Novel properties of photons and hadrons in strong magnetic fields

In collaboration with K. Hattori and S. Ozaki

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First of all, and most importantly,

I came here to say

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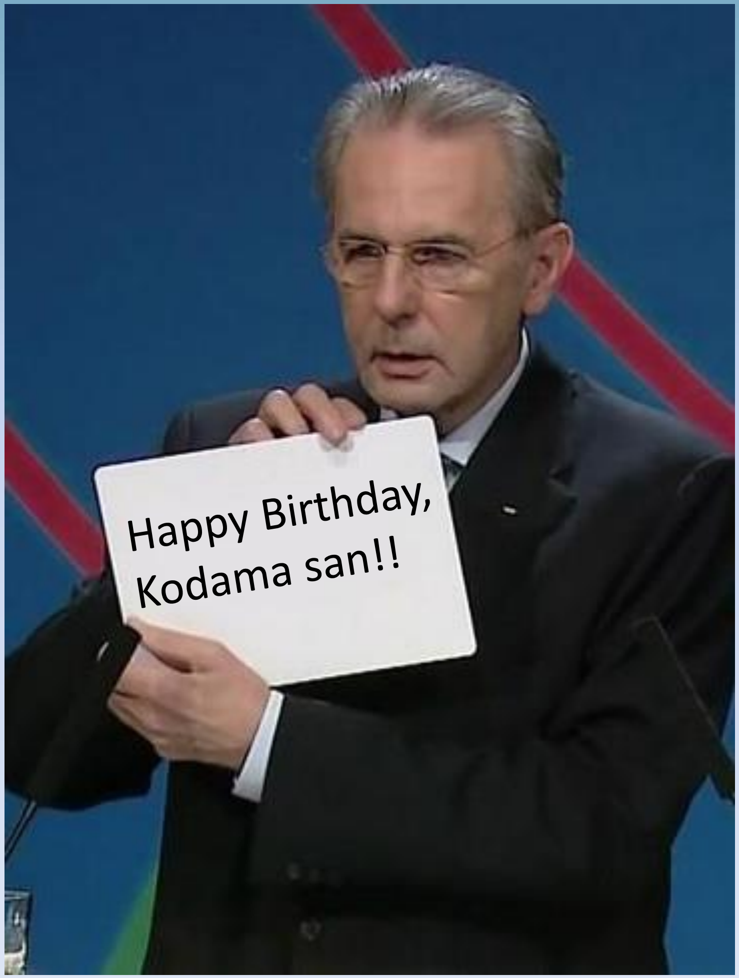


Count Rogge, 8th president of IOC

``Rogge generator”
http://app-sale.info/rogge/
Insert your favorite words!

Aim of the talk

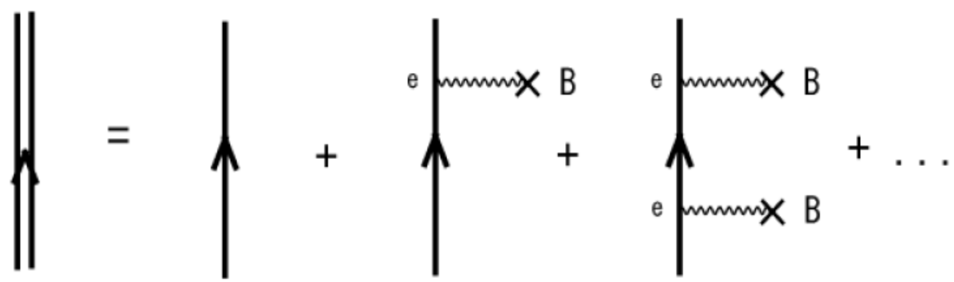
- present a possibility/attempts to investigate pre-equilibrium stages of heavy-ion collisions by using <u>strong field physics</u>
- Show an example of observables which are related to information at the very early stages.

What is strong field physics?

- **Characteristic phenomena that occur under strong gauge fields**
  (EM fields and Yang-Mills fields)

- **Typically, weak-coupling but non-perturbative**

  ex) electron propagator in a strong magnetic field



$$1 + O\left(\frac{eB}{m_e^2}\right) + O\left(\left(\frac{eB}{m_e^2}\right)^2\right)$$

$$eB_c \equiv m_e^2$$

$$eE_c \sim m_e^2$$

Schwinger's critical field

must be resumed when $B >> B_c$

→ **"Nonlinear QED effects"**

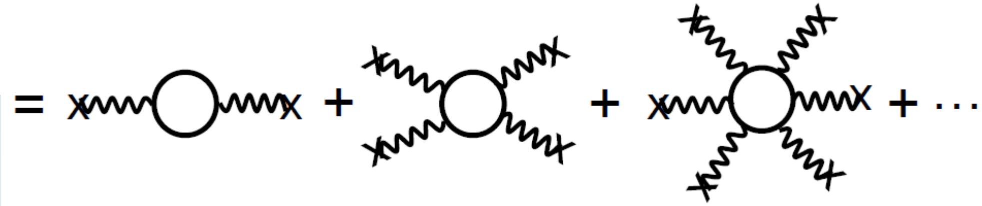
- **A new interdisciplinary field:** involving high-intensity LASER physics, hadron physics (heavy-ion physics), condensed matter physics (exciton), astrophysics (neutron stars, magnetars, early universe)

Examples of nonlinear QED effects

- **Euler-Heisenberg action**

effective potential of constant EM fields

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2-\mathfrak{B}^2) + \frac{e^2}{h\,c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\,\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2\,i\,(\mathfrak{E}\mathfrak{B})}\right)+\mathrm{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2\,i\,(\mathfrak{E}\mathfrak{B})}\right)-\mathrm{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}$$



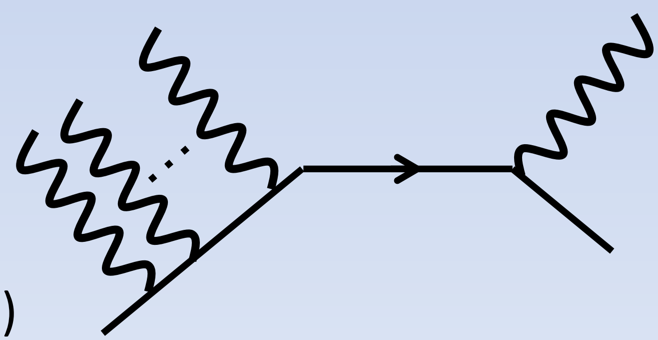
nonlinear w.r.t. *E* and *B*

- **Nonlinear Compton effects**

$$e + n\gamma \rightarrow e' + m\gamma$$

Multiple absorption of photons

(experimentally confirmed SLAC E144 (1996))



Plan

- **Introduction**

  strong magnetic field in heavy-ion collisions

- **Photons in strong B**

  vacuum birefringence and decay into e+e- pair

  photon's HBT interferometry in HIC

- **Neutral pions in strong B**

  new decay mode : π^0 +B $\rightarrow$ e+e- "Bee decay"

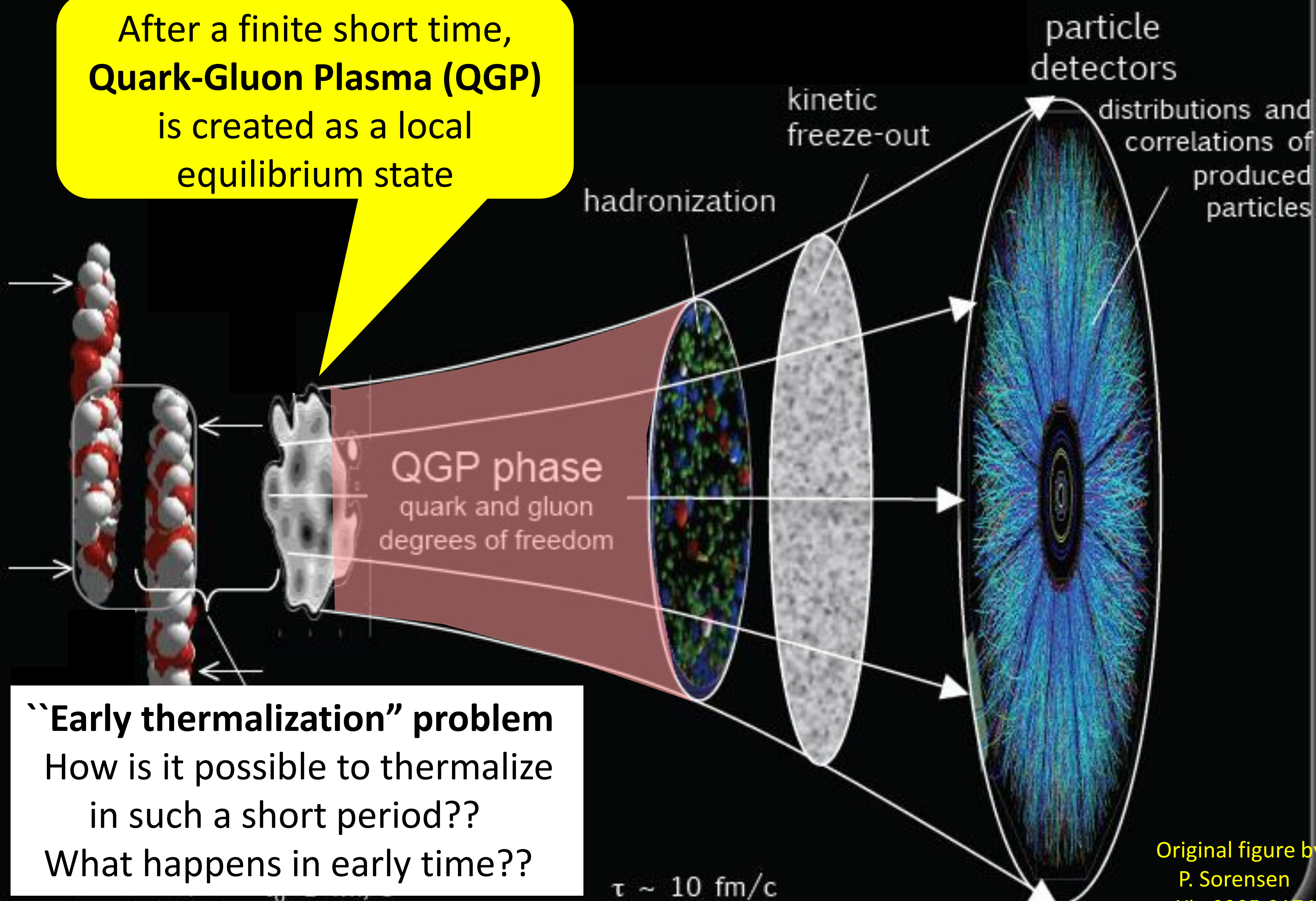
  photon conversion into π^0 in strong B

