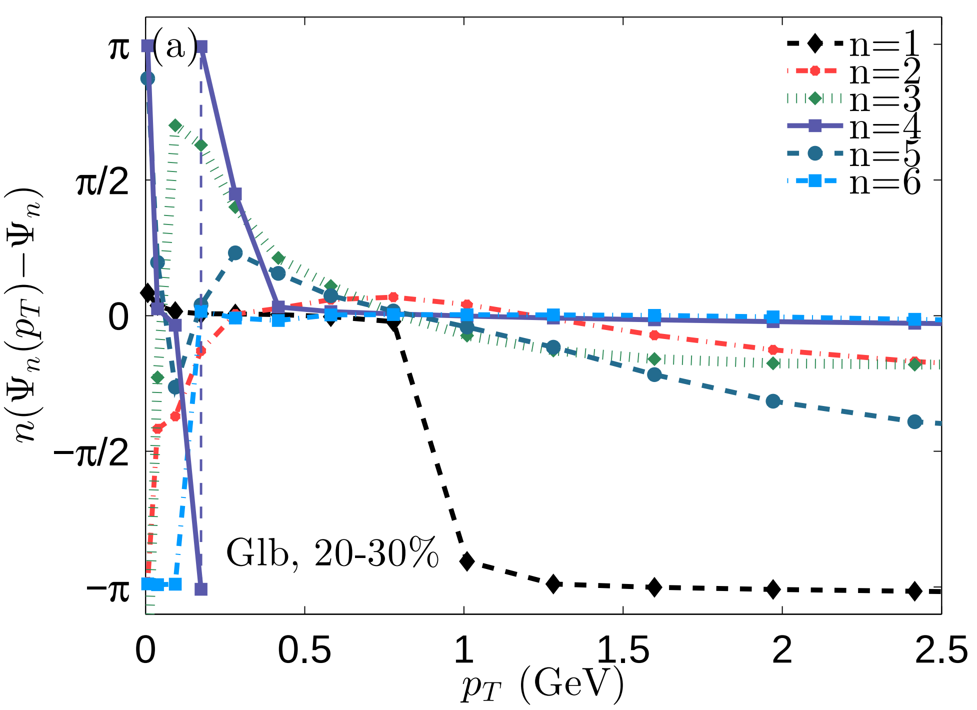
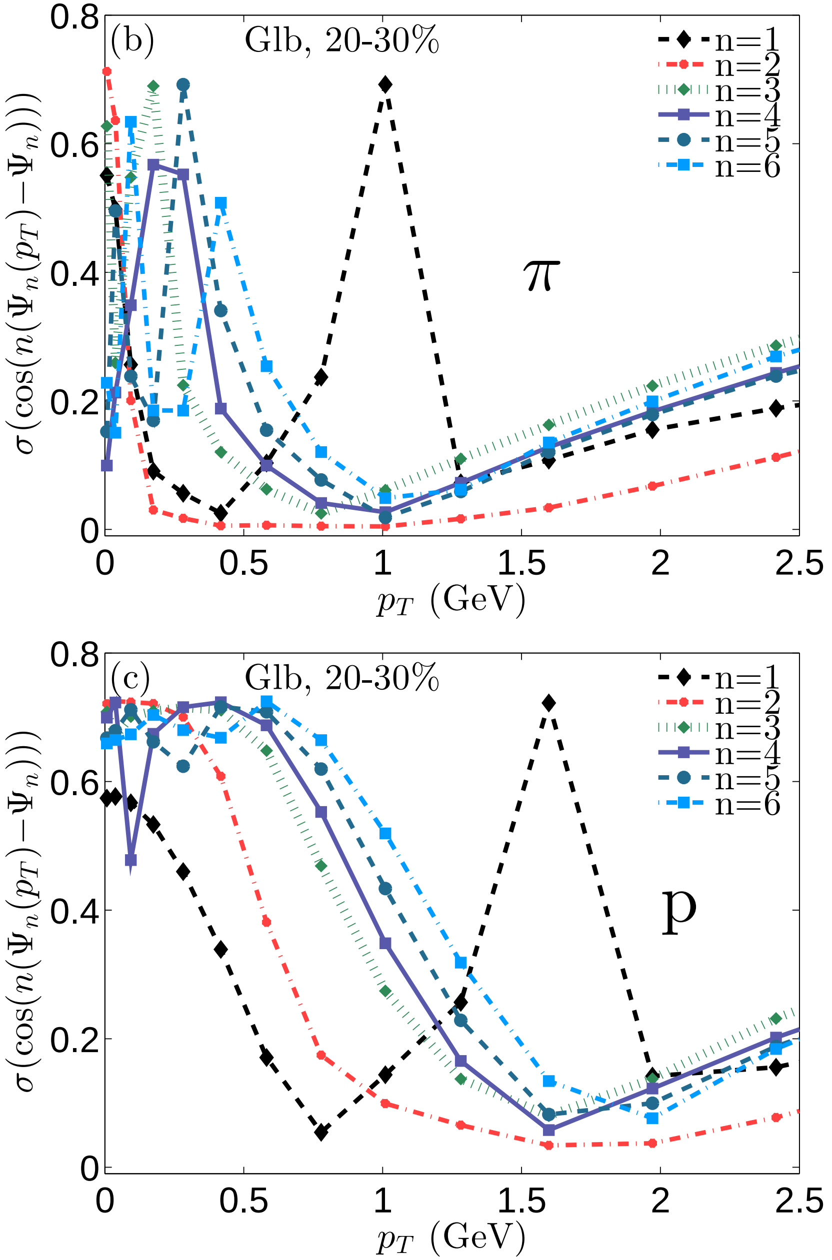
- $\Psi_n(p_T) - \Psi_n$ fluctuates from event to event.

