



RANP 2013

Takeshi Kodama's Fest

$Z_c^+(3900)$ decay width from
QCD sum rules

M. Nielsen
Universidade de São Paulo

in coll. with J.M.Dias, F.S. Navarra & C.M. Zanetti



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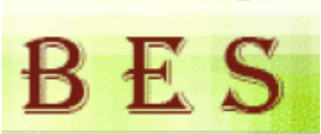
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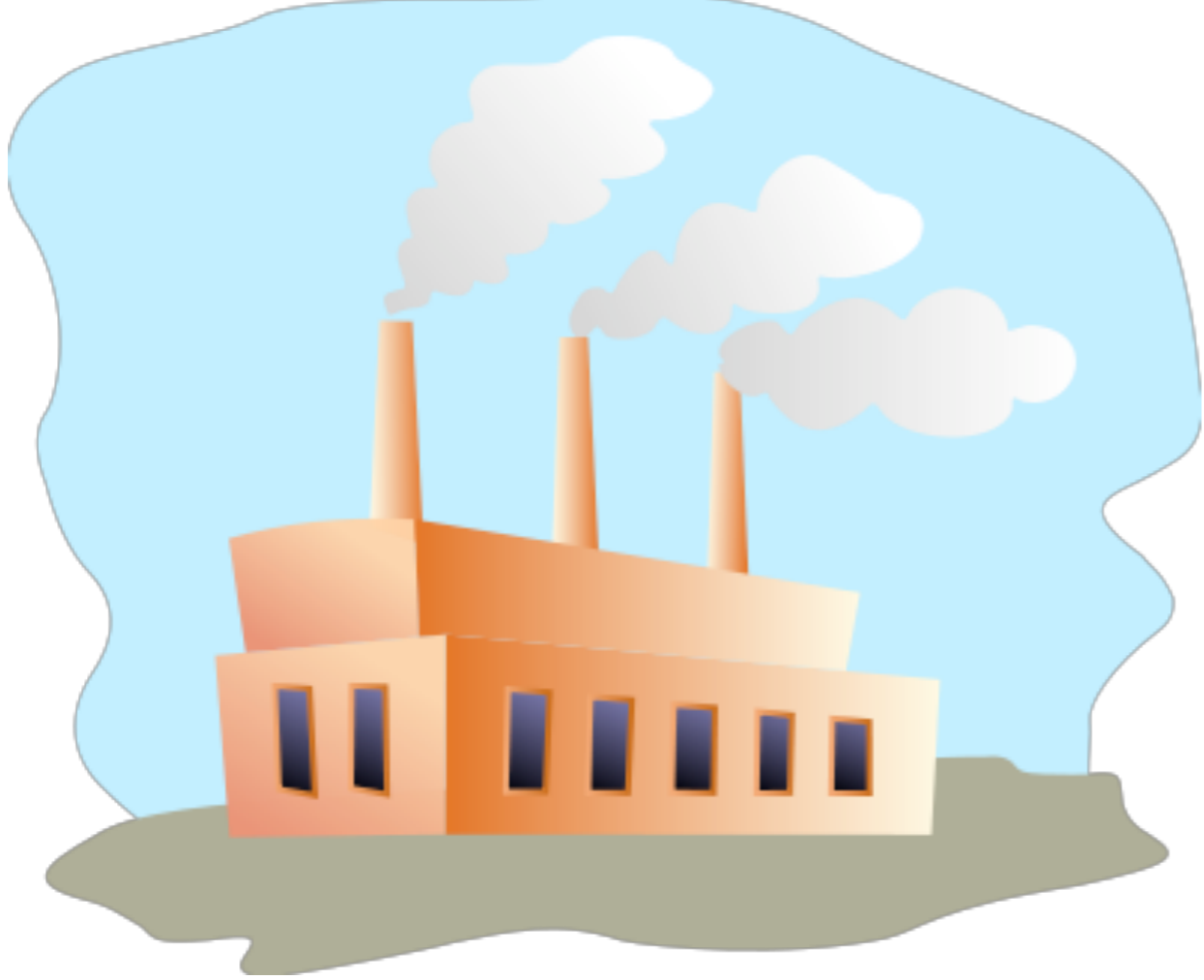
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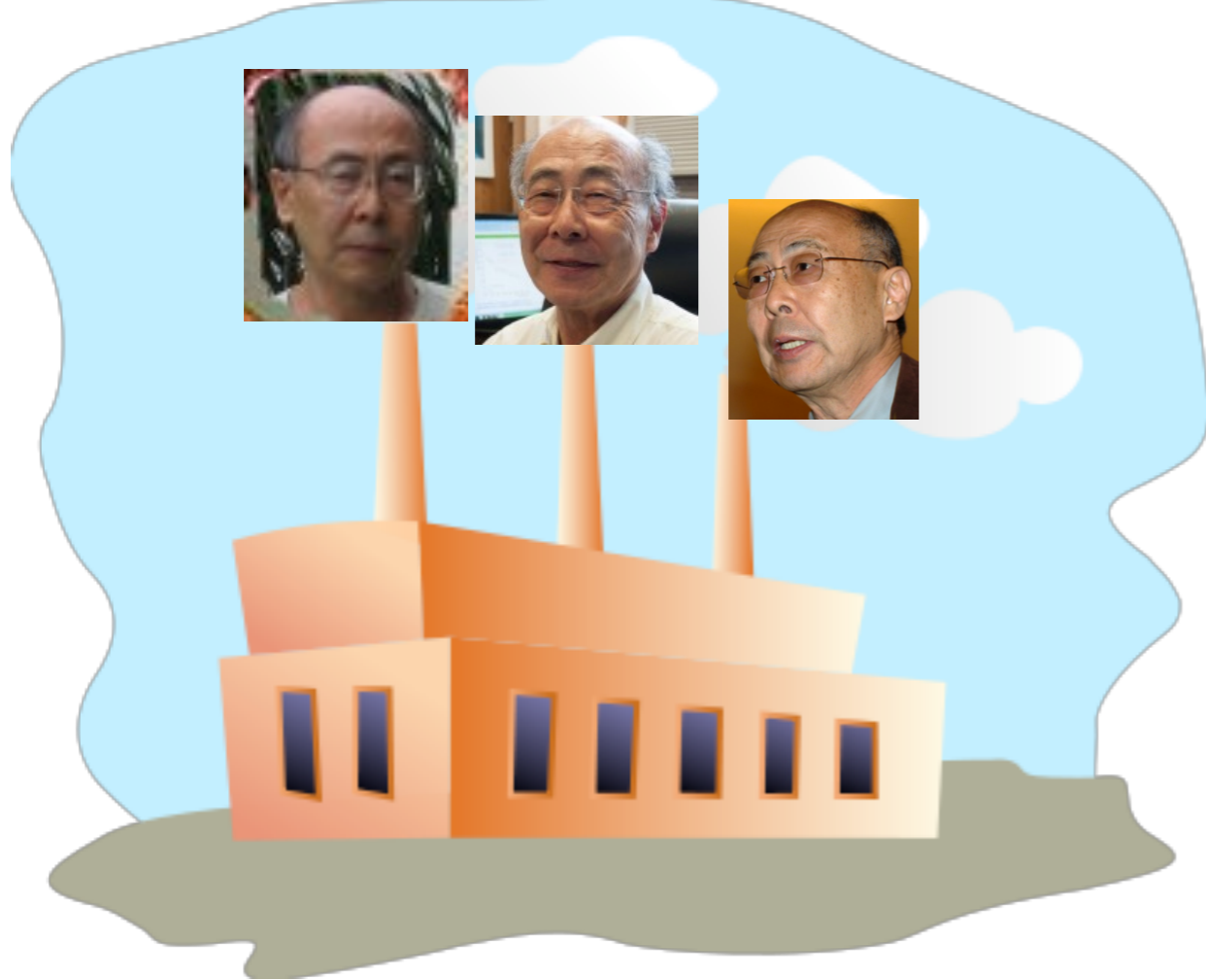
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Some Charmonium states discovered at the B factories

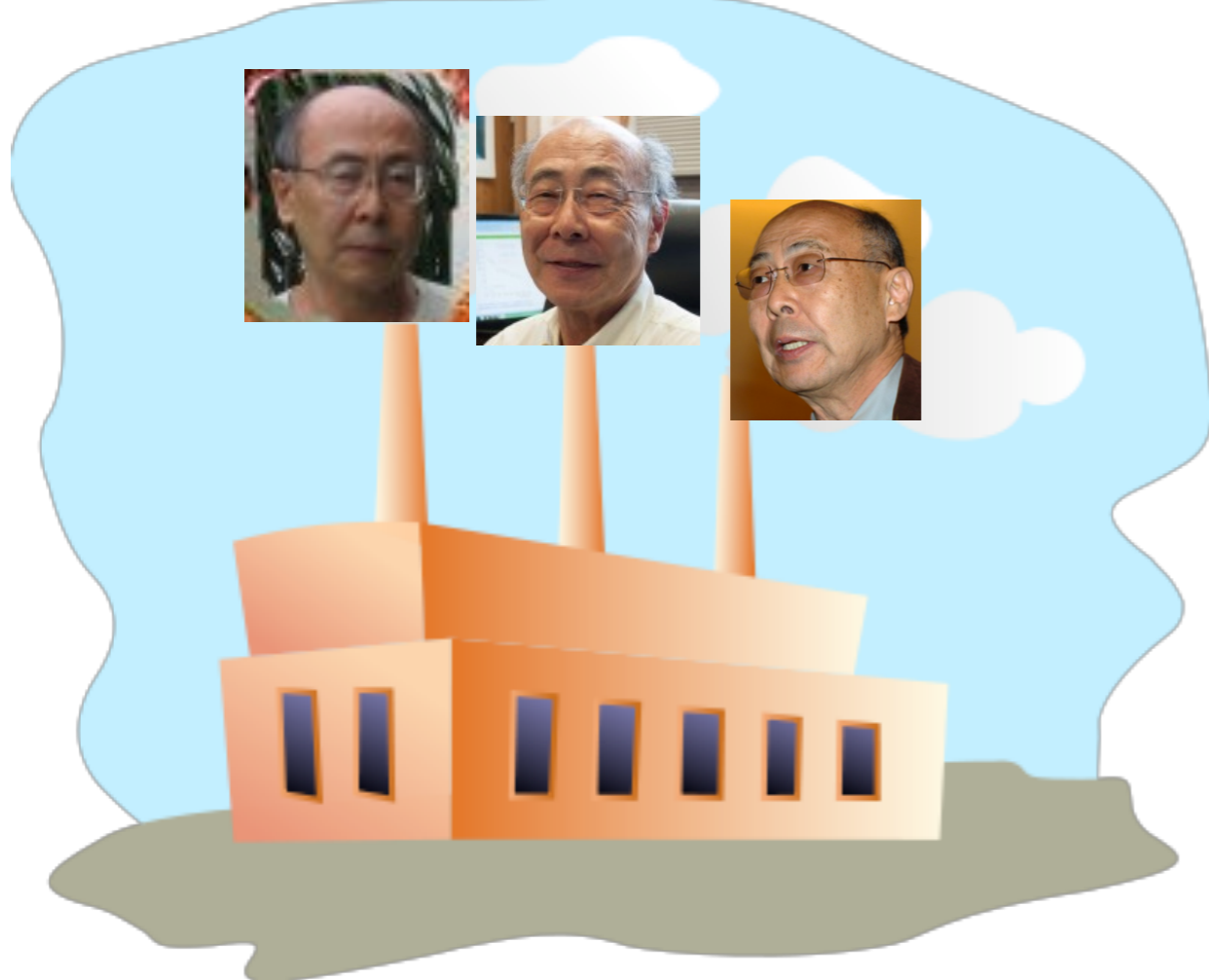
 <p>X(3872) 2003</p>    	 <p>Y(4260) 2005</p>  	 <p>Z⁺(4430) 2007</p>  
 <p>Y(4360) 2006</p> 	<p>Y(4660) 2007</p> 	<p>Z₁⁺(4050) 2008</p> 
<p>Z₂⁺(4250) 2008</p> 	 <p>Y(4140) 2009</p>   <p>$\gamma\gamma$</p>	<p>Z_c⁺(3900) 2013</p>  



B factories



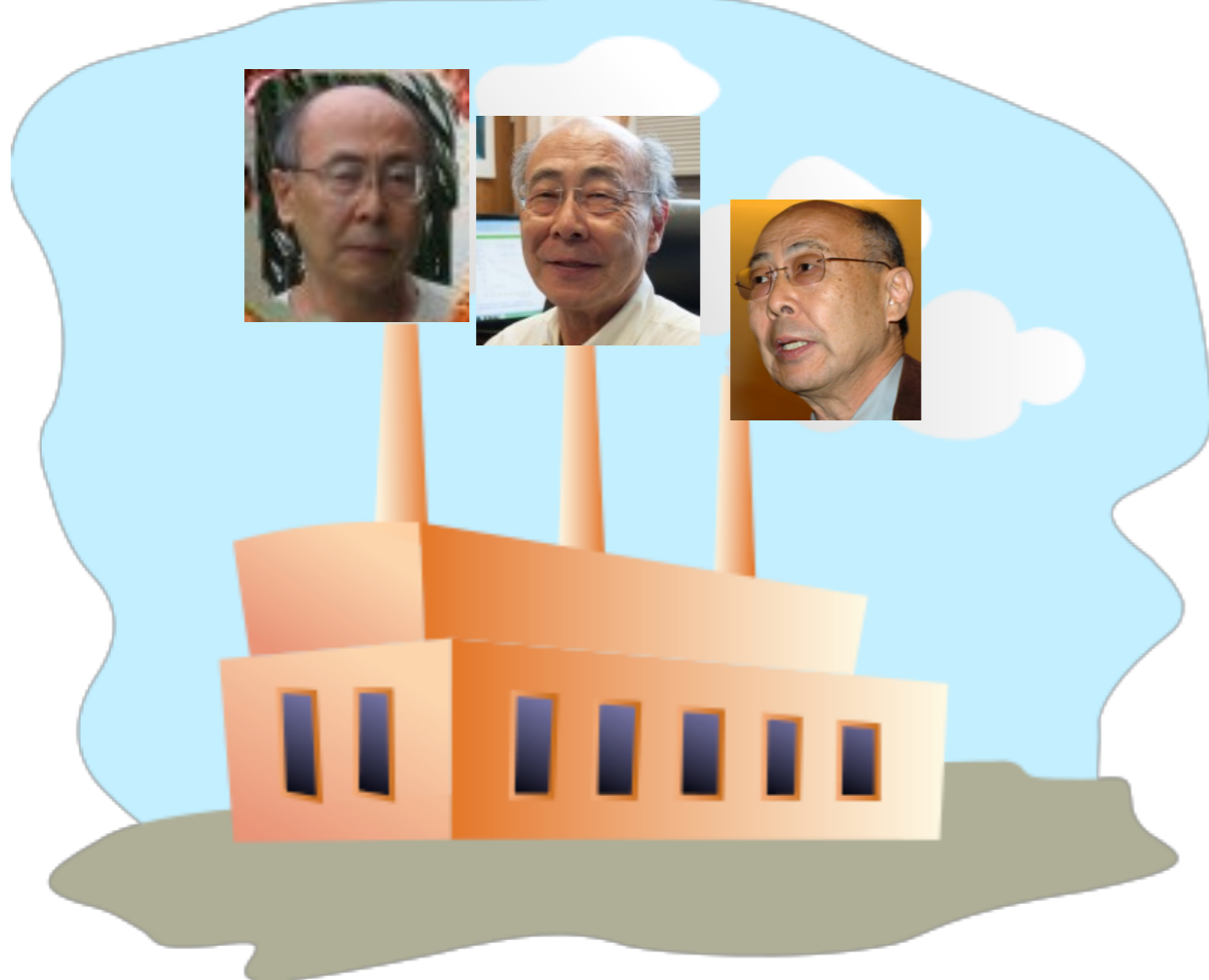
B factories



B factories

$e^+ e^-$ colliders with CM energy of 10.6 GeV





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$e^+ e^-$ colliders with CM energy of 10.6 GeV