- **Summary**

  other possibilities?

Heavy-ion Collisions: Little Bang

After a finite short time,
Quark-Gluon Plasma (QGP)
is created as a local
equilibrium state

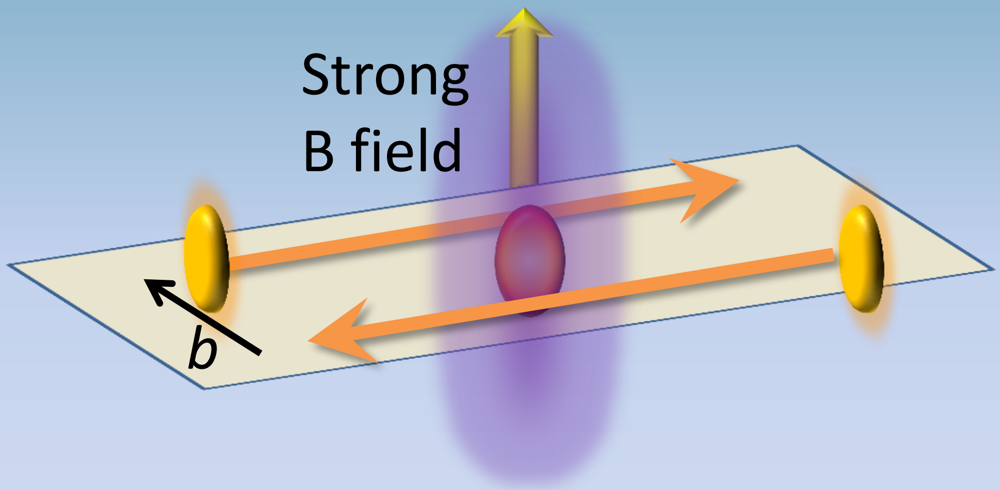


``Early thermalization" problem
How is it possible to thermalize
in such a short period??
What happens in early time??

Original figure by
P. Sorensen
arXiv:0905.0174

Strong magnetic fields in HICs

- Non-central HICs at RHIC and LHC provide **STRONGEST** magnetic fields.



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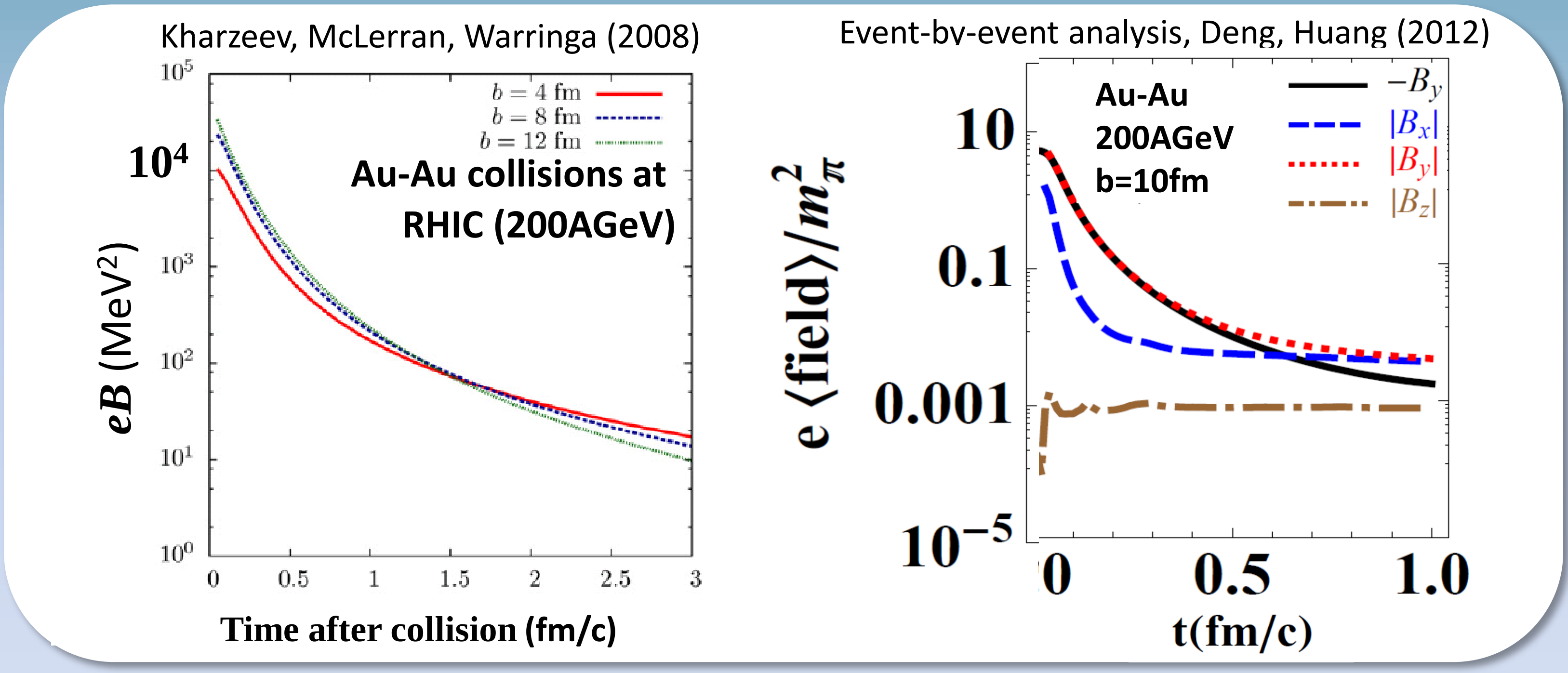

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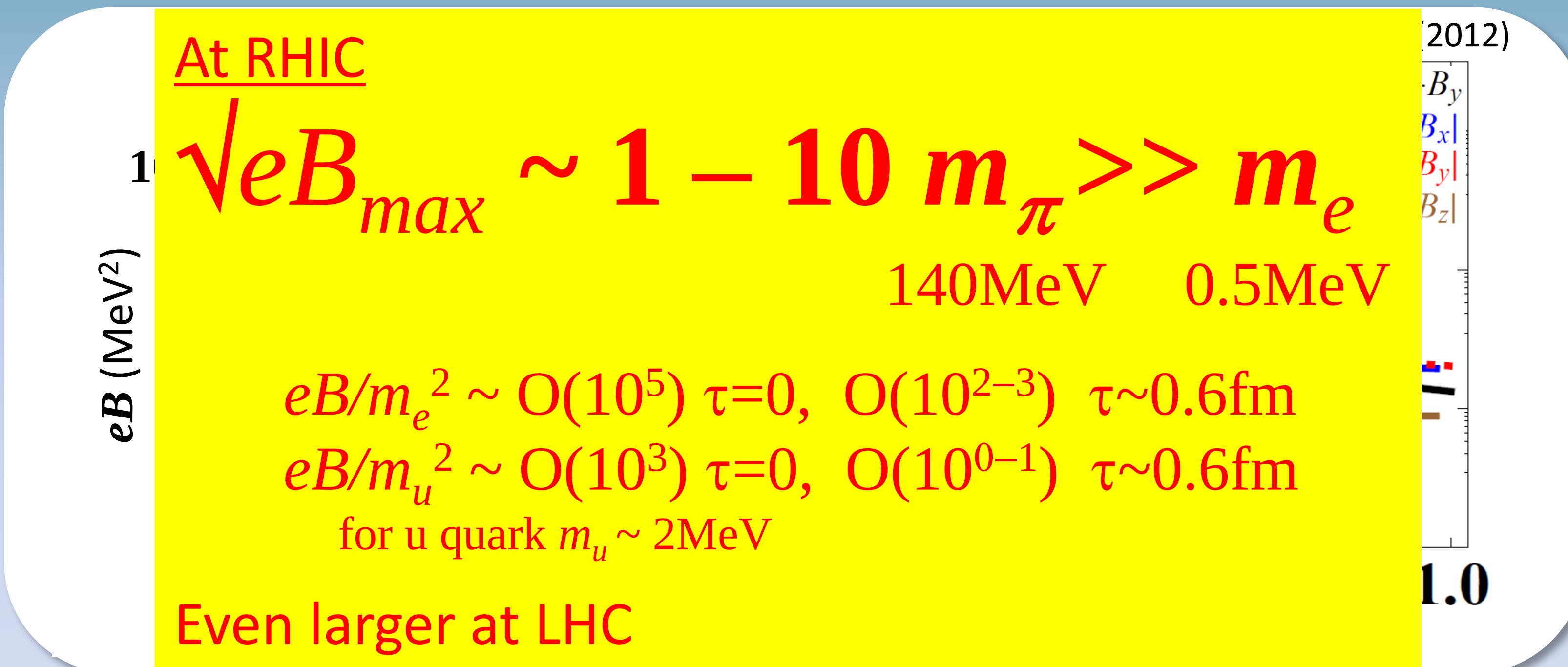