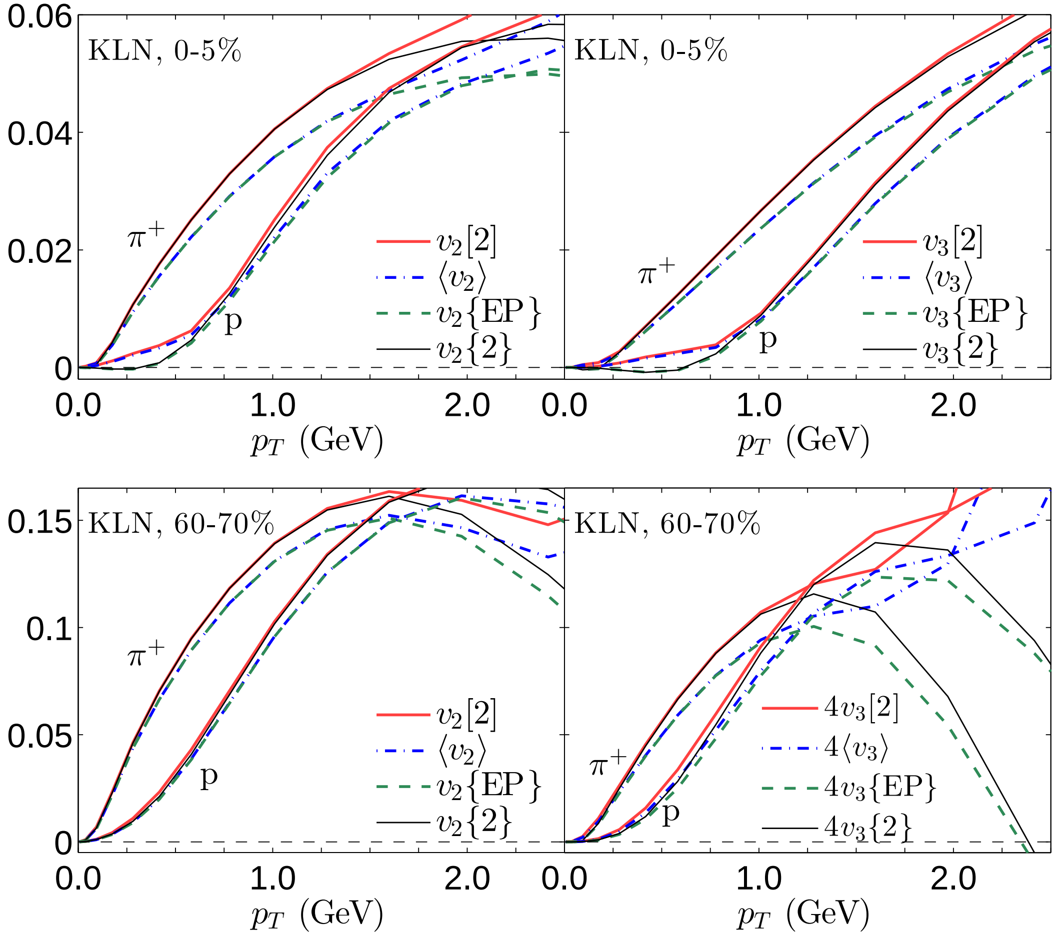
p_T-dependent flow angles and their fluctuations