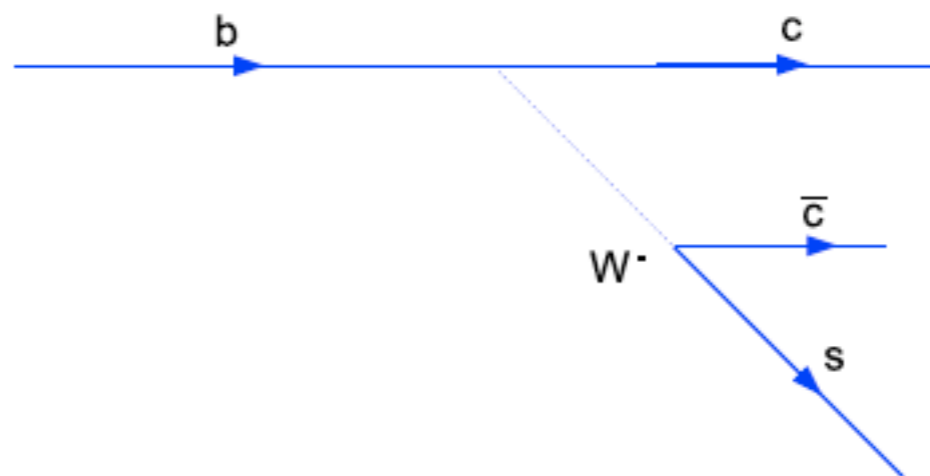


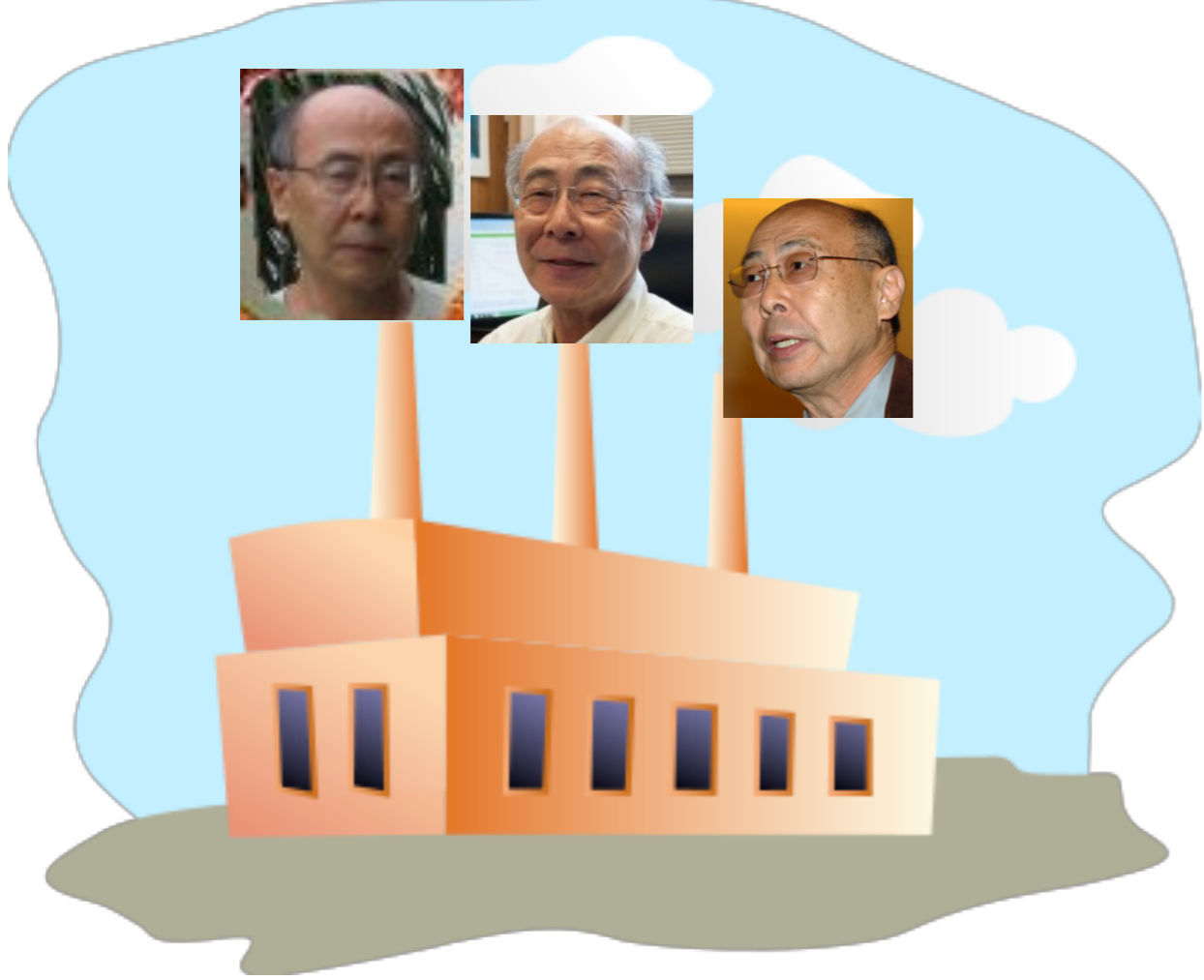
B meson: $b\bar{q}$, $q\bar{b}$

$$b \rightarrow W^- c :$$

50% of the times \rightarrow

$$b \rightarrow c\bar{c}s$$





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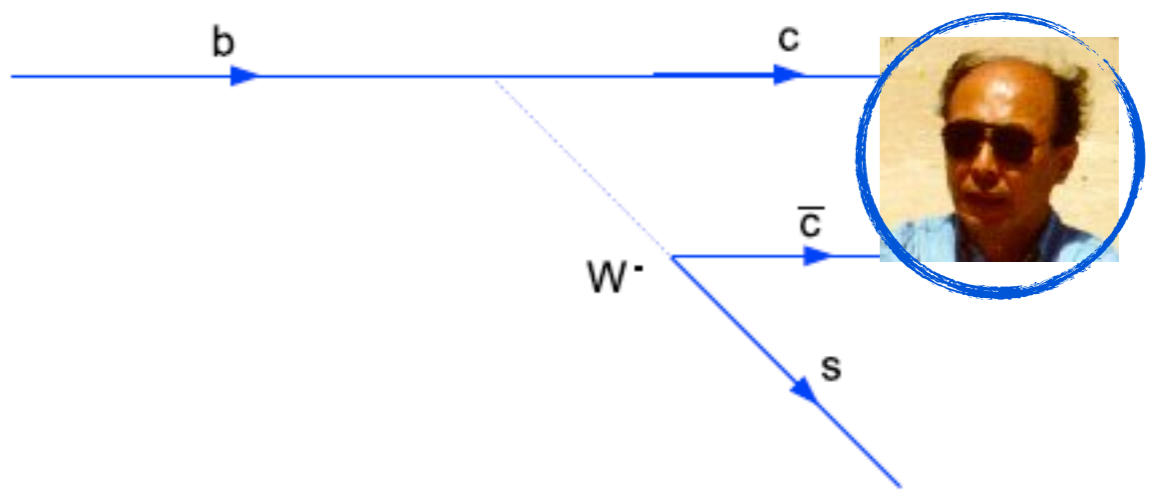


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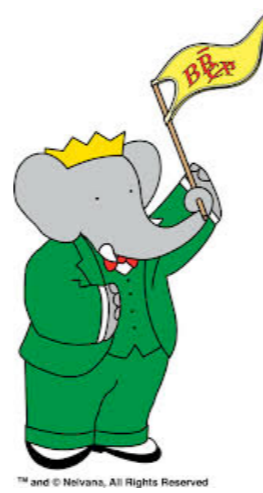


<p style="text-align: center;">X(3872)</p> <p style="text-align: center;"><i>$B^\pm \rightarrow K^\pm (J/\psi \pi^+ \pi^-)$</i></p>	<p style="text-align: center;">Y(4260)</p> <p style="text-align: center;"><i>$e^+ e^- \rightarrow \gamma_{IRS} (J/\psi \pi^+ \pi^-)$</i></p>	<p style="text-align: center;">Z⁺(4430)</p> <p style="text-align: center;"><i>$\bar{B}^0 \rightarrow K^- (\psi' \pi^+)$</i></p>
<p style="text-align: center;">Y(4360)</p> <p style="text-align: center;"><i>$e^+ e^- \rightarrow \gamma_{IRS} (\psi' \pi^+ \pi^-)$</i></p>	<p style="text-align: center;">Y(4660)</p> <p style="text-align: center;"><i>$e^+ e^- \rightarrow \gamma_{IRS} (\psi' \pi^+ \pi^-)$</i></p>	<p style="text-align: center;">Z₁⁺(4050)</p> <p style="text-align: center;"><i>$\bar{B}^0 \rightarrow K^- (\chi_{c1} \pi^+)$</i></p>
<p style="text-align: center;">Z₂⁺(4250)</p> <p style="text-align: center;"><i>$\bar{B}^0 \rightarrow K^- (\chi_{c1} \pi^+)$</i></p>	<p style="text-align: center;">Y(4140)</p> <p style="text-align: center;"><i>$B^+ \rightarrow K^+ (\phi J/\psi)$</i></p>	<p style="text-align: center;">Z_c⁺(3900)</p> <p style="text-align: center;"><i>$Y(4260) \rightarrow (J/\psi \pi^+) \pi^-$</i></p>

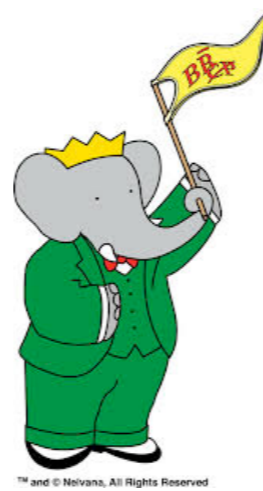
Common features

- All these states decay into J/ψ (ψ') \rightarrow they have a $c\bar{c}$ pair in their quark components
- Their masses are not compatible with quark model calculations for charmonium states
- Absence of open charm production in their decays is inconsistent with $c\bar{c}$ interpretation
- Candidates for exotic (not quark-antiquark) states

X(3872)



X(3872)



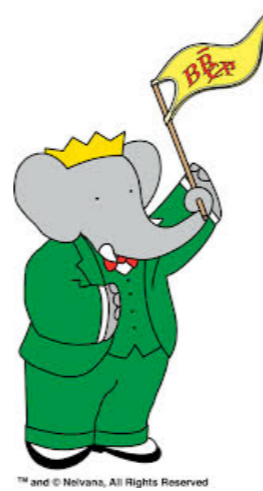
$$M(D^{*0}\bar{D}^0) = (3871 \pm 1)$$

X(3872): molecular ($D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0$) state (Swanson, Close, Voloshin, Wong ...)

Tornqvist (ZPC61(94)) predict a $\bar{D}D^*$ molecule with $J^{PC} = 0^{-+}$ or 1^{++}

Maiani et al. (PRD71 (05)) tetraquark $J^{PC} = 1^{++}$ state

X(3872)

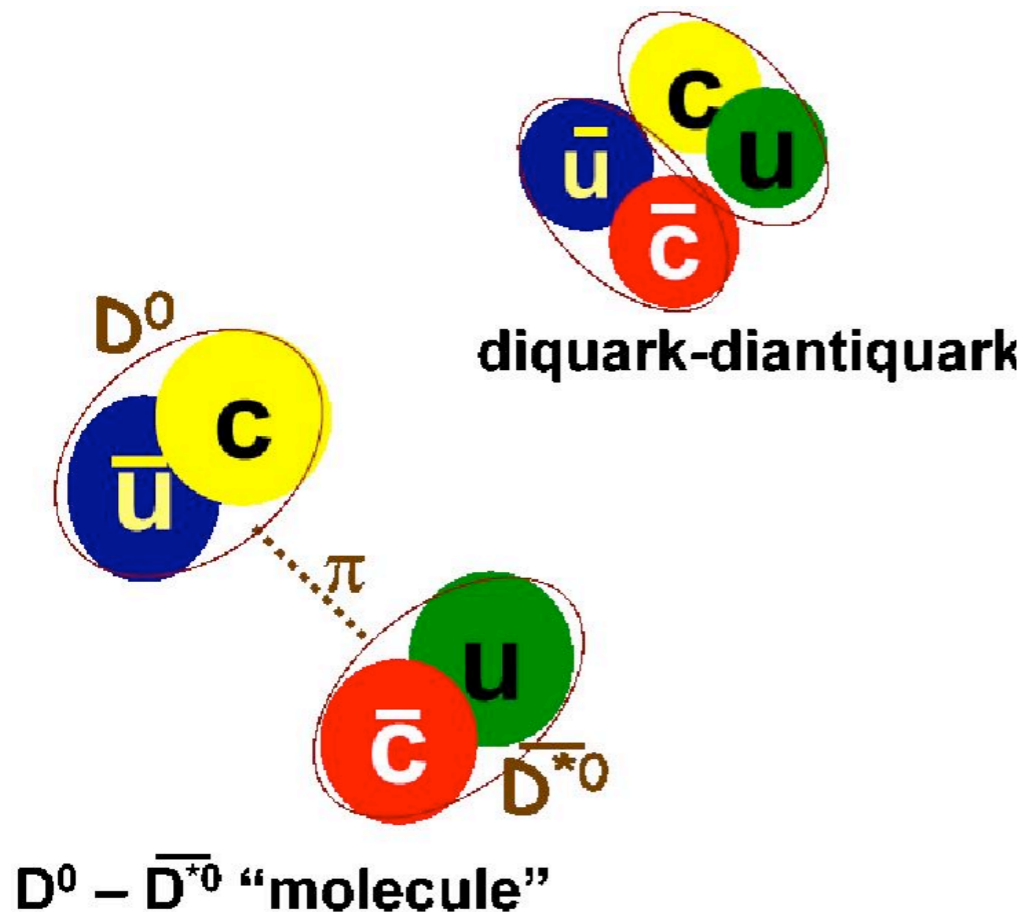


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molecular and tetraquark interpretations differ by the way quarks are organized in the state



$Z^+(4430)$

charged state \rightarrow
not a $c\bar{c}$!