At RHIC

$$\sqrt{eB_{max}} \sim \mathbf{1 - 10}\ m_{\pi} >> m_e$$

140MeV 0.5MeV

$eB/m_e^2 \sim O(10^5)$ $\tau=0$, $O(10^{2-3})$ $\tau\sim 0.6\text{fm}$

$eB/m_u^2 \sim O(10^3)$ $\tau=0$, $O(10^{0-1})$ $\tau\sim 0.6\text{fm}$

for u quark $m_u \sim 2\text{MeV}$

Even larger at LHC

- **Decay very fast**:

  Strong field physics will be most prominent in very early time!

  (though the fields are still strong enough even at QGP formation time)

Very strong fields exist at very early time in HIC

“Strong field physics” can be a good probe of early time dynamics in HICs

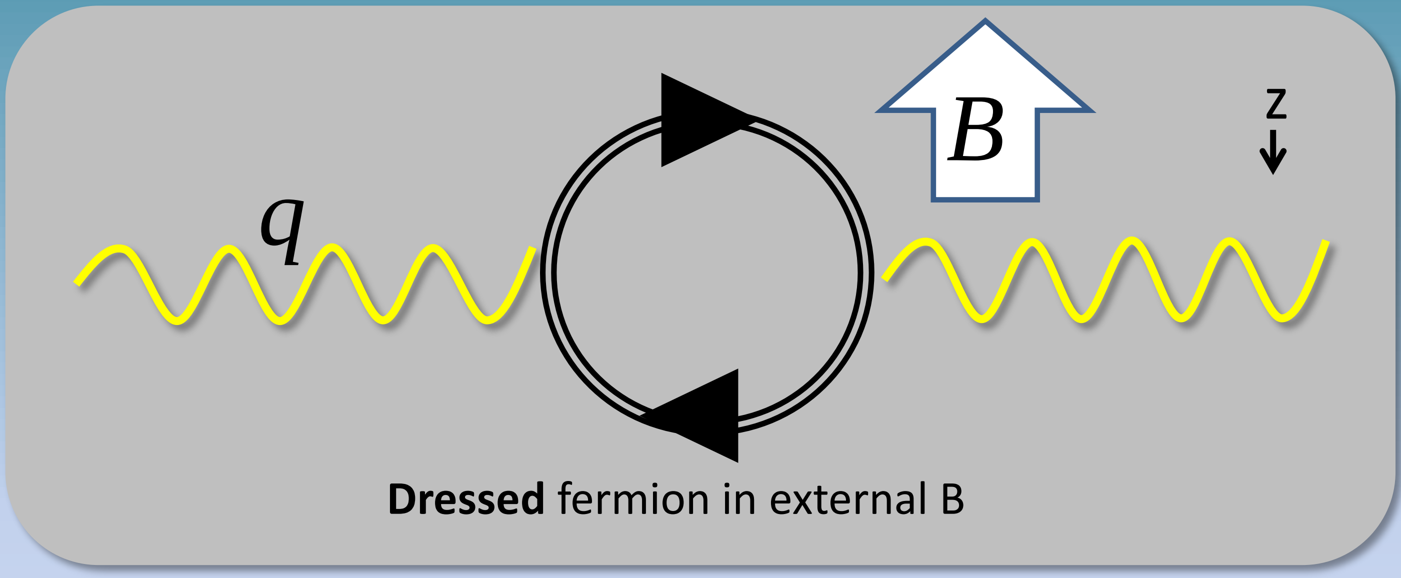
Can provide new insights into unsolved problem of “early thermalization”

We discuss

- Novel properties of **photons** and **neutral pions** in strong magnetic fields
- Possible **observable** effects in HICs

- HICs create many photons and neutral pions.
- Both are **charge neutral**. But can be affected through fermion (quark or electron) one loop.

Photons in strong magnetic fields



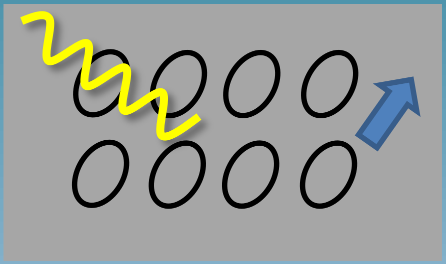
- **Properties of a photon propagating in a magnetic field**

  ← vacuum polarization tensor $\Pi^{\mu\nu}(q,B)$

- **Old but new problem** [Weisscopf 1936, Baier-Breitenlohner 1967, Narozhnyi 1968, Adler 1971]

  - Polarization tensor $\Pi^{\mu\nu}(q,B)$ has been known in *integral* form
  - Analytic representation obtained very recently [Hattori-Itakura 2013]

Magnetic vacuum as a media



Propagating photon in strong magnetic field

= probing magnetic vacuum “polarized” by external fields

~ photon couples to virtual excitation of vacuum (cf: exciton-polariton)

B dependent anisotropic response of a fermion (Landau levels)

- discretized transverse vs unchanged longitudinal motion
  - → Two different refractive indices : ***VACUUM BIREFRINGENCE***
- energy conservation gets modified
  - → Pol. Tensor can have imaginary part : ***PHOTON DECAY INTO e+e- PAIR***

(lots of astrophysical applications)

$$\Pi^{\mu\nu}_{\rm ex}(q) = \chi_0(q^2\eta^{\mu\nu} - q^\mu q^\nu) + \underbrace{\chi_1(q_\parallel^2\eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu) + \chi_2(q_\perp^2\eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu)}_{\text{present only in external fields}}$$

II parallel to B

⊥ transverse to B

$$\eta_\parallel^{\mu\nu} = diag(1,0,0,-1)$$

$$\eta_\perp^{\mu\nu} = diag(0,-1,-1,0)$$

$$\begin{aligned} q^\mu &= (q^0, q_\perp, 0, q^3) \\ q_\parallel^\mu &= (q^0, 0, 0, \ q^3) \\ q_\perp^\mu &= (0, q_\perp, 0, 0) \end{aligned}$$

Vacuum birefringence

- **Maxwell eq. with the polarization tensor :**

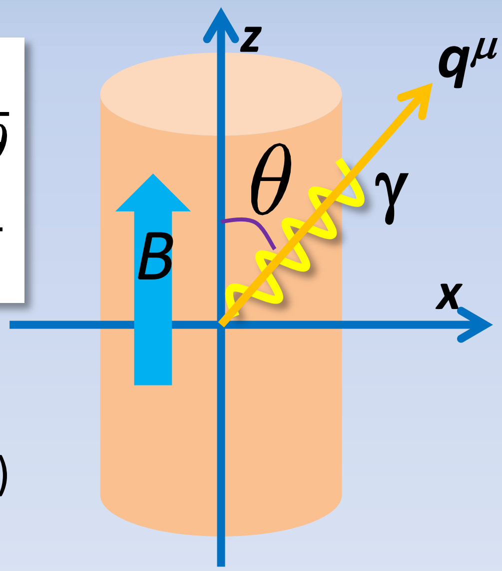
$$\left(q^2\eta^{\mu\nu} - q^\mu q^\nu + \hat{\Pi}^{\mu\nu}_{\rm ex}\right) A_\nu(q) = 0$$

$$\Pi^{\mu\nu}_{\rm ex}(q) = \chi_0(q^2\eta^{\mu\nu} - q^\mu q^\nu) + \chi_1(q_\parallel^2\eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu) + \chi_2(q_\perp^2\eta_\perp^{\mu\nu} - q_\perp^\mu q_\perp^\nu)$$

- **Dispersion relation of two physical modes gets modified**

→ Two refractive indices : "Birefringence"

$$n^2 \equiv \frac{|\boldsymbol{q}|^2}{\omega^2} \Rightarrow \begin{cases} n_1^2 = \dfrac{1+\chi_0+\chi_1}{1+\chi_0+\chi_1\cos^2\theta} \\ n_2^2 = \dfrac{1+\chi_0}{1+\chi_0+\chi_2\sin^2\theta} \end{cases}$$



1. Compute χ_0, χ_1, χ_2 analytically at the one-loop level

Hattori-Itakura Ann. Phys. 330 (2013)