- Except for directed flow (n=1), $\Psi_n(p_T)-\Psi_n$ fluctuates most strongly at low p_T
- Directed flow angle $\Psi_1(p_T)$ flips by 180° at $p_T\sim 1$ GeV for charged hadrons (pions) and at $p_T\sim 1.5$ GeV for protons (momentum conservation)



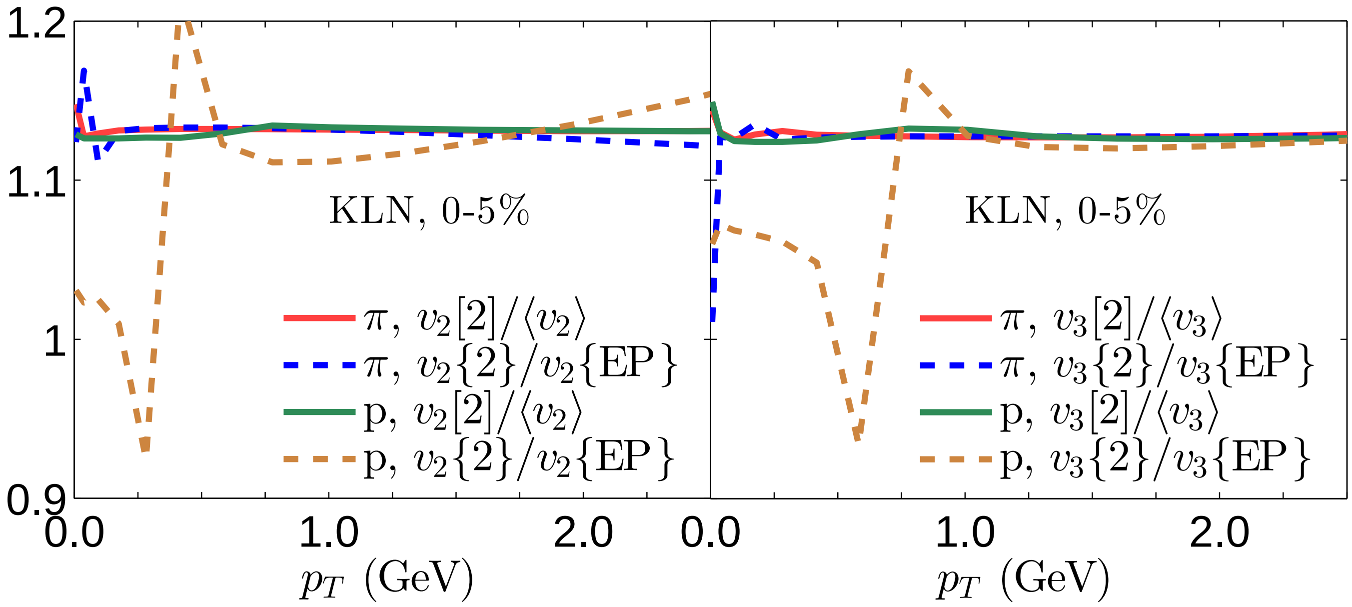
Elliptic and triangular flow comparison (I)



In central collisions, angular fluctuations suppress $v_n\{\mathrm{EP}\}(p_T)$ and $v_n\{2\}(p_T)$ below the mean and rms flows at low p_T (clearly visible for protons)

This effect disappears in peripheral collisions, but a similar effect then takes over at higher p_T, for both pions and protons.