PRL100(08)142001



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PRL100(08)142001



arXiv:0905.2869

searched $Z^-(4430)$ in 4 decay modes:

no conclusive
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Belle



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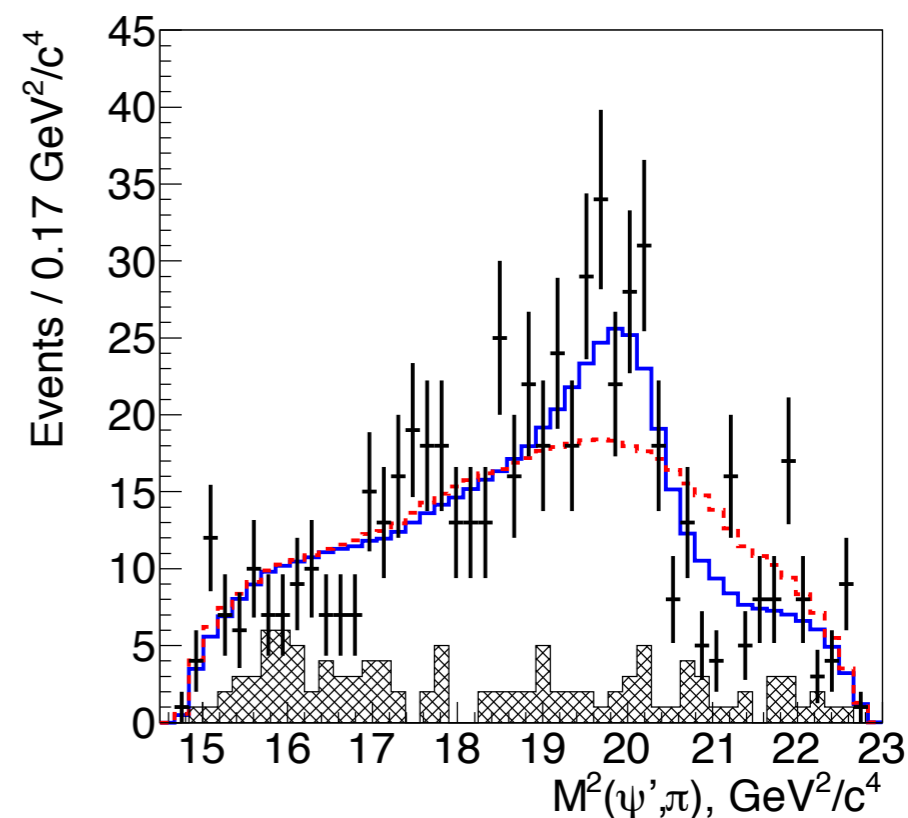
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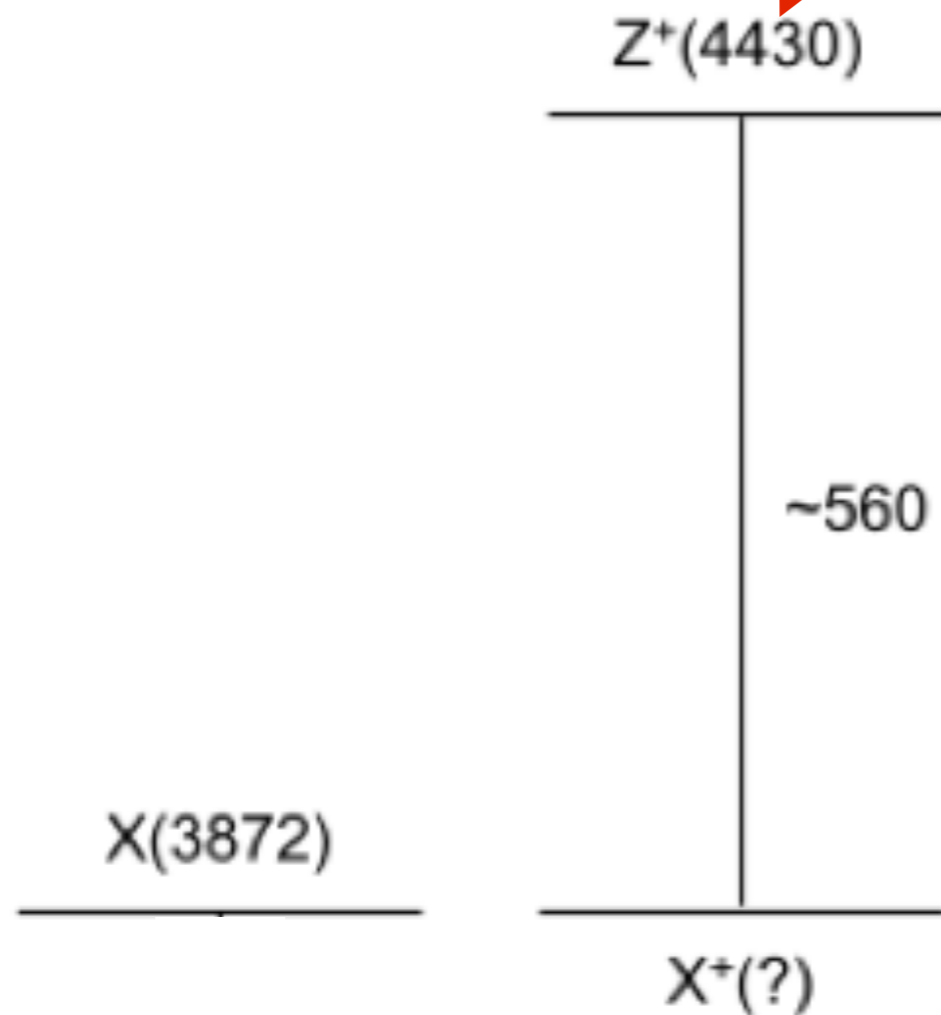


arXiv:1306.4894

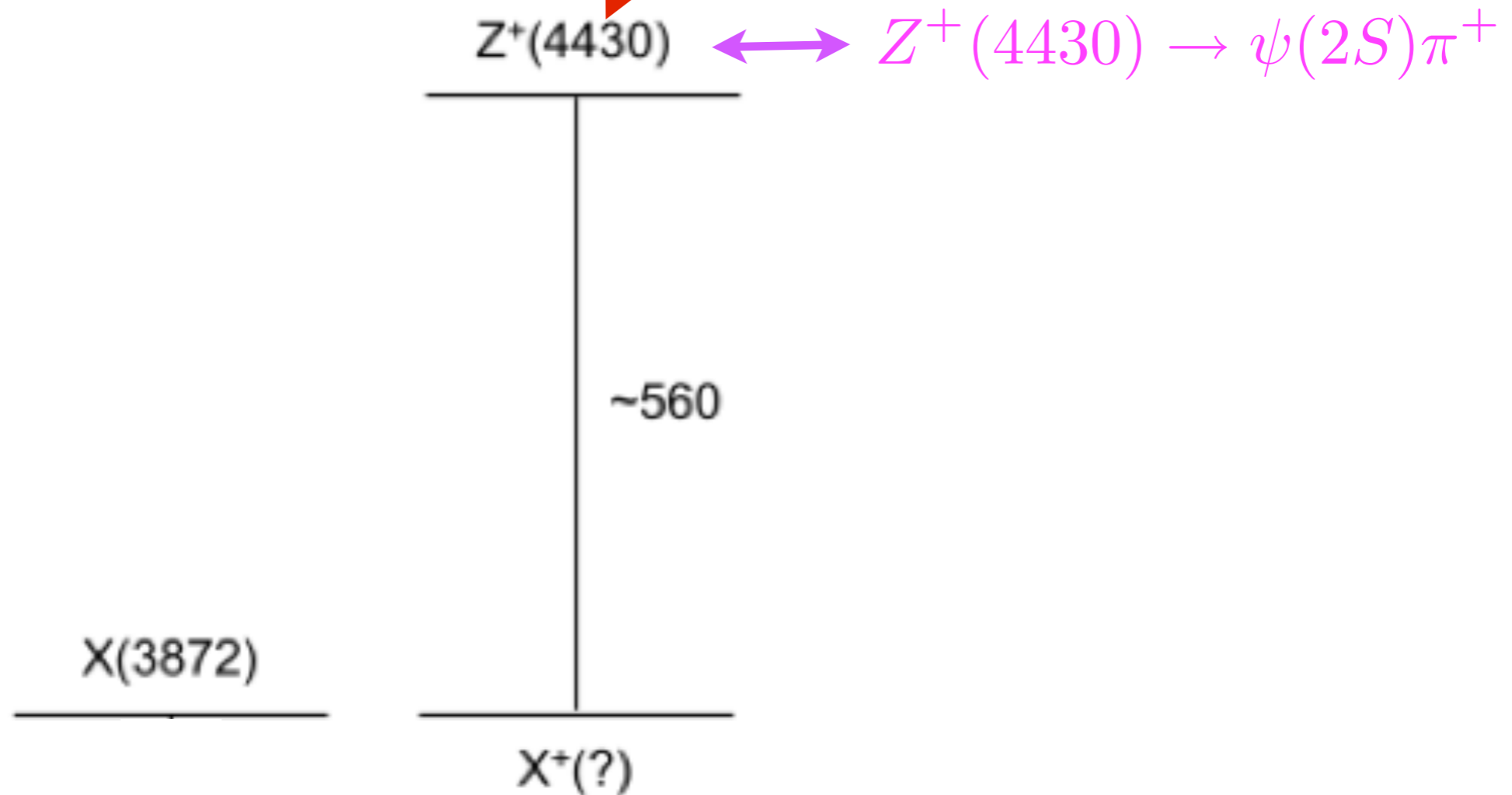
confirm the
observation of
 $Z^+(4430)$ with
 6.1σ and $J^+ = 1^+$



Maiani et al. (arXiv:0708.3997) : four-quark radial excitation of the I^+ charged state (X(3872) partner)

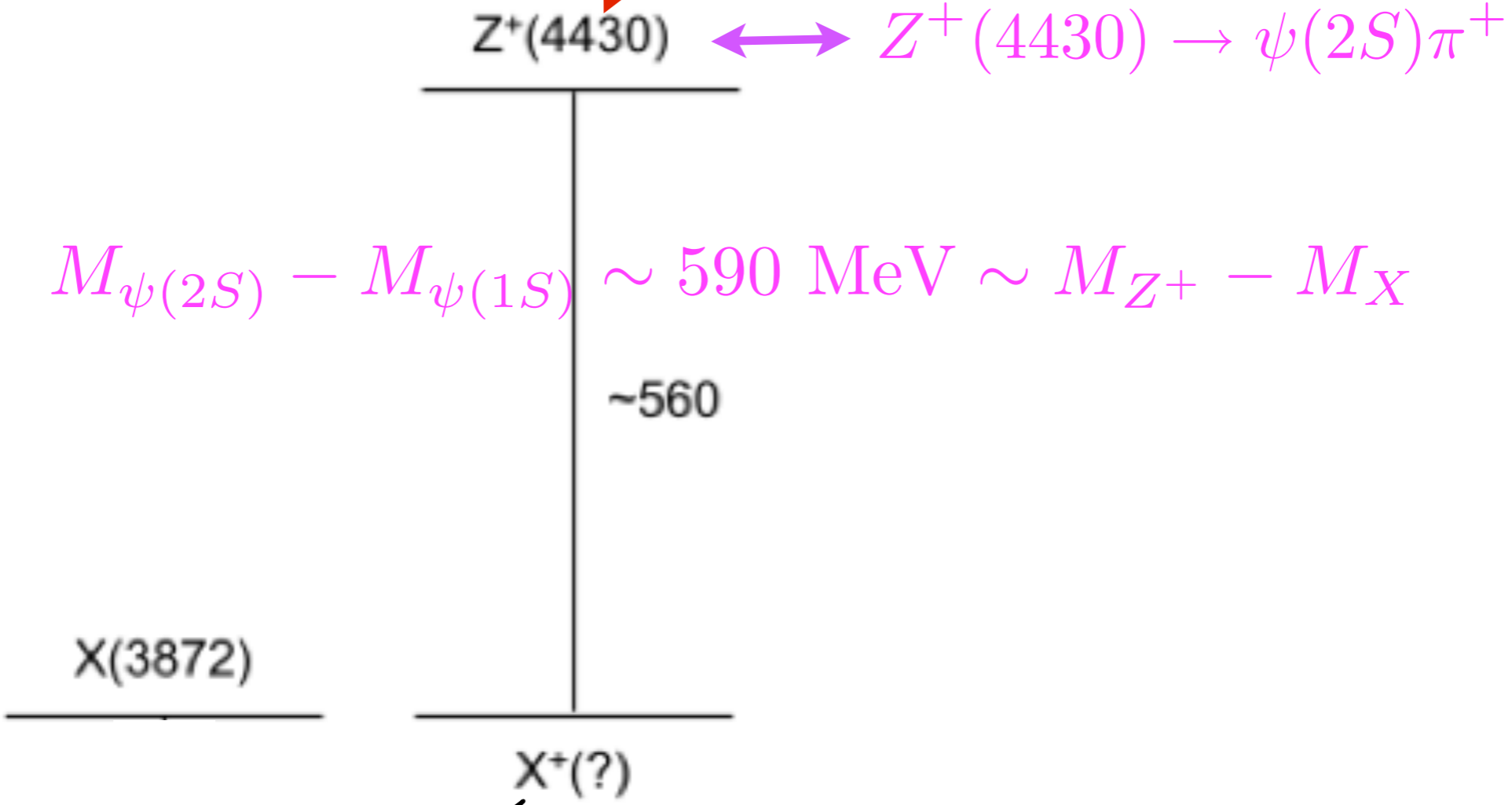


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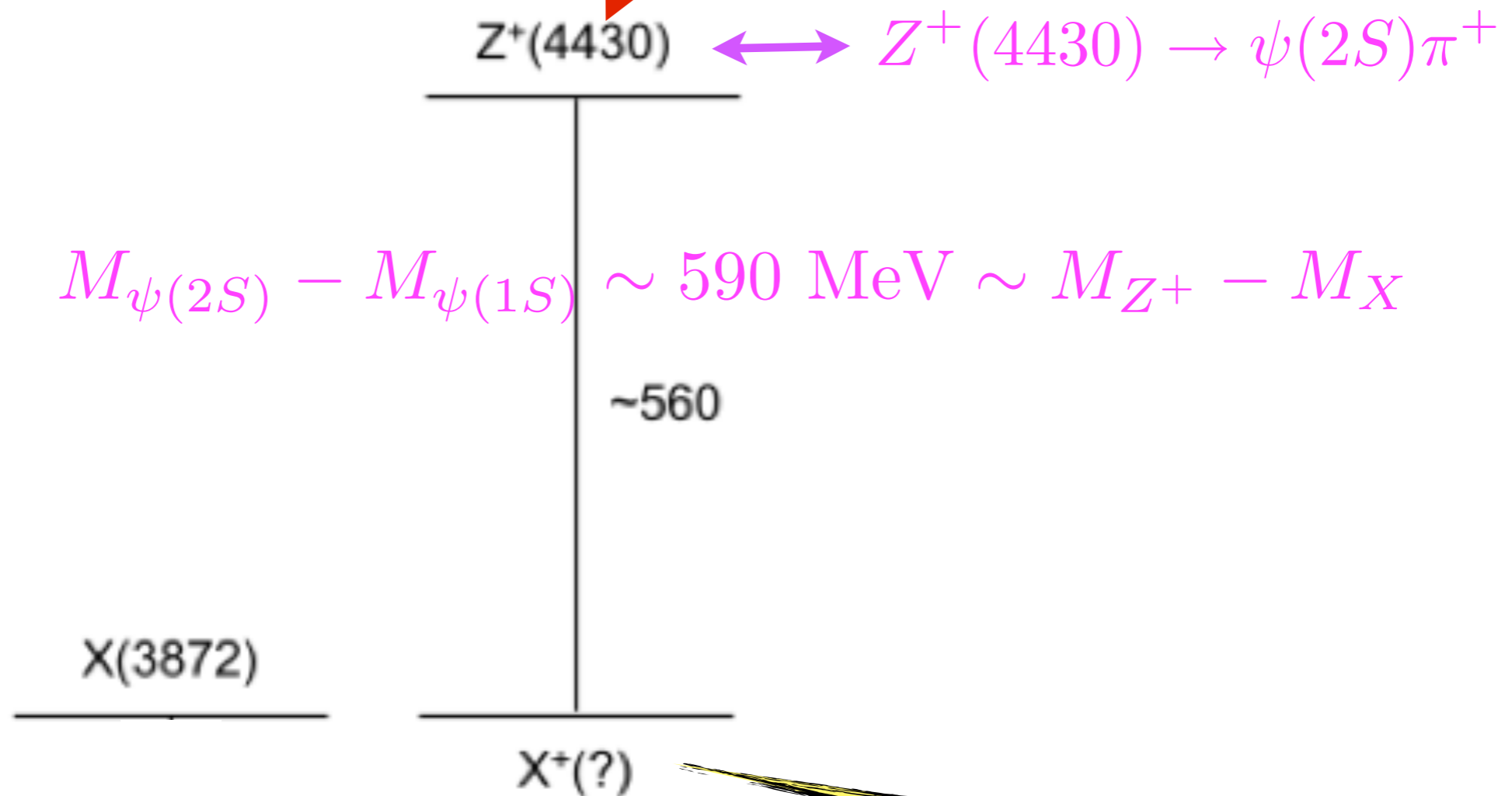
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who is the charged partner of X(3872)?