2. Solve them self-consistently w.r.t *n in LLL approx.*

Hattori-Itakura Ann. Phys. 334 (2013)

Analytic representation of $\Pi^{\mu\nu}(q,B)$

Representation in double integral w.r.t. proper times

$$\chi_i(r_\parallel^2, r_\perp^2; B_\mathrm{r}) = \frac{\alpha}{4\pi}\int_{-1}^{1} d\beta \int_0^\infty d\tau \, \frac{\Gamma_i(\tau,\beta)}{\sin\tau} \, \mathrm{e}^{-iu\cos(\beta\tau)} \, \mathrm{e}^{i\eta\cot\tau} \, \mathrm{e}^{-i\phi_\parallel \tau} \, ,$$

$$B_\mathrm{r} = B/B_c, \; r_\parallel^2 = \frac{q_\parallel^2}{4m^2} \text{ and } r_\perp^2 = \frac{q_\perp^2}{4m^2} = -\frac{|q_\perp|^2}{4m^2}$$

$$\eta \equiv -2r_\perp^2/B_\mathrm{r} \text{ and } u \equiv \eta/\sin\tau$$

$$\phi_\parallel(r_\parallel^2, B_\mathrm{r}) = \frac{1}{B_\mathrm{r}}\left\{ 1 - (1-\beta^2)\, r_\parallel^2 \right\} ,$$

$$\Gamma_0(\tau,\beta) = \cos(\beta\tau) - \beta\sin(\beta\tau)\cot\tau \, ,$$

$$\Gamma_1(\tau,\beta) = (1-\beta^2)\cos\tau - \Gamma_0(\tau,\beta) \, ,$$

$$\Gamma_2(\tau,\beta) = 2\frac{\cos(\beta\tau) - \cos\tau}{\sin^2\tau} - \Gamma_0(\tau,\beta) \quad .$$

Analytic representation of $\Pi^{\mu\nu}(q,B)$

$$\chi_i = \frac{\alpha B_{\rm r}}{4\pi} \sum_{n=0}^{\infty} (2-\delta_{n0}) \left[\sum_{\ell=0}^{\infty} \Omega_{\ell i}^{n(0)} + \sum_{\ell=1}^{\infty} \Omega_{\ell i}^{n(1)} + \sum_{\ell=2}^{\infty} \Omega_{\ell i}^{n(2)} \right],$$

$$\Omega_{\ell 0}^{n(0)} = (1-\delta_{n0})\, C_\ell^{n-1}(\eta)\, F_\ell^n(\xi, B_{\rm r}) - n\eta^{-1}\, C_\ell^n(\eta)\, G_\ell^n(\xi, B_{\rm r}),$$

$$\Omega_{\ell 0}^{n(1)} = (1+\delta_{n0})\, C_{\ell-1}^{n+1}(\eta)\, F_\ell^n(\xi, B_{\rm r}) - n\eta^{-1}\, C_{\ell-1}^n(\eta)\, G_\ell^n(\xi, B_{\rm r}),$$

$$\Omega_{\ell 0}^{n(2)} = 0.$$

$$\Omega_{\ell 1}^{n(0)} = C_\ell^n(\eta)\,\{F_\ell^n(\xi, B_{\rm r}) - H_\ell^n(\xi, B_{\rm r})\} - \Omega_{\ell 0}^{n(0)},$$

$$\Omega_{\ell 1}^{n(1)} = C_{\ell-1}^n(\eta)\,\{F_\ell^n(\xi, B_{\rm r}) - H_\ell^n(\xi, B_{\rm r})\} - \Omega_{\ell 0}^{n(1)},$$

$$\Omega_{\ell 1}^{n(2)} = 0,$$

$$\Omega_{\ell 2}^{n(0)} = -\Omega_{\ell 0}^{n(0)},$$

$$\Omega_{\ell 2}^{n(1)} = D_\ell^{n(1)}(\eta)\, F_\ell^n(\xi, B_{\rm r}) - \Omega_{\ell 0}^{n(1)},$$

$$\Omega_{\ell 2}^{n(2)} = D_\ell^{n(2)}(\eta)\, F_\ell^n(\xi, B_{\rm r}).$$

$$D_\ell^{n(1)}(\eta) = -8 \sum_{\lambda=0}^{\ell-1} (\ell-\lambda)\,\{(1-\delta_{n0})\, C_\lambda^{n-1}(\eta) - C_\lambda^n(\eta)\},$$

$$D_\ell^{n(2)}(\eta) = -8 \sum_{\lambda=0}^{\ell-2} (\ell-\lambda-1)\,\{(1+\delta_{n0})\, C_\lambda^{n+1}(\eta) - C_\lambda^n(\eta)\}.$$

$$C_\ell^n(\eta) \equiv e^{-\eta} \frac{\ell!}{(\ell+n)!}\, \eta^n \left[L_\ell^n(\eta)\right]^2.$$

$$F_\ell^n(r_\parallel^2, B_{\rm r}) = \int_{-1}^{1} \frac{d\beta}{r_\parallel^2\beta^2 - nB_{\rm r}\beta + (1-r_\parallel^2) + (2\ell+n)B_{\rm r}} \equiv I_{\ell\Delta}^n(r_\parallel^2)$$

$$G_\ell^n(r_\parallel^2, B_{\rm r}) = \frac{1}{2r_\parallel^2} \left[\Xi_\ell^n(B_{\rm r}) + nB_{\rm r}\, I_{\ell\Delta}^n(r_\parallel^2) \right],$$

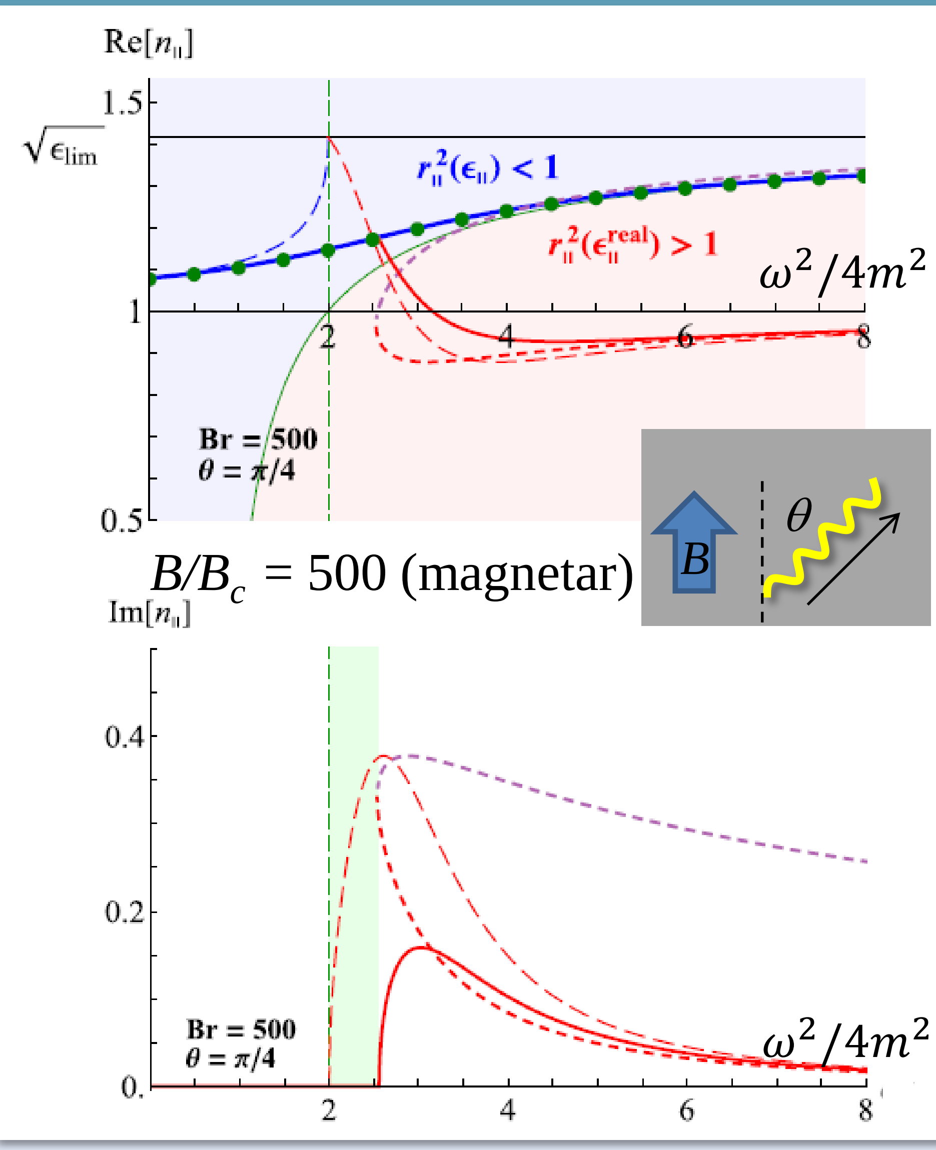
$$H_\ell^n(r_\parallel^2, B_{\rm r}) = \frac{1}{r_\parallel^2} \left[2 + \frac{nB_{\rm r}}{2r_\parallel^2}\, \Xi_\ell^n(B_{\rm r}) + \frac{1}{4r_\parallel^2} \left\{ (b^2-4ac) + (nB_{\rm r})^2 \right\} I_{\ell\Delta}^n(r_\parallel^2) \right],$$