Elliptic and triangular flow comparison (II): v_n ratios



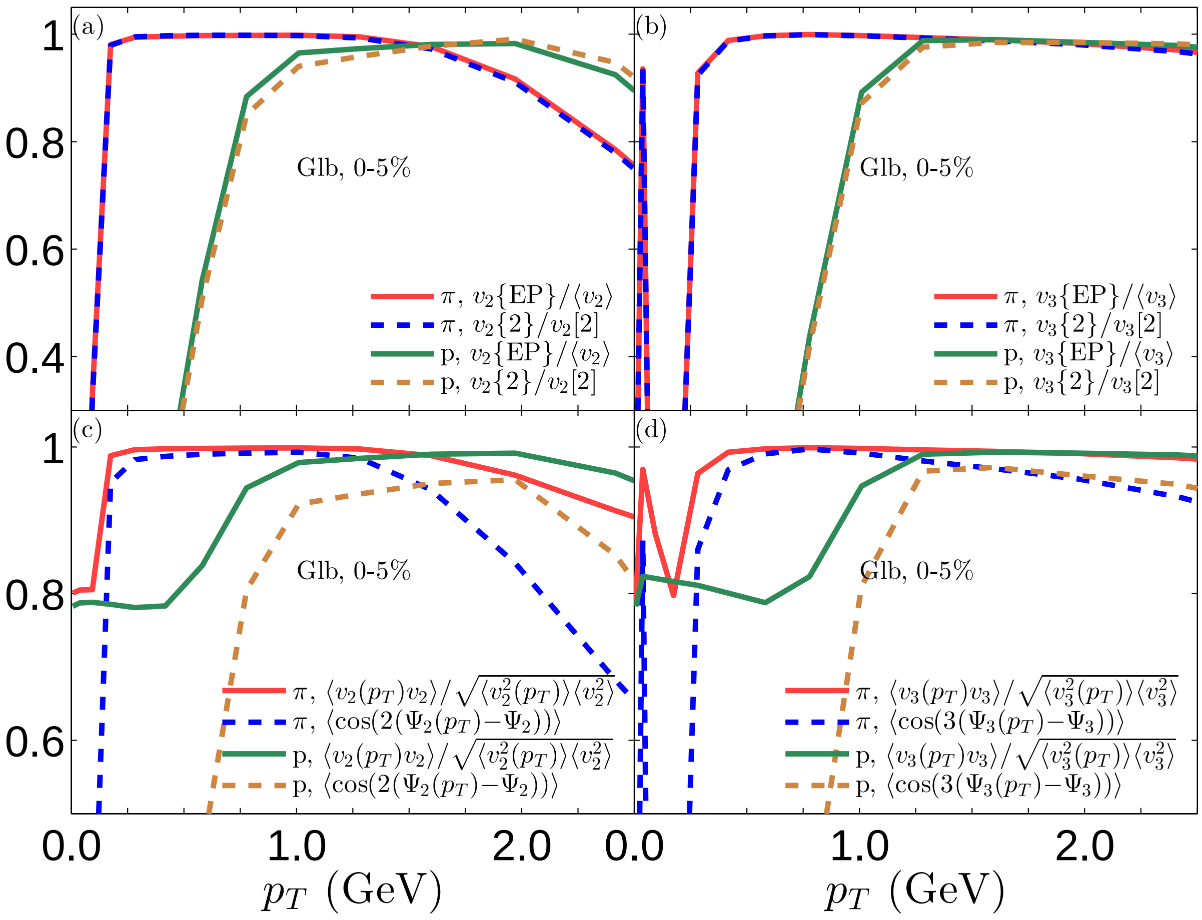
Except for where the numerator or denominator goes through zero, for central collisions these ratios are equal to $2/\sqrt{\pi} \approx 1.13$, independent of p_T. Expected if flow angles are randomly oriented (Bessel-Gaussian distribution for v_n, see Voloshin et al., PLB 659, 537 (2008)).

Not true in peripheral collisions, especially not for v_2 (Gardim et al., 1209.2323)

That this works even for $v_n\{2\}/v_n\{EP\}$ suggests an approximate factorization of angular fluctuation effects!