$Z_c^+(3900)$: most recent acquisition

BES III

arXiv:1303.5949



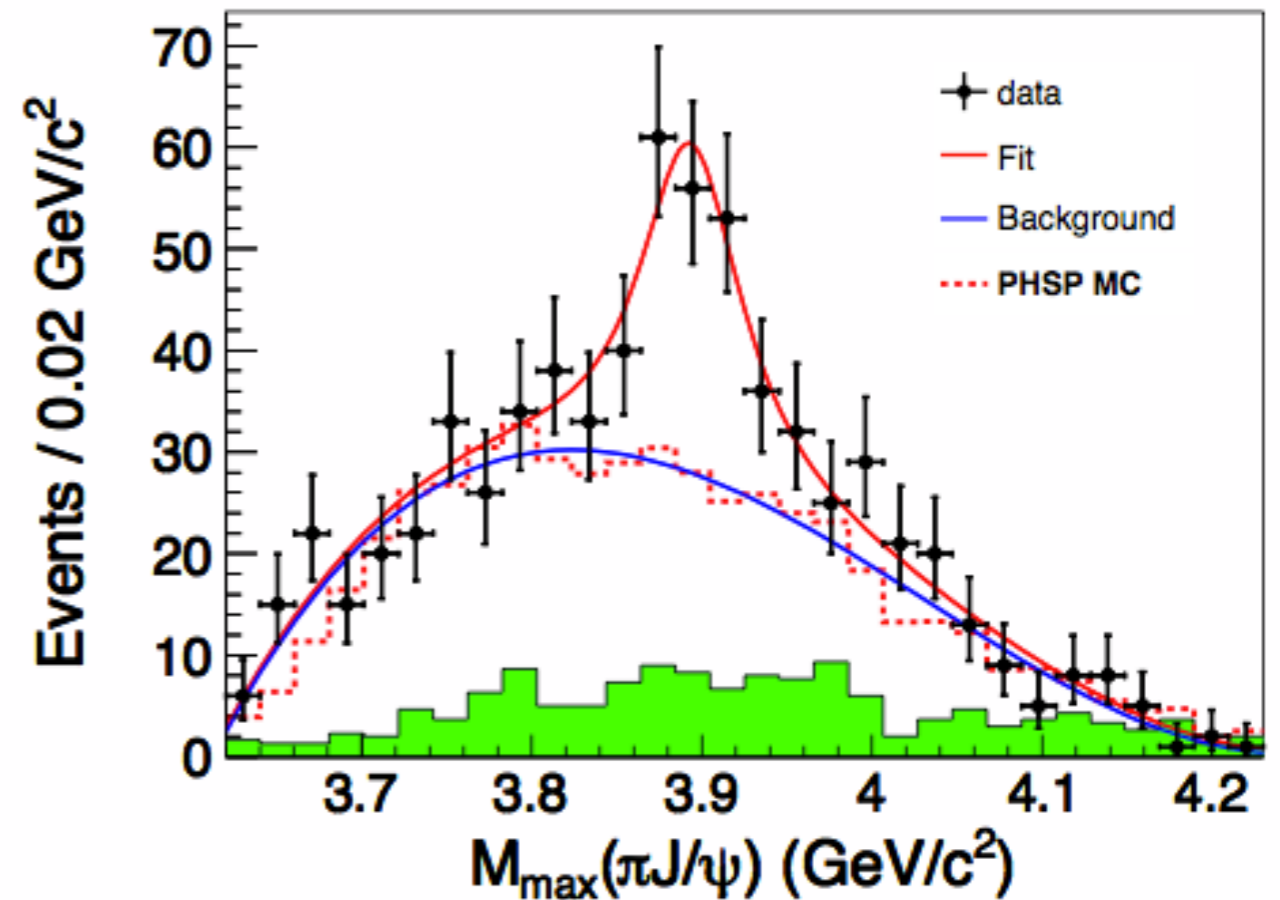
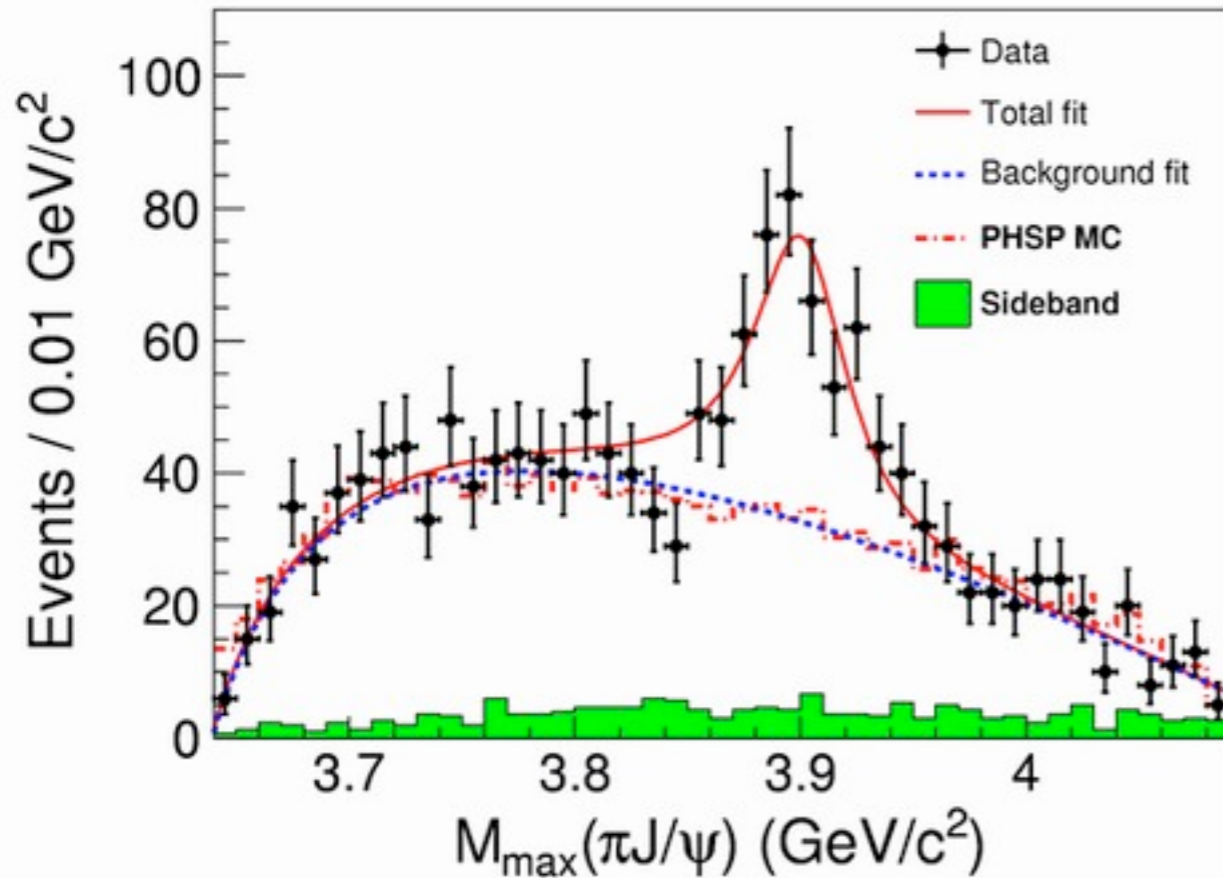
arXiv:1304.0121

$$M = (3890 \pm 3.6 \pm 4.9) \text{ MeV}$$

$$\Gamma = (46 \pm 10 \pm 20) \text{ MeV}$$

$$M = (3894.5 \pm 6.6 \pm 4.5) \text{ MeV}$$

$$\Gamma = (63 \pm 24 \pm 26) \text{ MeV}$$



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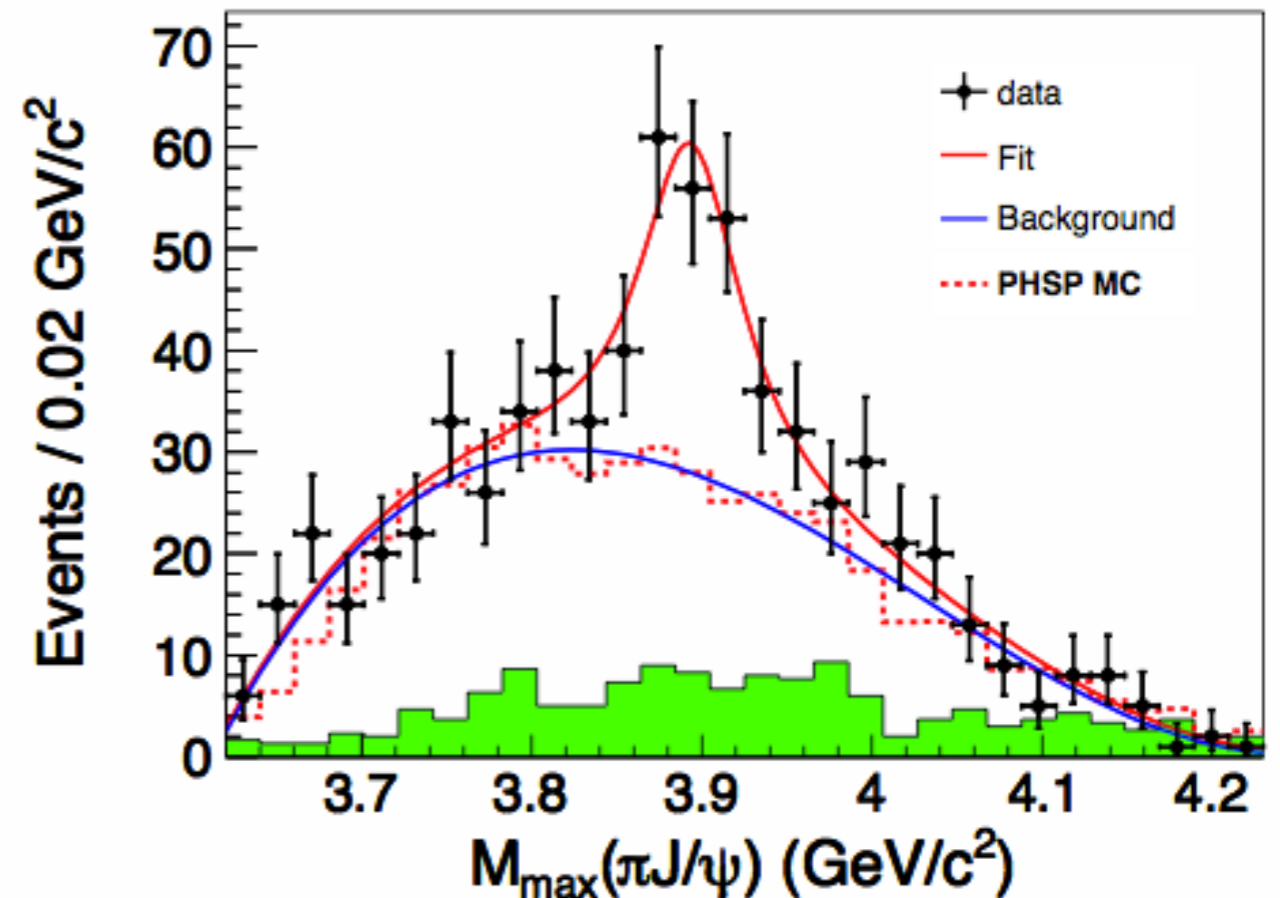
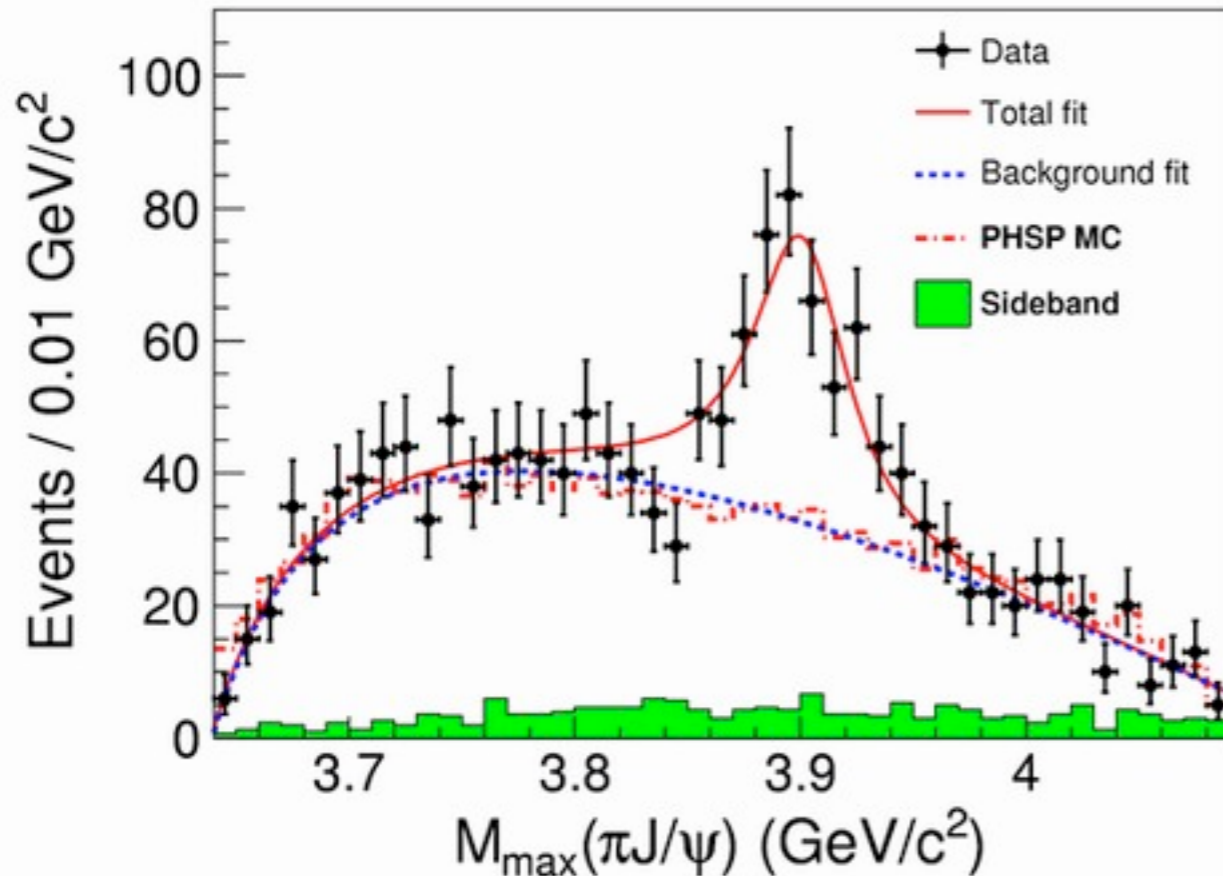
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$J^P = 1^+ \rightarrow$ charged partner of the $X(3872)$

QCD Sum Rule

Fundamental Assumption: Principle of Duality

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j(x)j^\dagger(0)] | 0 \rangle$$

Theoretical side

Phenomenological side

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Theoretical side

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j(x)j^\dagger(0)] | 0 \rangle = \sum_n C_n(Q^2) \hat{O}_n$$

Phenomenological side

$$\Pi(q^2) = - \int ds \frac{\rho(s)}{q^2 - s + i\epsilon} + \dots$$

$$\rho(s) = \lambda^2 \delta(s - m^2) + \rho_{cont}(s)$$

$$\langle 0 | j | H \rangle = \lambda$$

$$\rho_{cont}(s) = \rho^{OPE}(s) \Theta(s - s_0)$$

s_0 : continuum parameter

$$\Pi^{phen}(Q^2) \leftrightarrow \Pi^{OPE}(Q^2) \quad \longrightarrow \quad \text{Borel transform}$$

$$\lambda^2 e^{-m^2/M^2} = \int_{s_{min}}^{s_0} ds e^{-s/M^2} \rho^{OPE}(s)$$

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Good Sum Rule \Rightarrow Borel window such that:

- pole contribution $>$ continuum contribution
- good OPE convergence
- good Borel stability

$$m^2 = \frac{\int_{s_{min}}^{s_0} ds e^{-s/M^2} s \rho^{OPE}(s)}{\int_{s_{min}}^{s_0} ds e^{-s/M^2} \rho^{OPE}(s)}$$

QCD sum rules calculation for $X(3872)$

Matheus, Narison, MN, Richard: PRD75 (07)

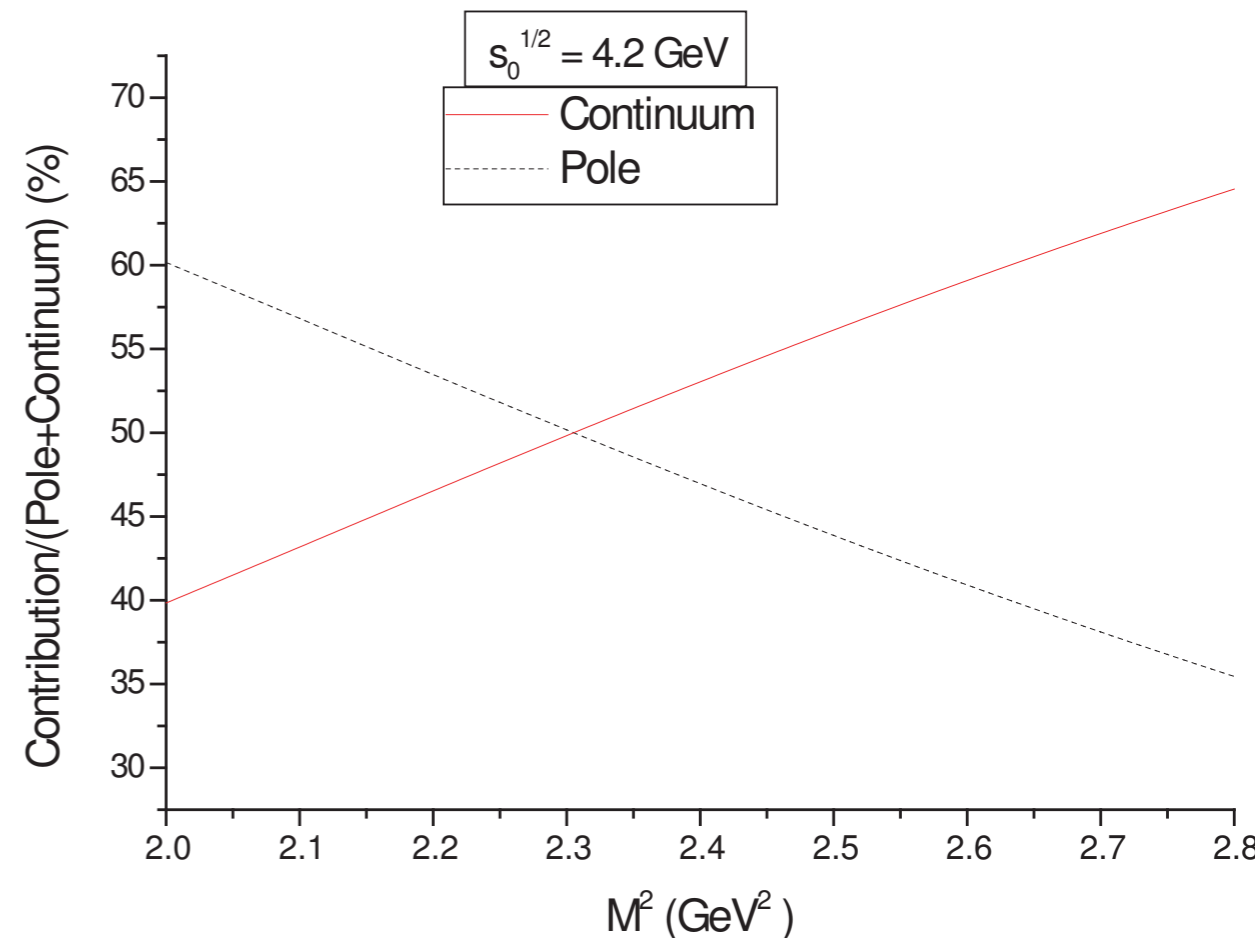
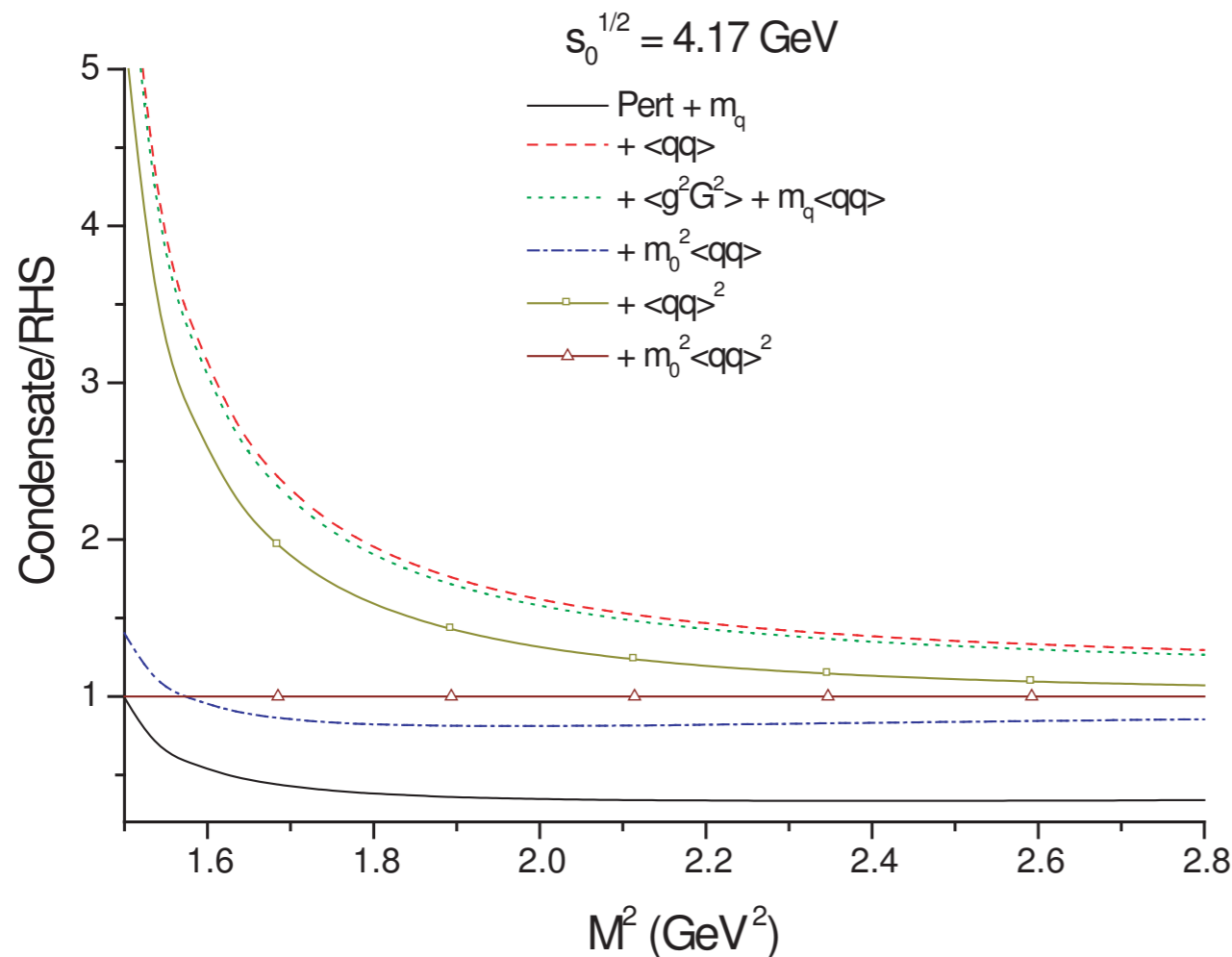
$$j_\mu = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(q_a^T C \gamma_5 c_b)(\bar{q}_d \gamma_\mu C \bar{c}_e^T) + (q_a^T C \gamma_\mu c_b)(\bar{q}_d \gamma_5 C \bar{c}_e^T)]$$

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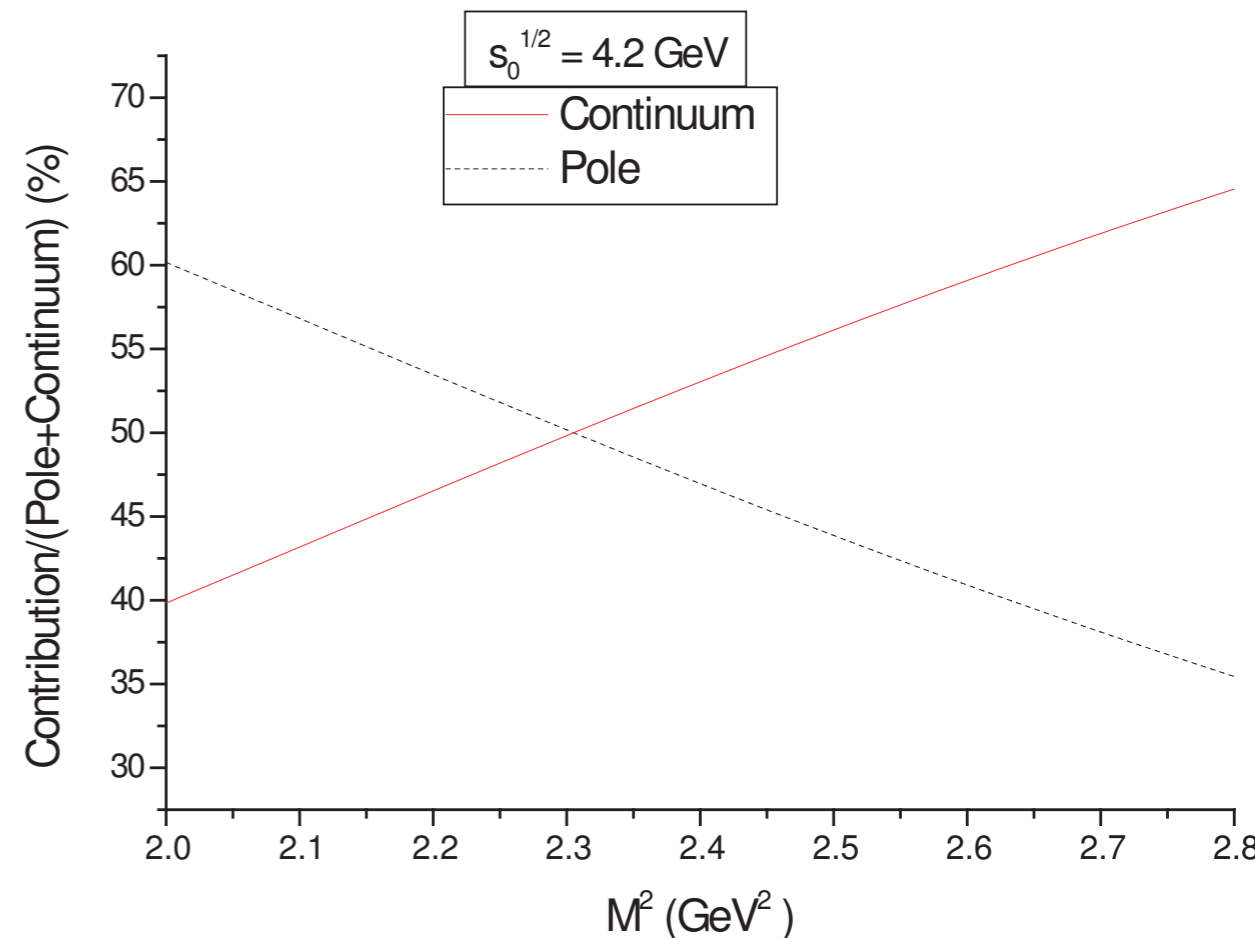
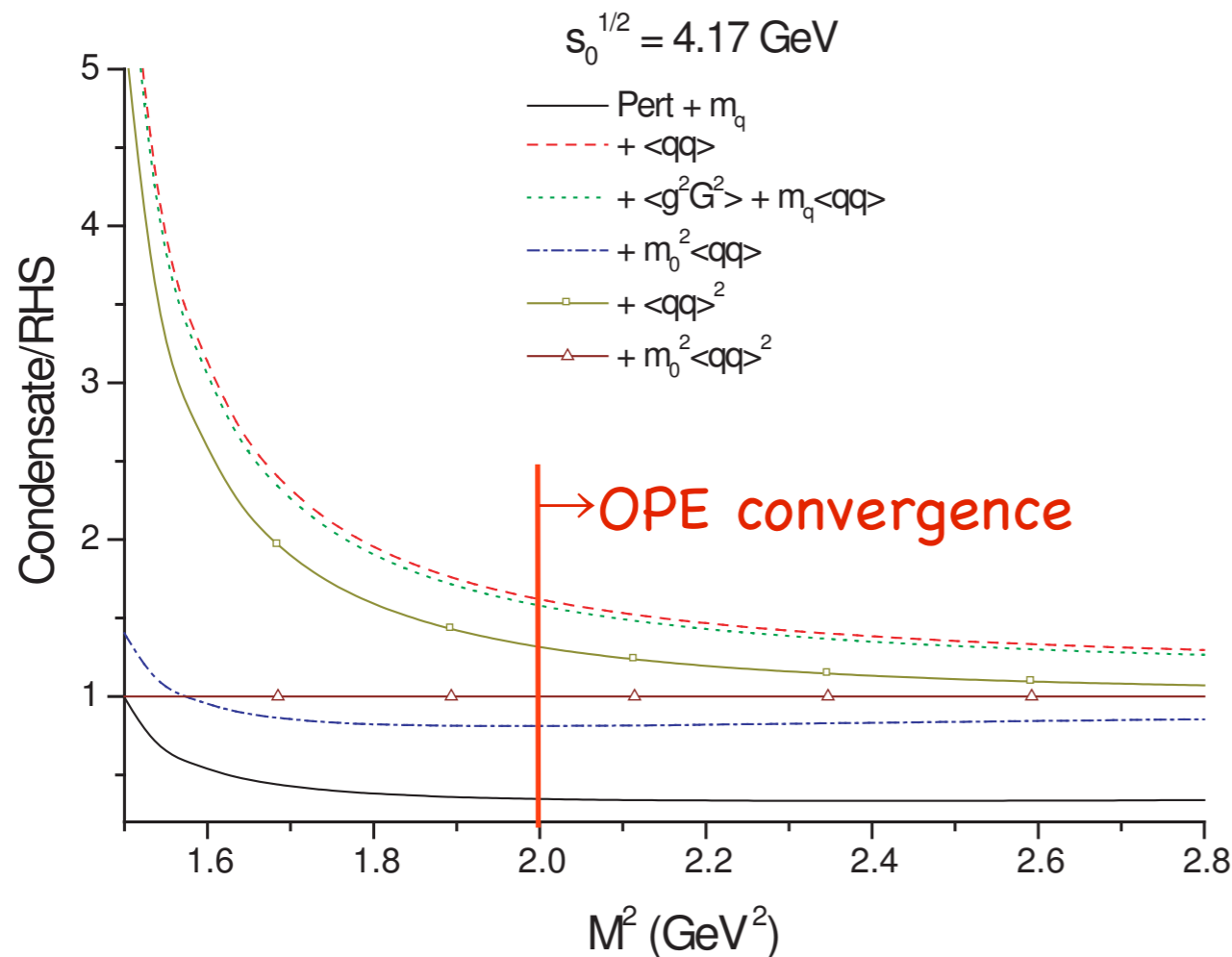