$$\Xi_\ell^n(B_{\rm r}) \equiv \ln\left|\frac{1+2\ell B_{\rm r}}{1+2(\ell+n)B_{\rm r}}\right| = \ln\left|\frac{m^2+2\ell eB}{m^2+2(\ell+n)eB}\right|$$

$$I_{\ell\Delta}^n(r_\parallel^2) = \begin{cases} \dfrac{1}{\sqrt{(r_\parallel^2-s_-^{\ell n})(r_\parallel^2-s_+^{\ell n})}} \cdot \dfrac{1}{2} \ln\left|\dfrac{a-c-\sqrt{b^2-4ac}}{a-c+\sqrt{b^2-4ac}}\right| & (r_\parallel^2 < s_-^{\ell n}) \\ \dfrac{1}{\sqrt{\left|(r_\parallel^2-s_-^{\ell n})(r_\parallel^2-s_+^{\ell n})\right|}} \left[\arctan\left(\dfrac{b+2a}{\sqrt{4ac-b^2}}\right) - \arctan\left(\dfrac{b-2a}{\sqrt{4ac-b^2}}\right)\right] & (s_-^{\ell n} < r_\parallel^2 < s_+^{\ell n}) \\ \dfrac{1}{\sqrt{(r_\parallel^2-s_-^{\ell n})(r_\parallel^2-s_+^{\ell n})}} \cdot \dfrac{1}{2} \left[\ln\left|\dfrac{a-c-\sqrt{b^2-4ac}}{a-c+\sqrt{b^2-4ac}}\right| + 2\pi i \right] & (s_+^{\ell n} < r_\parallel^2). \end{cases}$$

- Infinite summation w.r.t. n and l = summation over two Landau levels
- Numerically confirmed by Ishikawa, et al. arXiv:1304.3655 [hep-ph]
- couldn't find the same results starting from propagators with Landau level decomposition

Refractive index



- **Need to self-consistently solve the equation** (effects of back-reaction)
- **Use LLL solution for simplicity**

  → $\chi_0 = \chi_2 = 0, \chi_1 \neq 0$

$$n_{\parallel}^2 = \frac{1+\chi_1}{1+\chi_1\cos^2\theta}, \quad \chi_1 = \chi_1(q_{\parallel}^2, q_{\perp}^2, B)$$

$$n_{\perp}^2 = 1$$

$$\begin{cases} q_{\parallel}^2 = \omega^2 - q_z^2 = \omega^2(1 - n_{\parallel}^2\cos^2\theta) \\ q_{\perp}^2 = -|q_{\perp}|^2 = -\omega^2 n_{\parallel}^2 \sin^2\theta \end{cases}$$

- **Refractive index $n_{||}$ deviates from 1 and increases with increasing ω**

  cf: air n = 1.0003, water n = 1.333

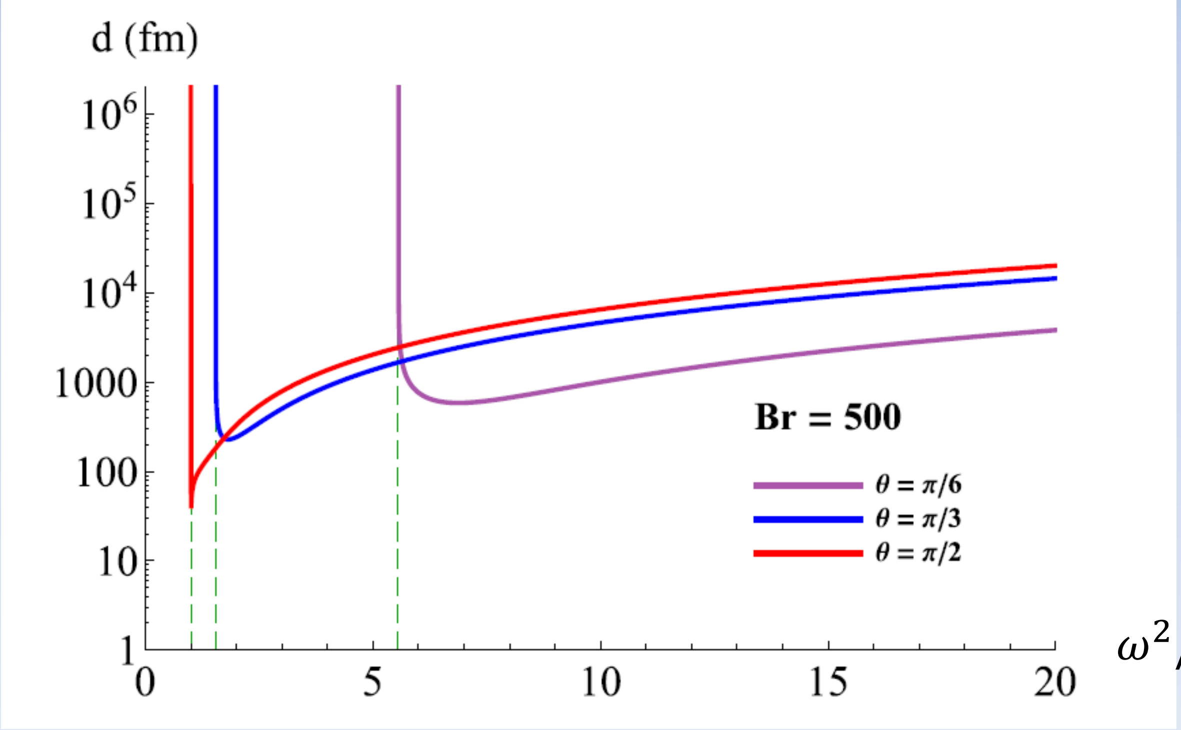
- **New branch at high energy is accompanied by an imaginary part → decay into an e+e- pair**

Decay length

Amplitude of an incident photon decays exponentially characterized by the decay length

$$d \equiv \frac{1}{2\omega\kappa} = \frac{1}{2\omega n_{\mathrm{imag}}}.$$

Surviving length ~ life time



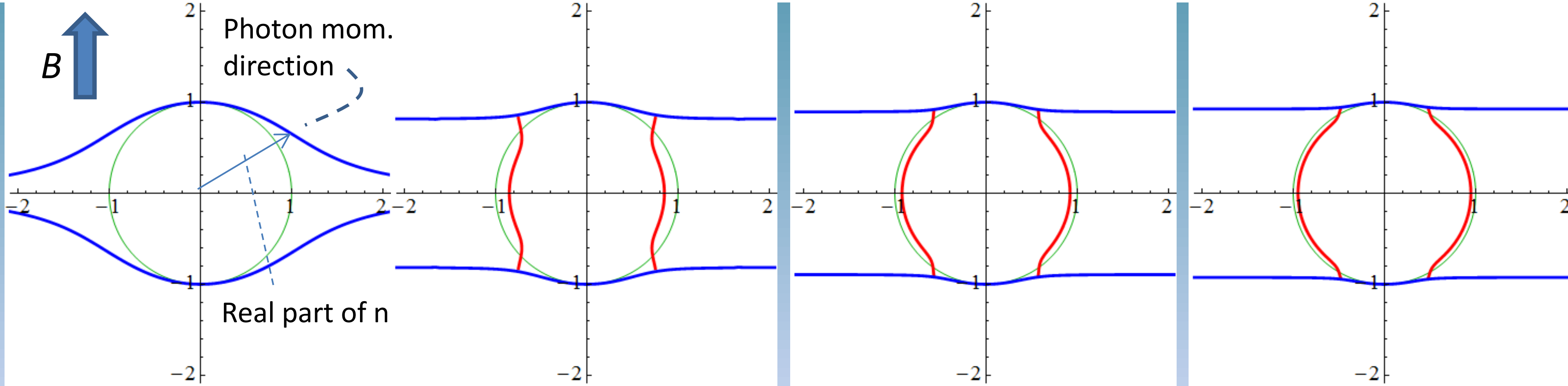
Very short length
→ relevant for magnetars

Even shorter in HIC
→ relevant for very soft photons generating anisotropic distribution