Elliptic and triangular flow comparison (III): v_n ratios

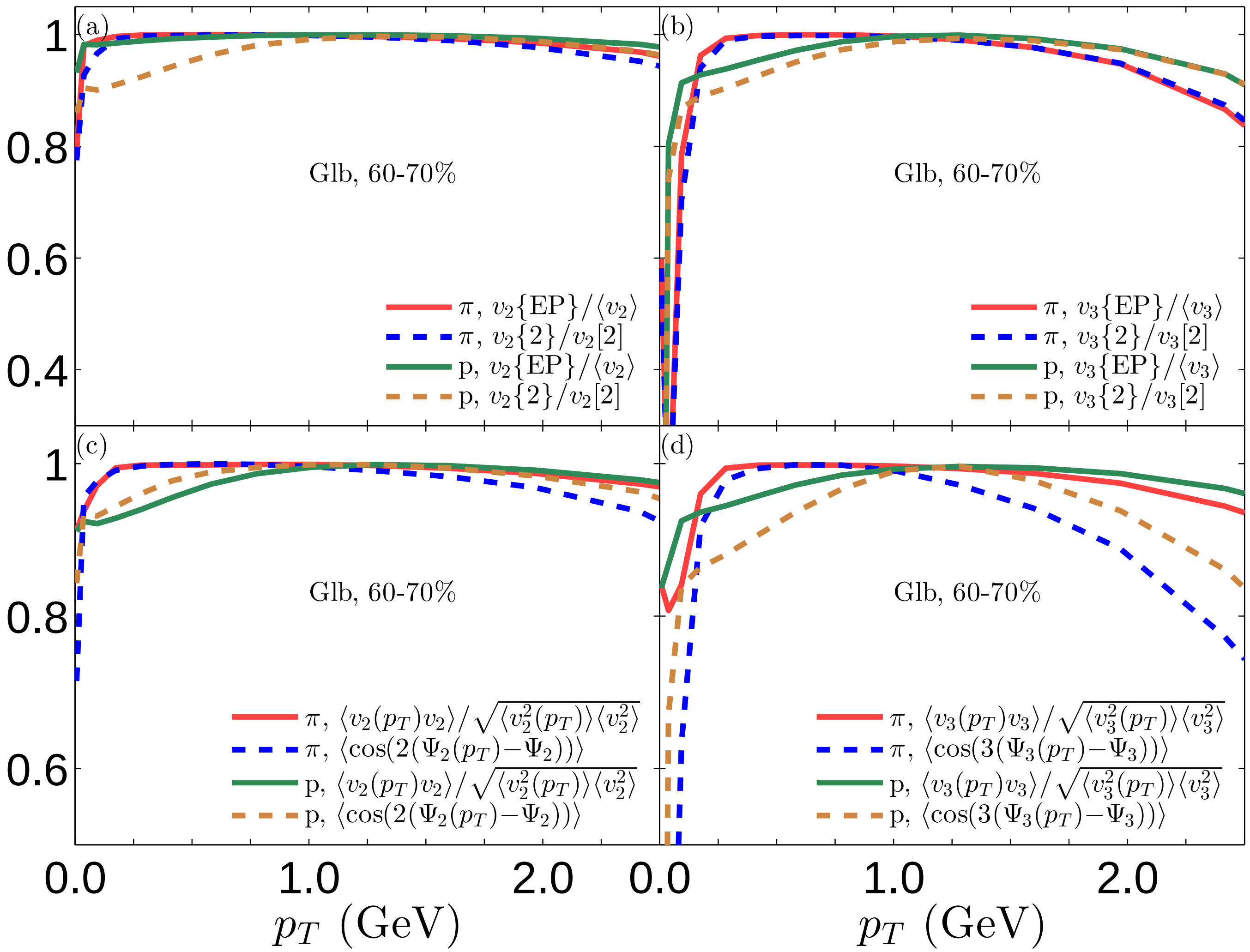
Central collisions:



- The angular fluctuation factor $\langle\cos[n(\Psi_n(p_T)-\Psi_n)]\rangle$ completely dominates the p_T-dependence of these ratios!
- Angular fluctuations have similar effect as poor event-plane resolution: they reduce v_n.
- Angular fluctuations are effective both at low and high p_T, but not at intermediate p_T.
- The window for seeing flow angle fluctuation effects at low p_T is smaller for pions than for protons.