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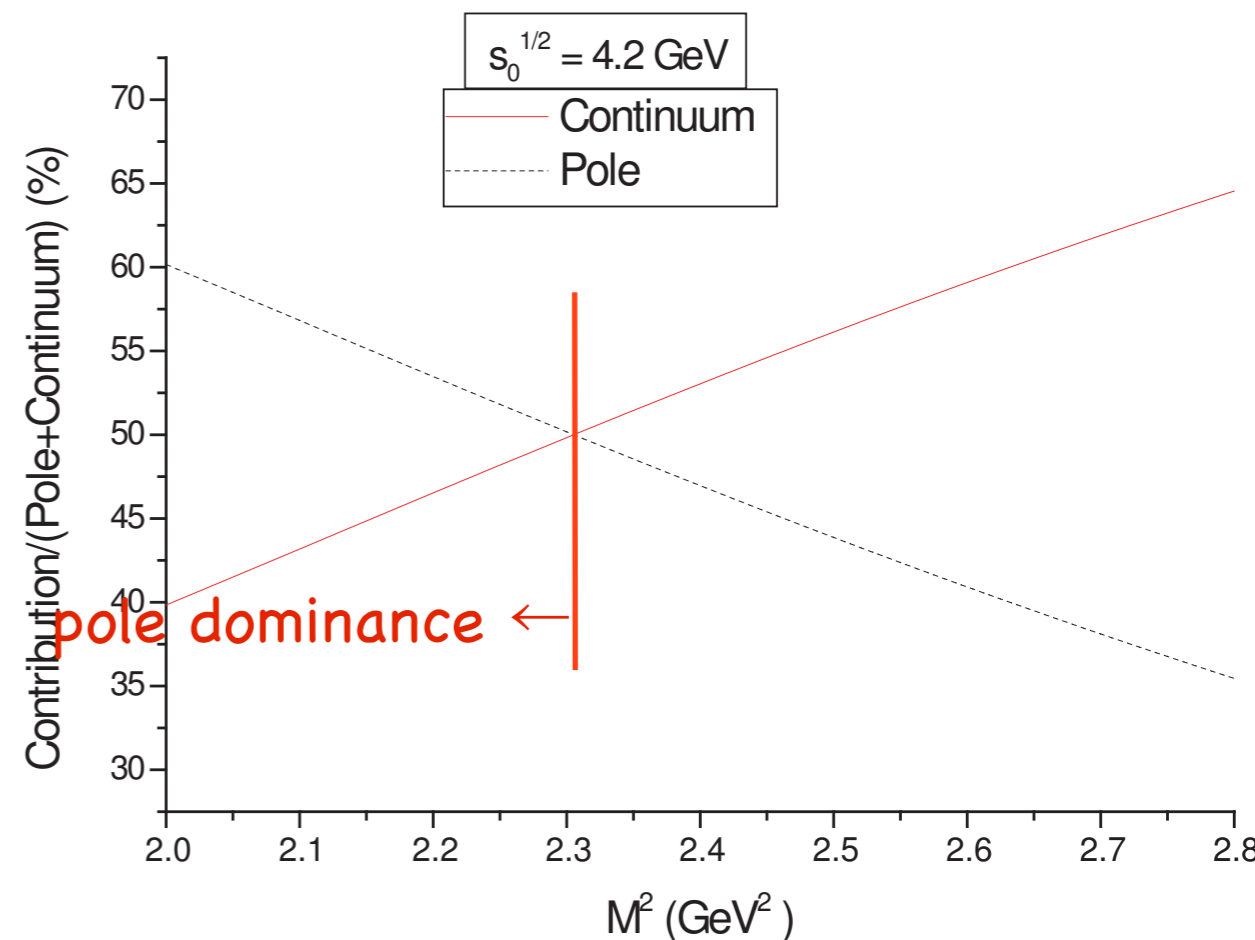
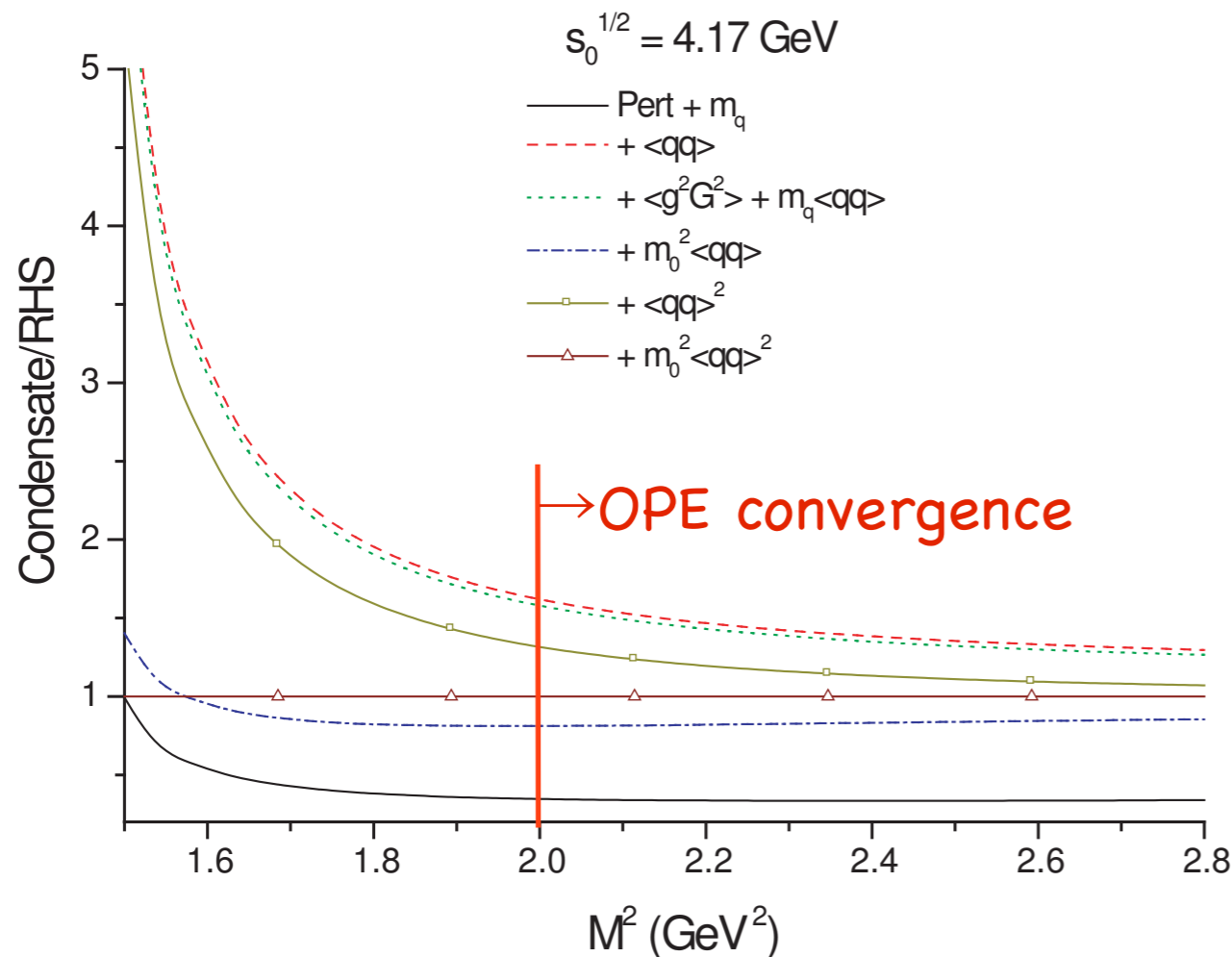


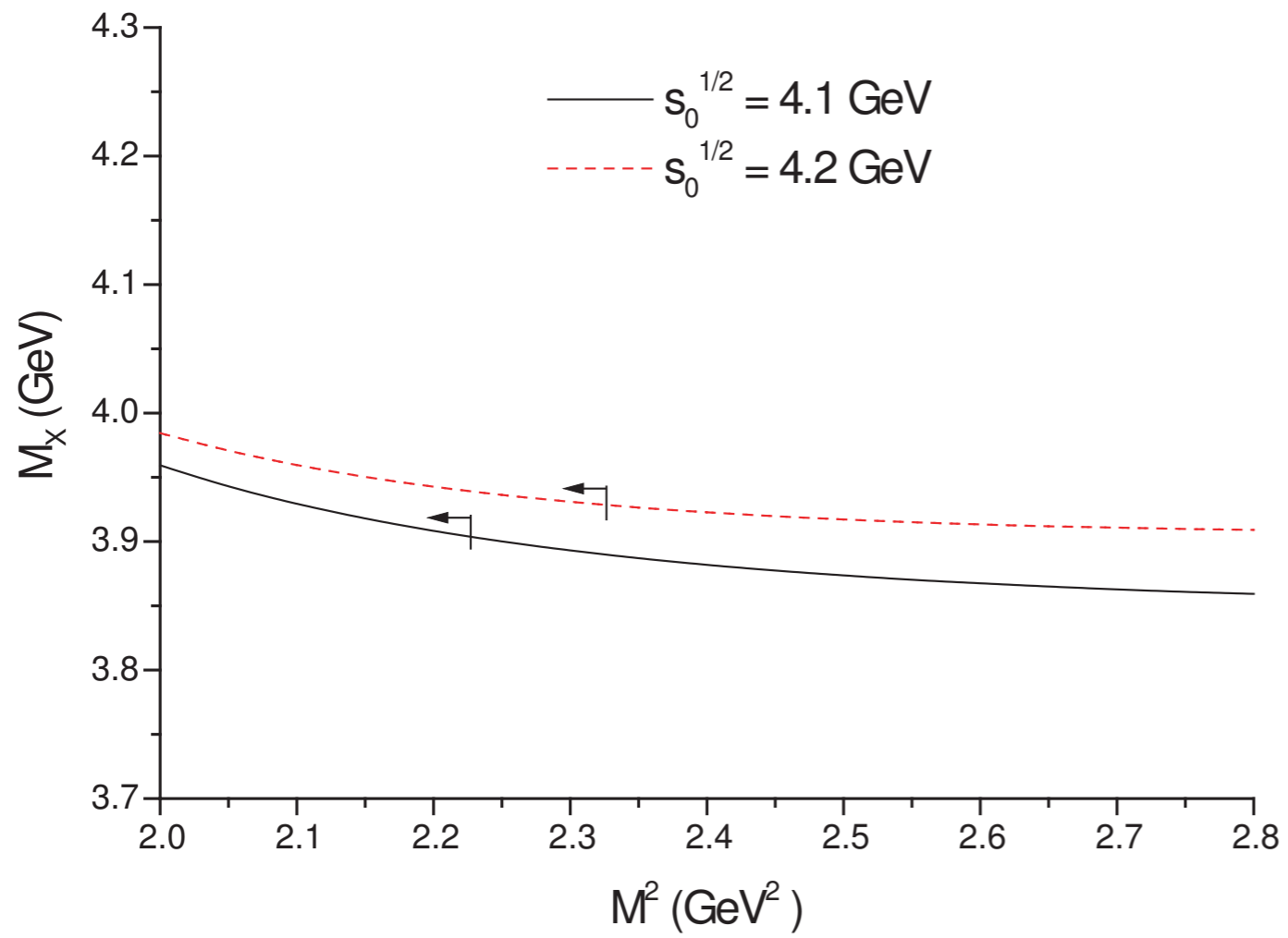
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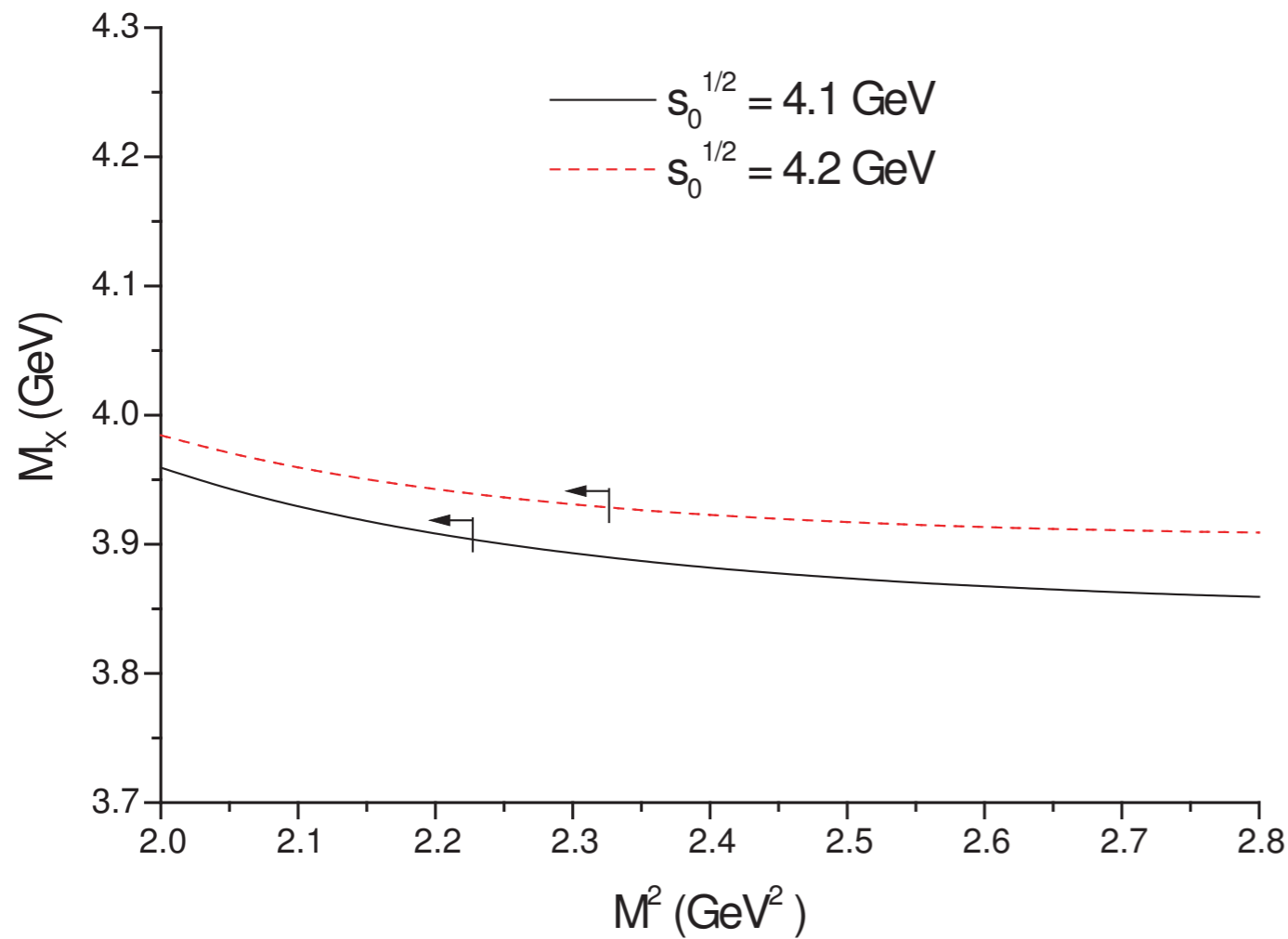
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$$m_X = (3.92 \pm 0.13) \text{ GeV}$$