Angle dependence at various photon energies

For magnetars → $B_{\mathrm{r}} = 500$

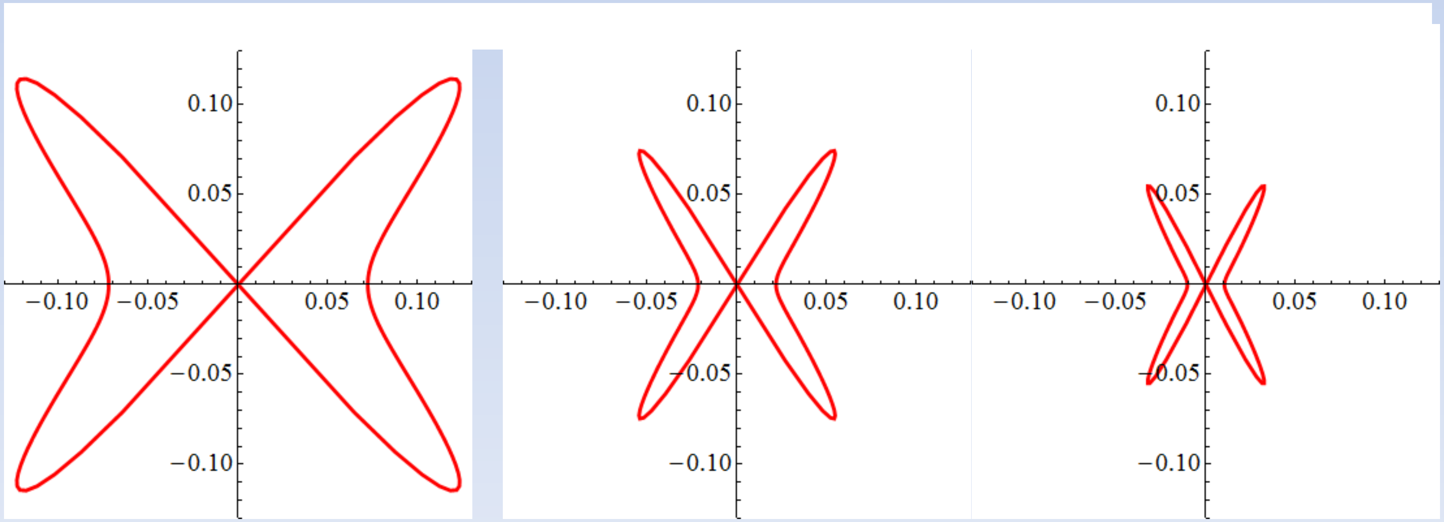
Real part



$\omega^2 = 1$ $\omega^2 = 3$ $\omega^2 = 5$ $\omega^2 = 7$

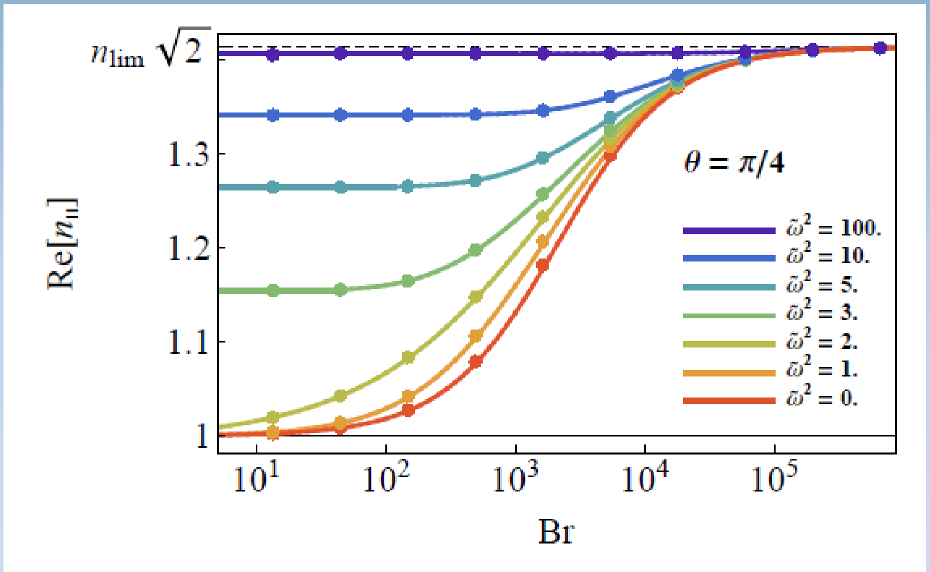
Imaginary part

No imaginary part

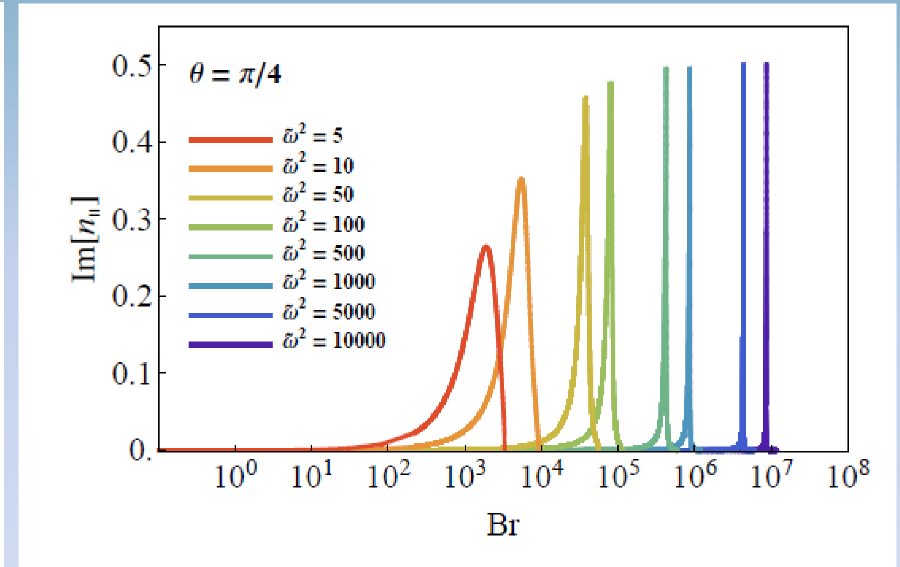


Effects are stronger with stronger magnetic fields or higher energy photons

Refractive index



Photon decay



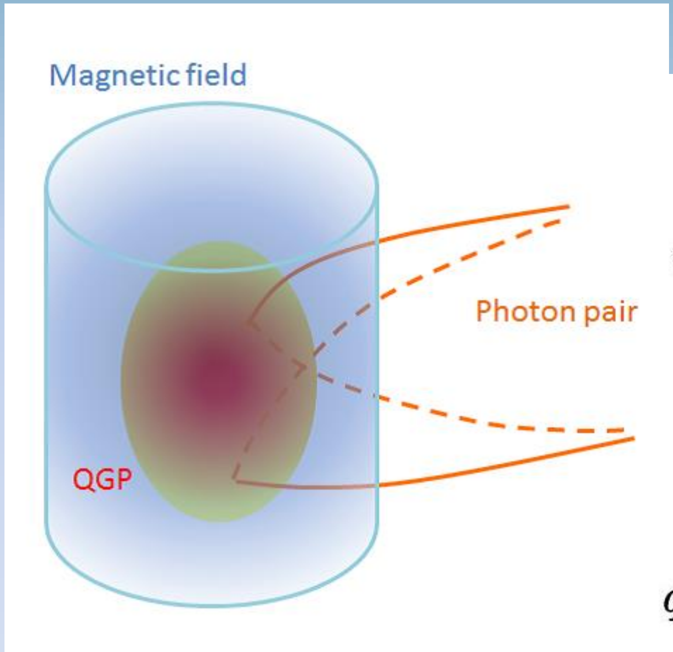
Br=*B*/*B*c = O(10^5) at RHIC $\tilde{\omega}^2$=10000 ←→ ω = 200MeV

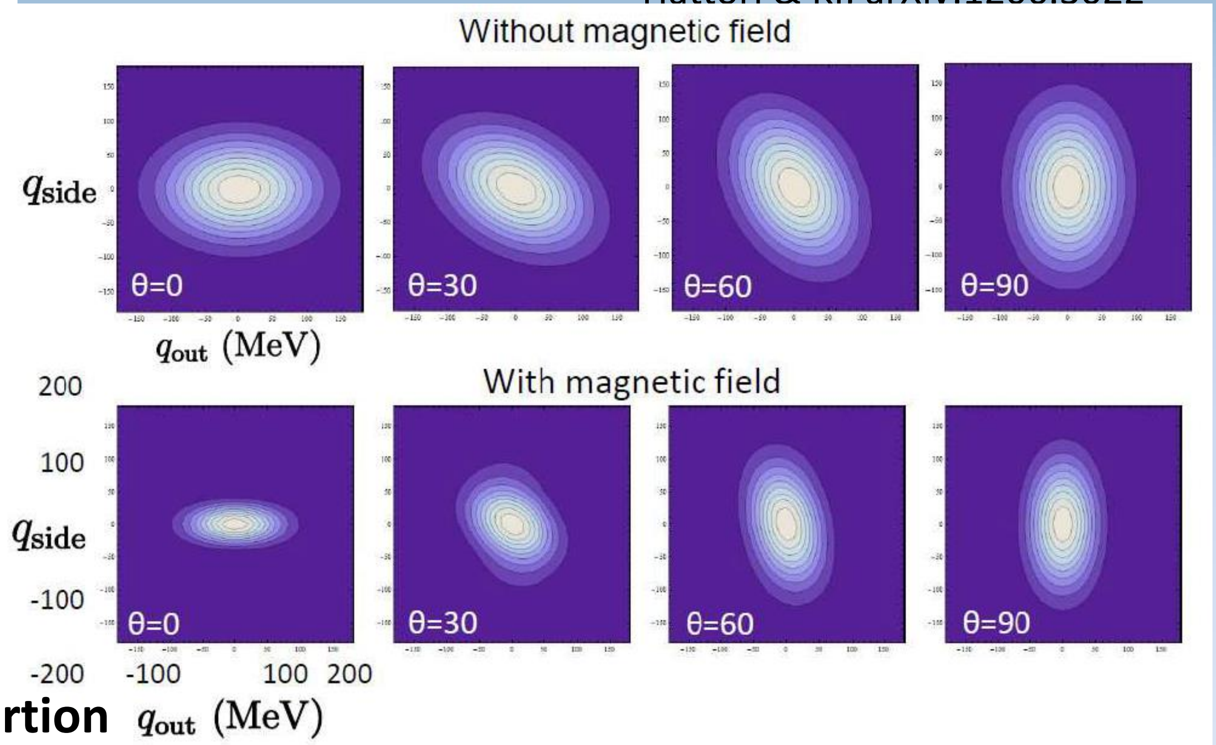
$$n_{\lim} = \sqrt{\epsilon_{\lim}} = |\cos\theta|^{-1}$$

Consequences in HIC

- **Generates elliptic flow (v_2) and higher harmonics (v_n)** (at very low momentum region)
- **Distorted photon ``HBT image"**

Based on a simple toy model with moderate modification
Hattori & KI. arXiv:1206.3022

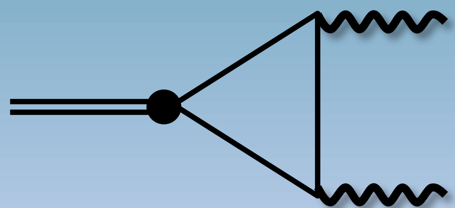




Magnification and distortion
← can determine the profile of photon source if spatial distribution of magnetic field is known.