Elliptic and triangular flow comparison (IV): v_n ratios

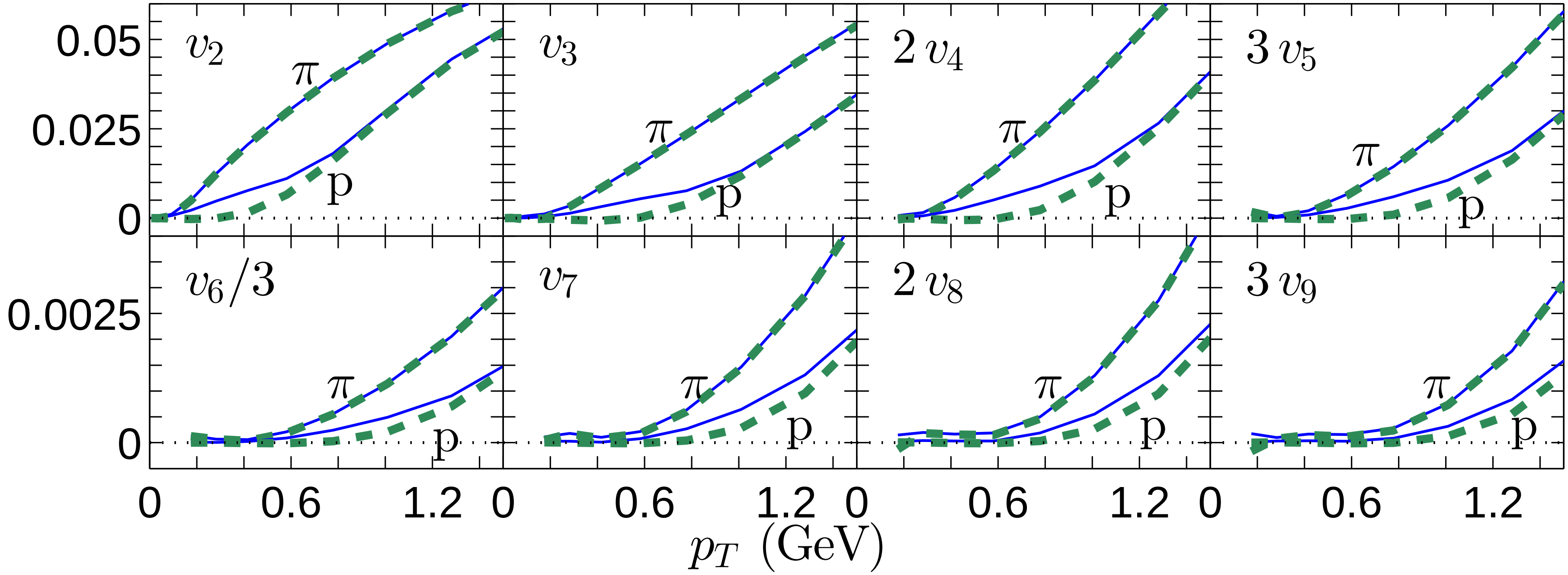
Peripheral collisions:



The window for seeing flow angle fluctuation effects at low p_T closes in peripheral collisions.

Flow angle fluctuation effects for higher order $v_n(p_T)$

Central collisions; solid: $\langle v_n(p_T)\rangle$; dashed: $v_n\{\mathrm{EP}\}(p_T)$:



As harmonic order n increases, suppression of $v_n\{\mathrm{EP}\}(p_T)$ (or $v_n\{2\}(p_T)$) from flow angle fluctuations for protons gets somewhat weaker but persists to larger p_T.

Test of factorization of two-particle spectra

Factorization $V_{n\Delta}(p_{T1},p_{T2}) := \langle\{\cos[n(\phi_1-\phi_2)]\}_{p_{T1}p_{T2}}\rangle \approx$ “$v_n(p_{T1}) \times v_n(p_{T2})$“ was checked experimentally as a test of hydrodynamic behavior, and found to hold to good approximation.

Gardim et al. (1211.0989) pointed out that event-by-event fluctuations break this factorization even if 2-particle correlations are exclusively due to flow.

They proposed to study the following ratio:

$$r_n(p_{T1},p_{T2}) := \frac{V_{n\Delta}(p_{T1},p_{T2})}{\sqrt{V_{n\Delta}(p_{T1},p_{T1})V_{n\Delta}(p_{T2},p_{T2})}} = \frac{\langle v_n(p_{T1})v_n(p_{T2})\cos[n(\Psi_n(p_{T1})-\Psi_n(p_{T2}))]\rangle}{v_n[2](p_{T1})v_n[2](p_{T2})}.$$

Even in the absence of flow angle fluctuations, this ratio is <1 due to v_n fluctuations (Schwarz inequality), except for $p_{T1}=p_{T2}$.

But it additionally depends on flow angle fluctuations.

To assess what share of the deviation from 1 is due to flow angle fluctuations, we can compare with