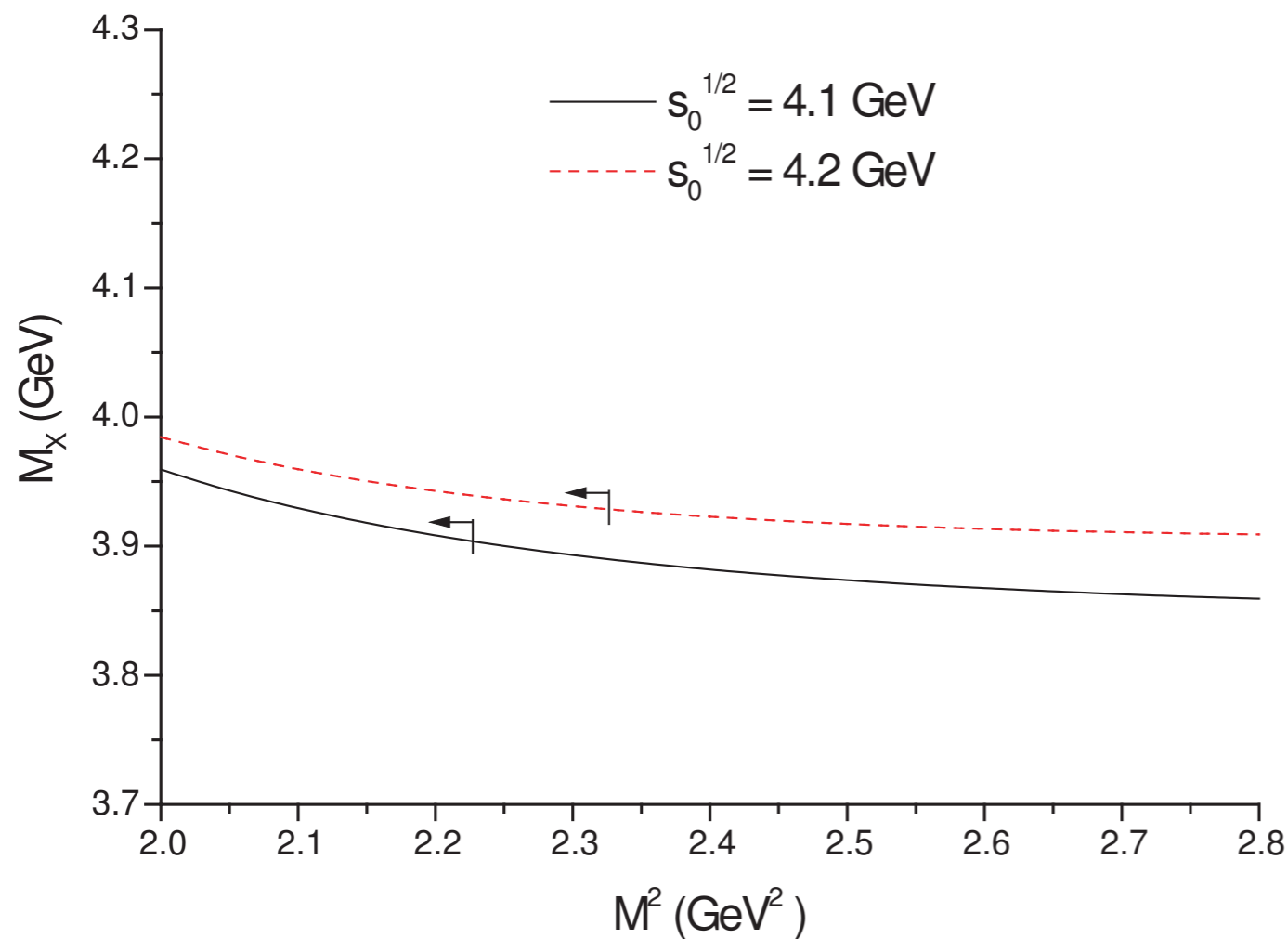


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Lee, MN, Wiedner: $D^0 \bar{D}^{*0}$ molecular current (arXiv:0803.1168)

$$j_\mu^{(q,mol)}(x) = \frac{1}{\sqrt{2}} \left[(\bar{q}_a(x) \gamma_5 c_a(x) \bar{c}_b(x) \gamma_\mu q_b(x)) - (\bar{q}_a(x) \gamma_\mu c_a(x) \bar{c}_b(x) \gamma_5 q_b(x)) \right]$$

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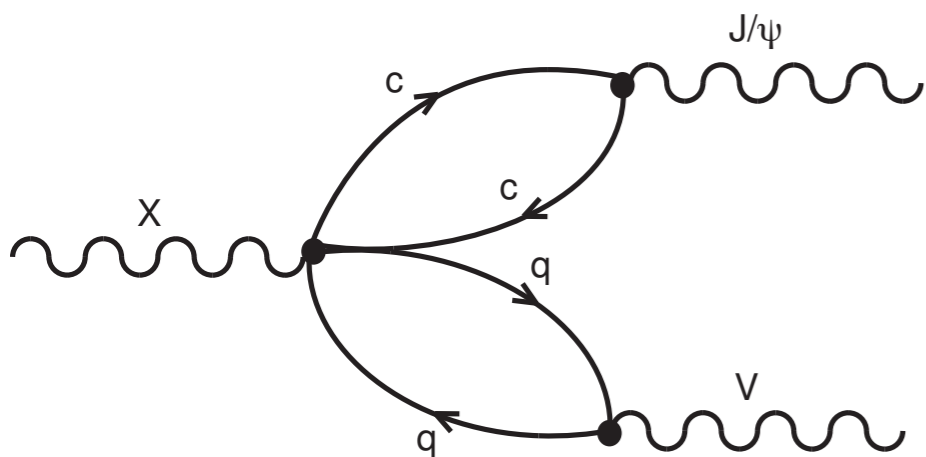
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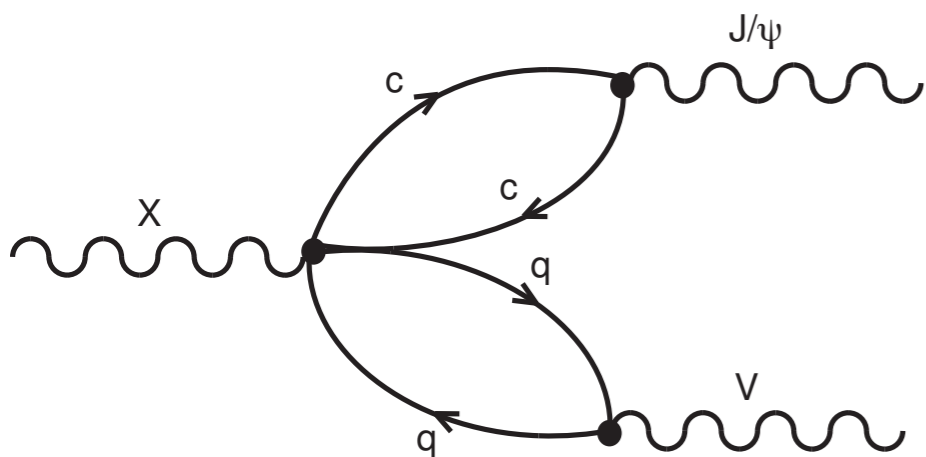
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same mass is obtained for $Z_c^+(3900)$

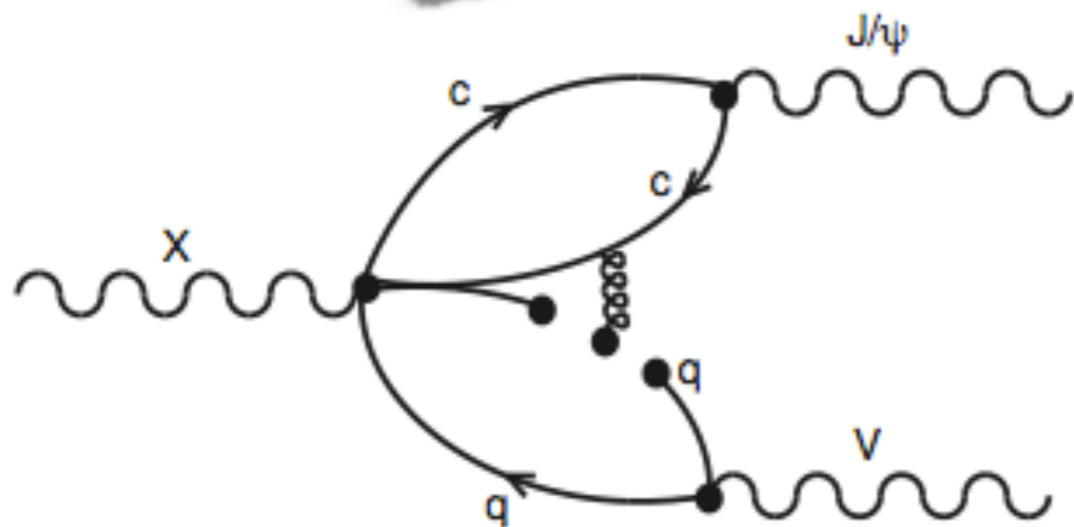


Problem: decay width $X \rightarrow J/\psi \pi \pi \pi$
 $\sim 50 \text{ MeV}$ (Navarra, MN, PLB639 (06)272)

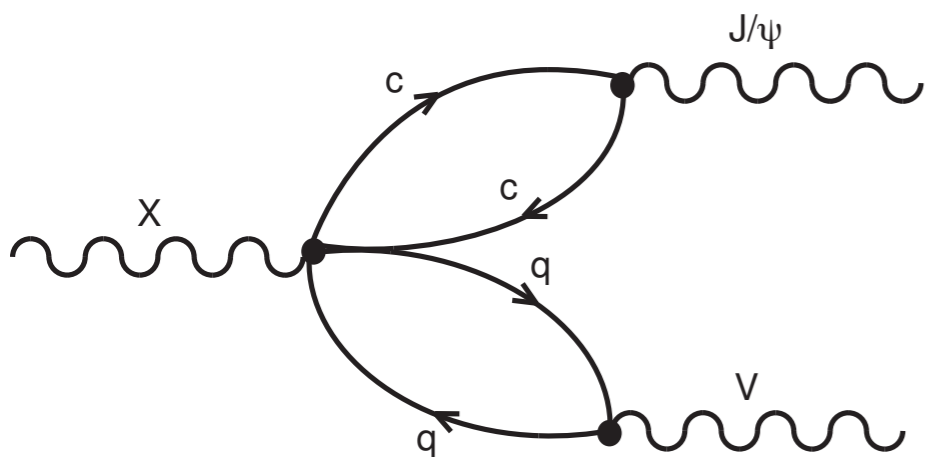


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How to solve this problem?

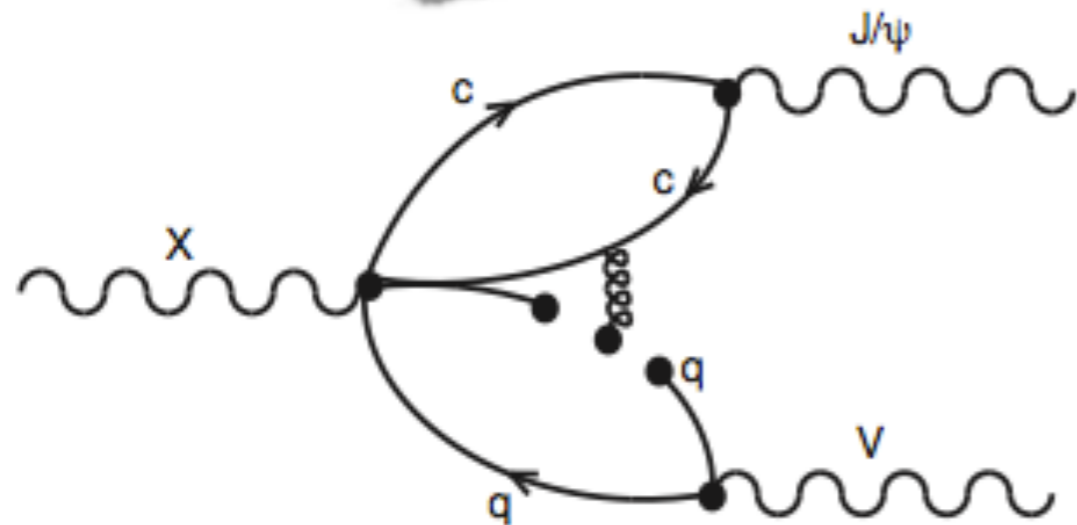


If $X(3872)$ is a genuine tetraquark state, only color-connected diagrams will contribute



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$$\Gamma_{CC}(X \rightarrow J/\psi (n\pi)) = (0.7 \pm 0.2) \text{ MeV.}$$

Navarra, MN,
 PLB639(06)272

Compatible with the experimental $X(3872)$
 width: $\Gamma < 2.3 \text{ MeV}$

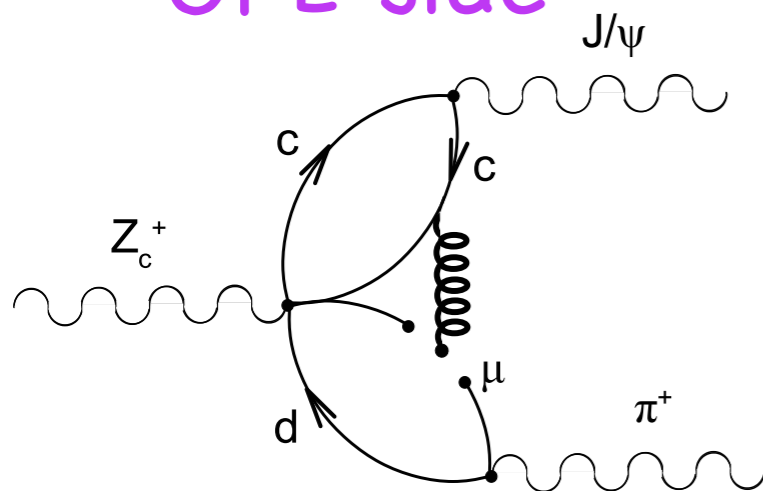
Decay width $Z^+ \rightarrow J/\psi \pi^+$

Dias, Navarra, MN, Zanetti
arXiv:1304.6433

$$\Pi_{\mu\nu\alpha}(p, p', q) = \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \Pi_{\mu\nu\alpha}(x, y)$$

$$\Pi_{\mu\nu\alpha}(x, y) = \langle 0 | T [j_\mu^\psi(x) j_{5\nu}^\pi(y) j_\alpha^\dagger(0)] | 0 \rangle$$

OPE side



Phen. side

$$\Pi_{\mu\nu\alpha}^{(phen)}(p, p', q) = \frac{\lambda_{Z_c} m_\psi f_\psi F_\pi g_{Z_c \psi \pi}(q^2) q_\nu}{(p^2 - m_{Z_c}^2)(p'^2 - m_\psi^2)(q^2 - m_\pi^2)} \left(-g_{\mu\lambda} + \frac{p'_\mu p'_\lambda}{m_\psi^2} \right) \left(-g_\alpha^\lambda + \frac{p_\alpha p^\lambda}{m_{Z_c}^2} \right) + \dots,$$

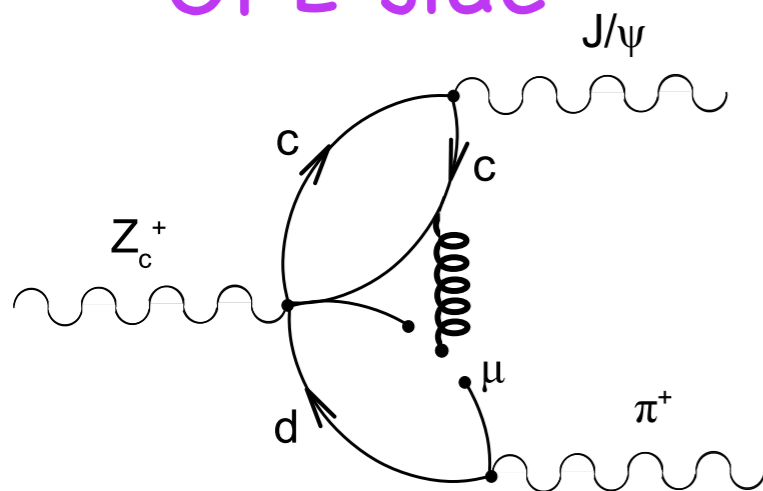
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coupling constant