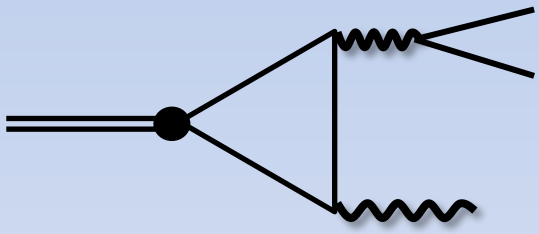
Neutral pion decay

- **Chiral anomaly** induces π^0 decay through triangle diagram



$$\pi^0 \to 2\gamma : \mathcal{O}\left(e^2\right)$$

Dominant (98.798 % in vacuum)



$$\pi^0 \to \gamma + e^+e^- : \mathcal{O}\left(e^3\right)$$

Dalitz decay (1.198 % in vacuum)
NLO contribution

99.996 %

- **Adler-Bardeen's theorem**

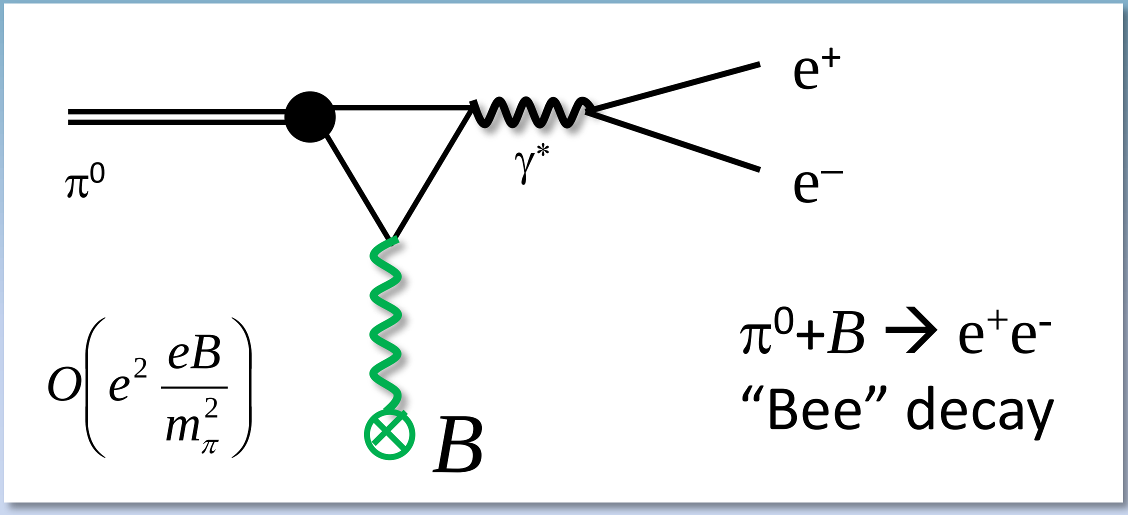
There is no radiative correction to the triangle diagram
Triangle diagram gives the exact result in all-order perturbation theory

→ only two photons can couple to π^0

Neutral pions in strong B

Hattori , KI, Ozaki, arXiv:1305.7224[hep-ph]

- There is only one diagram for a constant external field to be attached

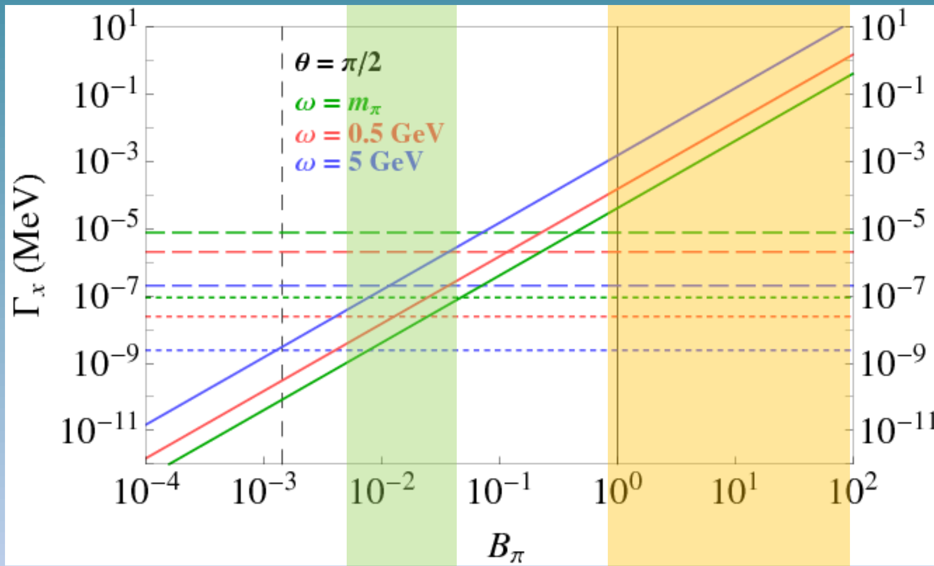


cf: axion
(very light, but
small coupling)

- Also implies
  -- conversion into γ with space-time varying B
  -- Primakoff process* ($\gamma^* + B \rightarrow \pi^0$): important in HIC
  -- mixing of π^0 and γ

* observed in nuclear Coulomb field

Decay rates of three modes



Solid : "Bee" decay
Dashed: 2γ decay
Dotted : Dalitz decay

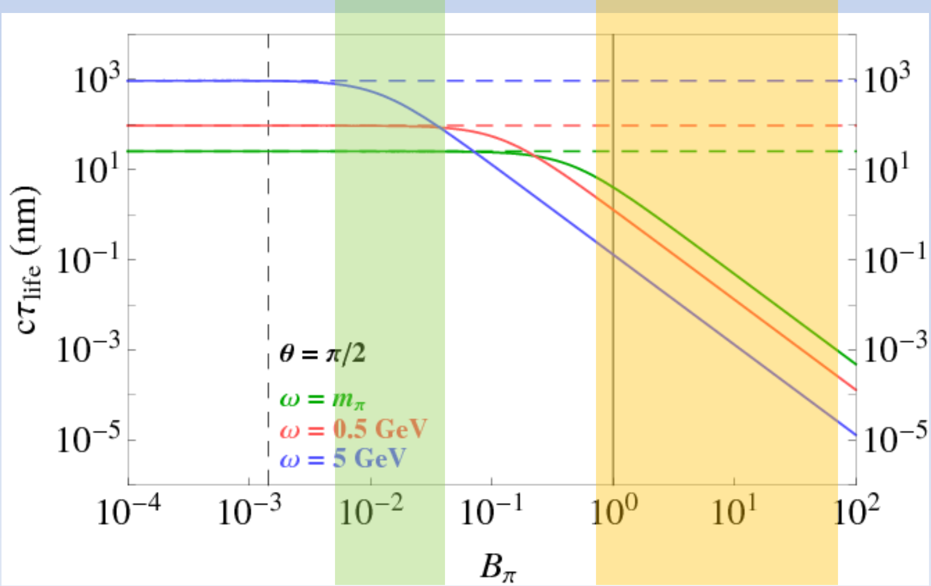
$$\Gamma_{Be^+e^-} = \frac{q^2 q_\parallel^2}{12\pi\omega_\pi}\left(\lambda\frac{eB}{q^2}\right)^2\left(1+\frac{2m^2}{q^2}\right)\sqrt{1-\frac{4m^2}{q^2}}$$

$$B_\pi = B/m_\pi^{\;2}$$

Mean lifetime

Magnetar

Heavy Ion Collision



← Picometer

← femtometer

$$\tau_{life} = \Gamma_{total}^{-1}$$

$$= \frac{1}{\Gamma_{2\gamma} + \Gamma_{Dalitz} + \Gamma_{Bee}}$$

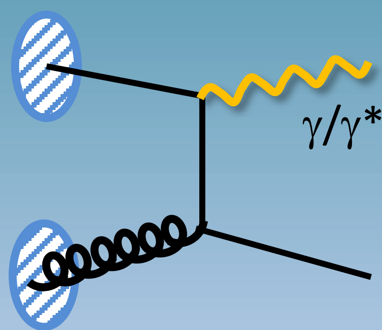
Energetic pions created in cosmic ray reactions will be affected