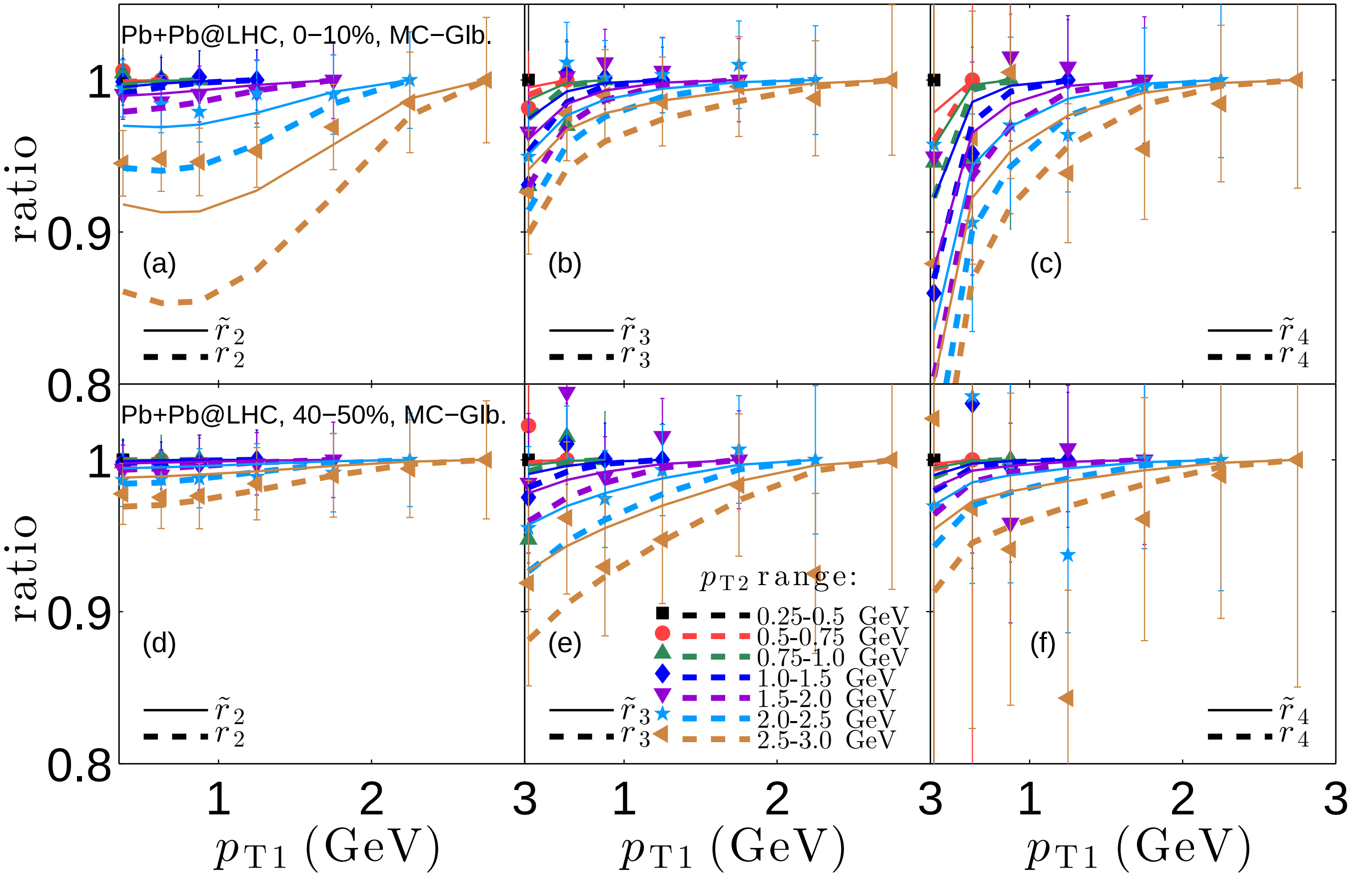
$$\tilde{r}_n(p_{T1},p_{T2}) := \frac{\langle v_n(p_{T1})v_n(p_{T2})\cos[n(\Psi_n(p_{T1})-\Psi_n(p_{T2}))]\rangle}{\langle v_n(p_{T1})v_n(p_{T2})\rangle}$$

which deviates from 1 **only** due to flow angle fluctuations. Again, this ratio approaches 1 for $p_{T1}=p_{T2}$.

Gardim et al. studied r_n for ideal hydro; we have studied r_n and $\tilde{r}_n$ for viscous hydro.

Breaking of factorization by e-by-e fluctuations (I)

Monte Carlo Glauber initial conditions, $\eta/s = 0.08 = 1/(4\pi)$:



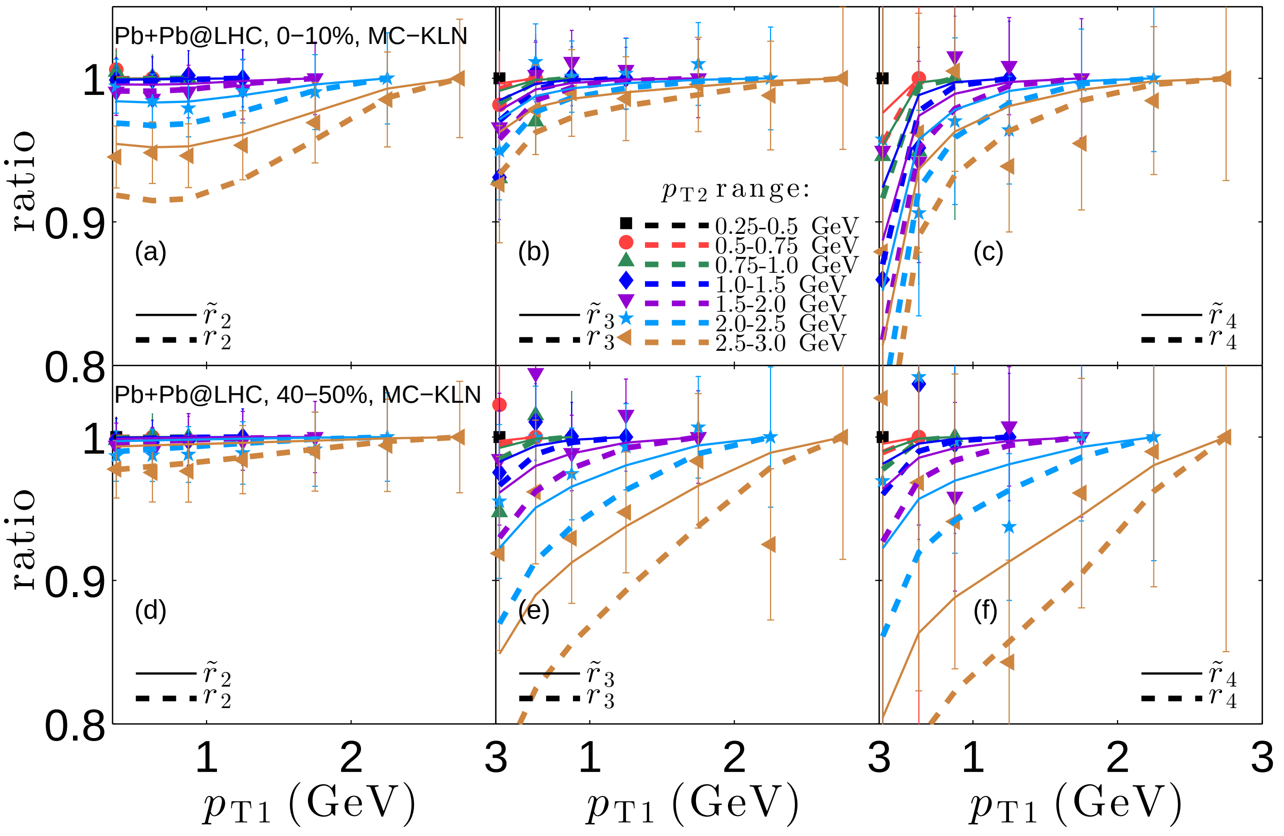
More than half of the factorization breaking effects are due to flow angle fluctuations.

In central collisions, $\eta/s = 0.08$ appears to overpredict the breaking of factorization (consistent with Gardim et al. who saw still larger effects for ideal hydro).

Factorization breaking effects appear to be larger for fluctuation-dominated flow harmonics.

Breaking of factorization by e-by-e fluctuations (II)

Monte Carlo KLN initial conditions, $\eta/s = 0.2 = 2.5/(4\pi)$:



In central collisions, factorization-breaking effects decrease with increasing η/s.

In peripheral collisions, larger η/s appears to cause a larger breaking of factorization, mostly due to flow angle fluctuations.

Data may indicate slight preference for larger η/s value, but more experimental precision and more detailed theoretical studies are needed to settle this. Analysis of ATLAS data in progress.

Conclusions

- Both the magnitudes v_n and the flow angles Ψ_n depend on p_T and fluctuate from event to event.
- In each event, the "p_T-averaged" (total-event) flow angles Ψ_n are identical for all particle species, but their p_T distribution differs from species to species.
- The mean v_n values and their p_T-dependence at RHIC and LHC have already been shown to put useful constraints on the QGP shear viscosity and its temperature dependence (see next talk by B. Schenke)
- **The effects of v_n and Ψ_n fluctuations can be separated experimentally by studying different V_n measures based on two-particle correlations.**
- Flow angle correlations are a powerful test of the hydrodynamic paradigm and will help to further constrain the spectrum of initial-state fluctuations and QGP transport coefficients.
- Studying event-by-event fluctuations of the anisotropic flows v_n and their flow angles Ψ_n as functions of p_T, as well as the correlations between different harmonic flows (both their magnitudes and angles), provides a rich data base for identifying the **"Standard Model of the Little Bang"**, by pinning down its initial fluctuation spectrum and its transport coefficients.