γ conversion into π^0 in HICs

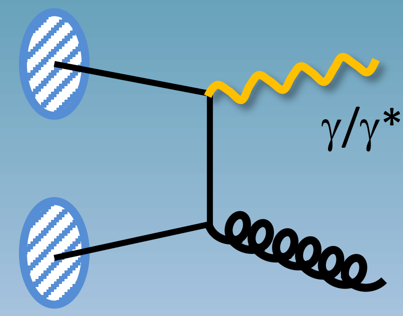
HICs create many high energy γs as well as γ^*s (decaying into dileptons)

nucleus

nucleus



Gluon Compton scattering in LO



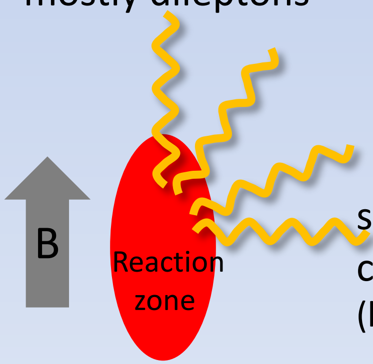
$q\bar{q}$ annihilation in LO

Some of γ^* will convert into π^0 in strong B, inducing **reduction of dilepton yield**

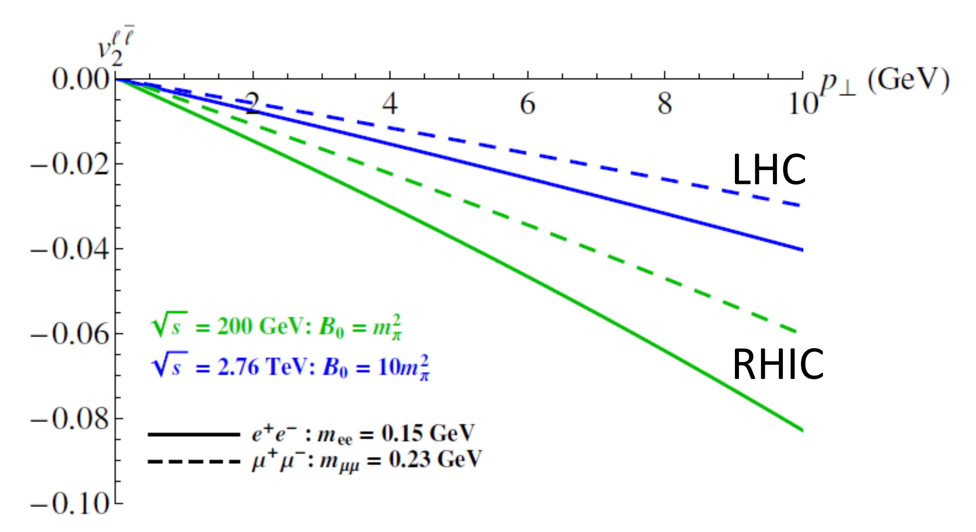
Conversion rate is strongest in perpendicular direction to B

→ negative elliptic flow of dileptons

mostly dileptons



some of them convert into π^0 (less dileptons)



- π^0 will get positive v2 but difficult to see
- Depends on time profile of B fields

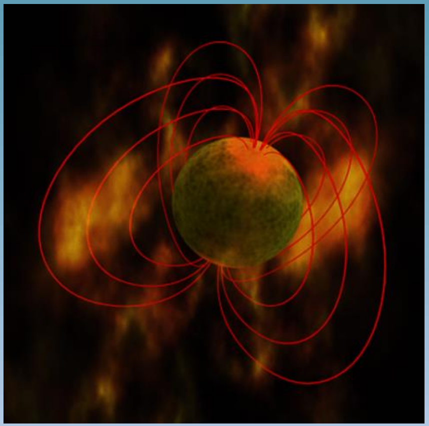
Summary

- Strong field physics can in principle provide useful information on early-time dynamics of HIC.
- Photons and neutral pions exhibit interesting phenomena in strong magnetic fields.
- Photons show birefringence and can decay into e+e- pairs. We obtained analytic representation of the polarization tensor and computed refractive indices.
- Chiral anomaly suggests that neutral pions can decay into e+e- without an accompanying photon, which becomes the dominant decay mode in strong magnetic fields.
- Conversion of a virtual photon into a neutral pion is also possible and can be seen as negative elliptic flow of dileptons in heavy-ion collisions.

How strong?

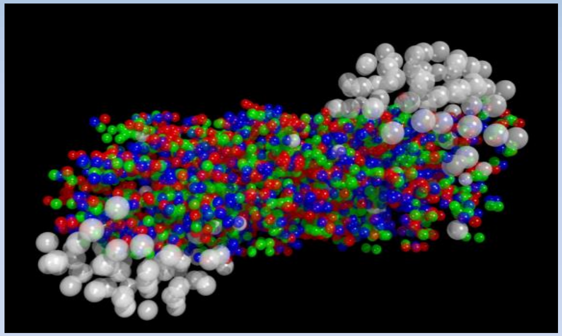
1 Tesla = 10^4 Gauss

10^{17}—10^{18} Gauss
$\sqrt{eB} \sim 1 - 10\ m_\pi$:
Noncentral heavy-ion coll. at RHIC and LHC
Also strong Yang-Mills fields $\sqrt{gB}$ ~ 1– a few GeV



10^{15}Gauss : Magnetars

4×10^{13} Gauss : "Critical" magnetic field of electrons
$\sqrt{eB_c} = m_e = 0.5\text{MeV}$



45 Tesla : strongest steady magnetic field
(High Mag. Field. Lab. In Florida)

10^8Tesla=10^{12}Gauss:
Typical neutron star surface

8.3 Tesla :
Superconducting magnets in LHC

Super critical magnetic field may have existed in very early Universe.
Maybe after EW phase transition? (cf: Vachaspati '91)

Time profile of magnetic fields

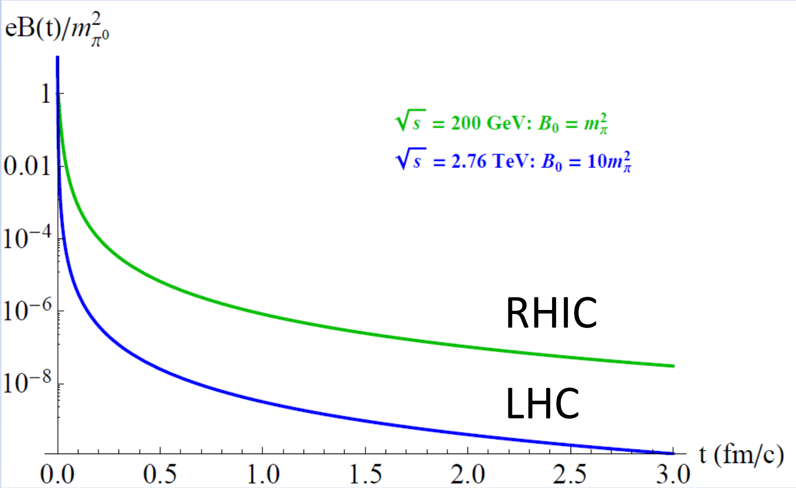
Mimicked by

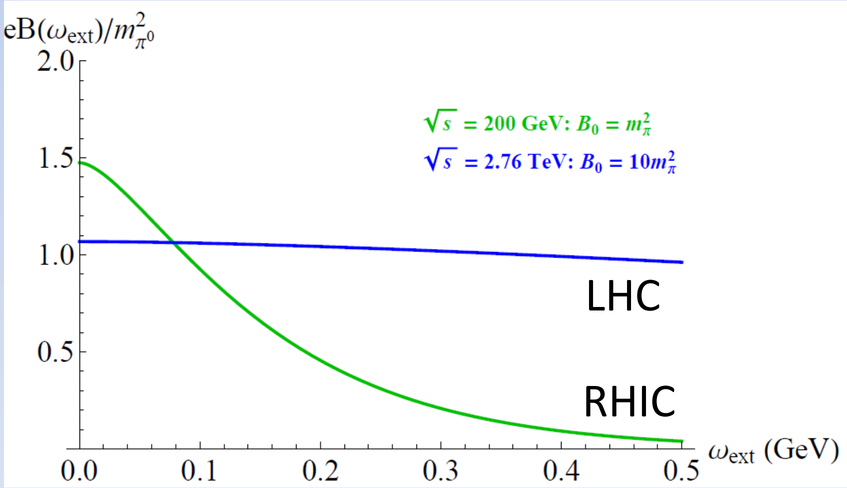
$$eB(t) = \kappa / (t^2 \sinh^2 y_b + \zeta^2)^{3/2}$$

$$\zeta = 1 \text{ fm}, \; eB(0) = m_\pi^2 \; (10 m_\pi^2)$$

$$\text{beam rapidity } y_b = 5.36 \; (7.98)$$

$$\sqrt{s} = 0.2 \; (2.76) \text{ TeV}$$





$$eB(\omega_{\rm ext}) = \sqrt{2/\pi} \cdot \kappa \, |\omega_{\rm ext}| / (\zeta \sinh^2 y_b) \cdot K_1 \left(\frac{\zeta \, |\omega_{\rm ext}|}{\sinh y_b} \right)$$