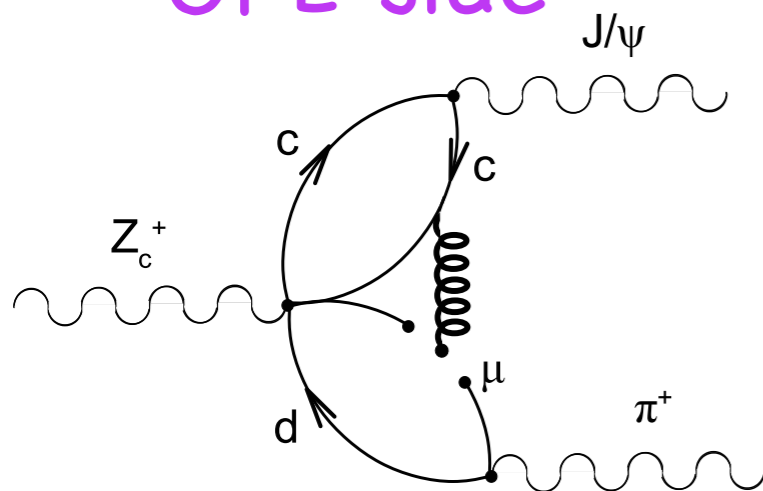
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OPE side



Phen. side

$$\Pi_{\mu\nu\alpha}^{(phen)}(p, p', q) = \frac{\lambda_{Z_c} m_\psi f_\psi F_\pi g_{Z_c\psi\pi}(q^2) q_\nu}{(p^2 - m_{Z_c}^2)(p'^2 - m_\psi^2)(q^2 - m_\pi^2)} \left(-g_{\mu\lambda} + \frac{p'_\mu p'_\lambda}{m_\psi^2} \right) \left(-g_\alpha^\lambda + \frac{p_\alpha p^\lambda}{m_{Z_c}^2} \right) + \dots,$$

$$\Gamma(Z_c^+(3900) \rightarrow J/\psi\pi^+) = \frac{p^*(m_{Z_c}, m_\psi, m_\pi)}{8\pi m_{Z_c}^2}$$

$$\times \frac{1}{3} g_{Z_c\psi\pi}^2 \left(3 + \frac{(p^*(m_{Z_c}, m_\psi, m_\pi))^2}{m_\psi^2} \right),$$

coupling constant

Phen. side:

$$A \left(e^{-m_\psi^2/M^2} - e^{-m_{Z_c}^2/M^2} \right) + B e^{-s_0/M^2} \quad \text{with} \quad A = \frac{g_{Z_c\psi\pi} \lambda_{Z_c} f_\psi F_\pi (m_{Z_c}^2 + m_\psi^2)}{2m_{Z_c}^2 m_\psi (m_{Z_c}^2 - m_\psi^2)}$$

OPE side:

$$\frac{\langle \bar{q} g \sigma \cdot G q \rangle}{12\sqrt{2}\pi^2} \int_0^1 d\alpha e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}$$

Phen. side:

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with

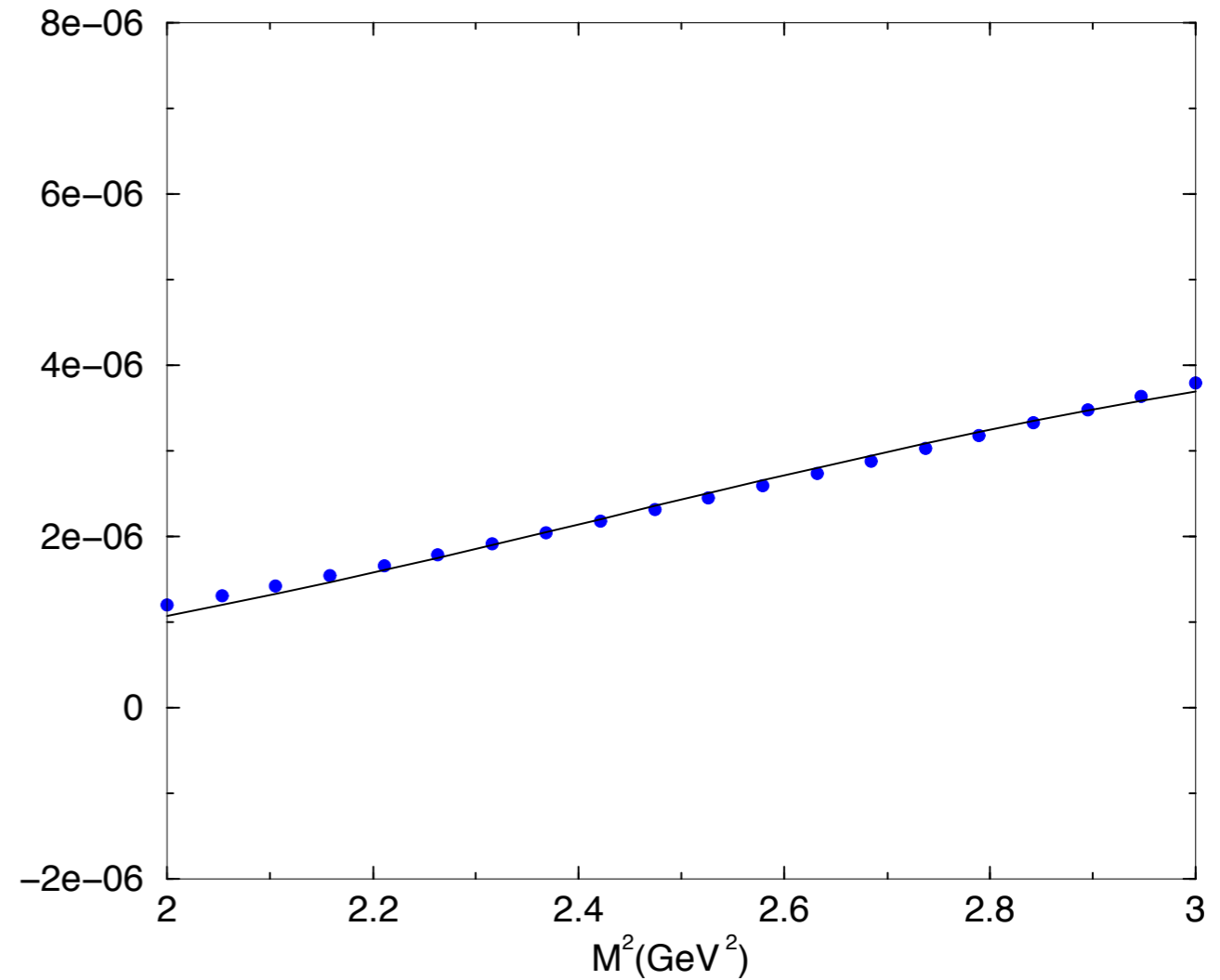
$$A = \frac{g_{Z_c \psi \pi} \lambda_{Z_c} f_\psi F_\pi (m_{Z_c}^2 + m_\psi^2)}{2m_{Z_c}^2 m_\psi (m_{Z_c}^2 - m_\psi^2)}$$

OPE side:

$$\frac{\langle \bar{q} g \sigma \cdot G q \rangle}{12\sqrt{2}\pi^2} \int_0^1 d\alpha e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}$$

$$g_{Z_c \psi \pi} = (3.89 \pm 0.56) \text{ GeV}$$

RHS X LHS (GeV⁵)



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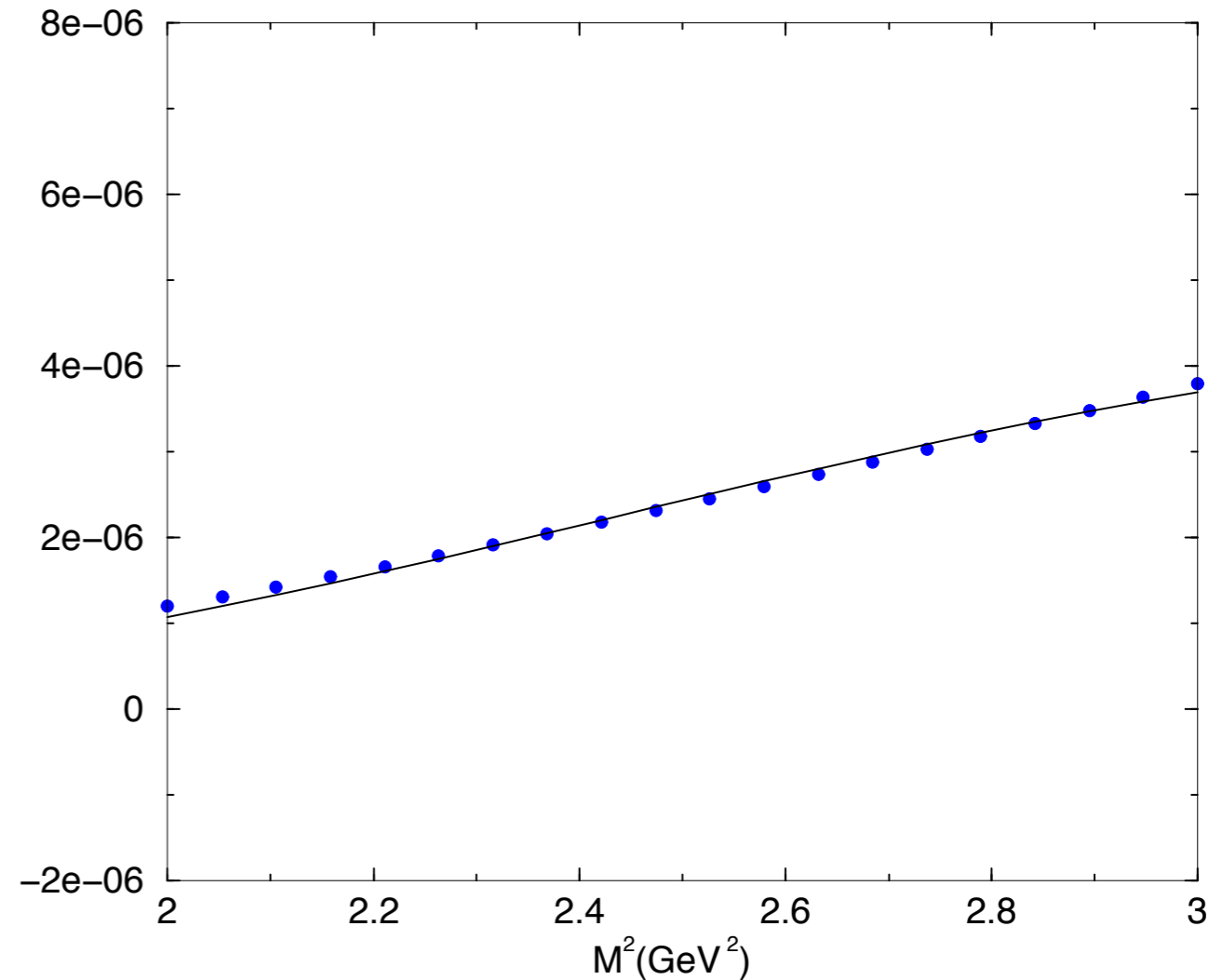
$$A = \frac{g_{Z_c \psi \pi} \lambda_{Z_c} f_\psi F_\pi (m_{Z_c}^2 + m_\psi^2)}{2m_{Z_c}^2 m_\psi (m_{Z_c}^2 - m_\psi^2)}$$

OPE side:

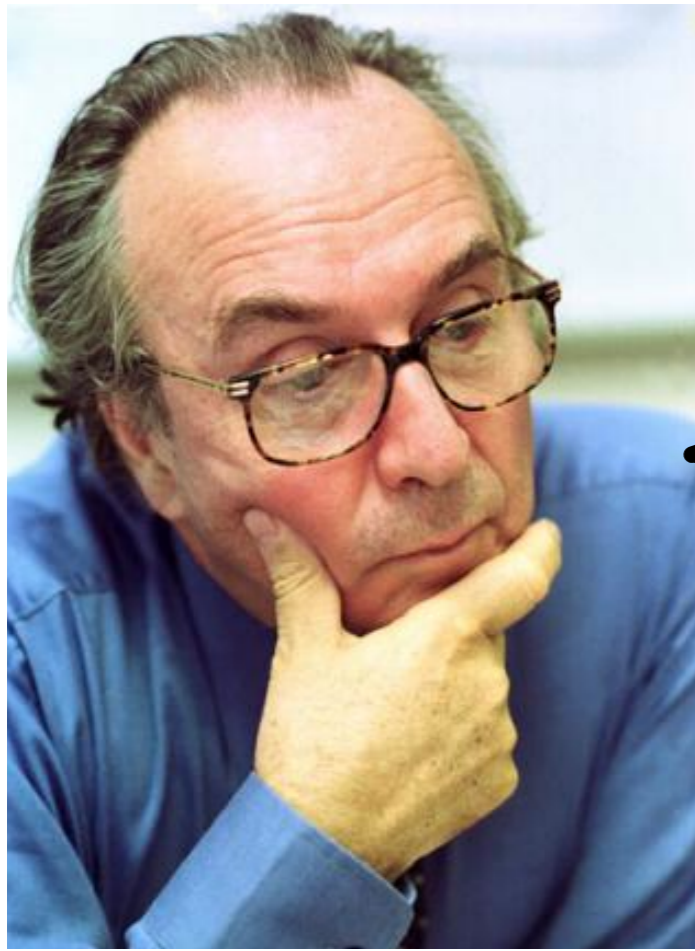
$$\frac{\langle \bar{q} g \sigma \cdot G q \rangle}{12\sqrt{2}\pi^2} \int_0^1 d\alpha e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}$$

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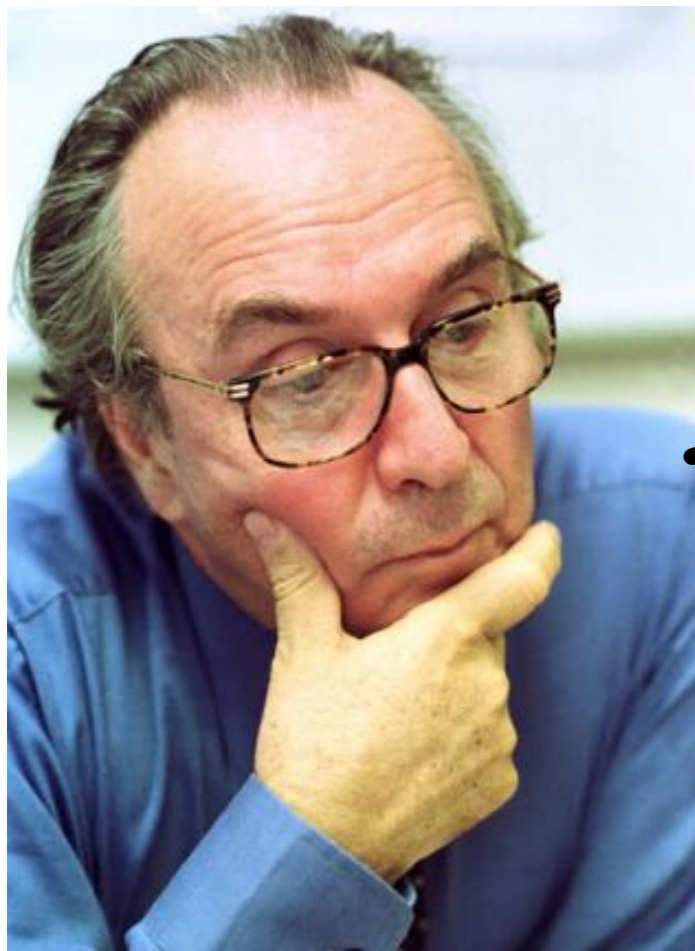


$$\Gamma(Z_c^+(3900) \rightarrow J/\psi \pi^+) = (29.1 \pm 8.2) \text{ MeV}$$



I knew it!

L. Maiani et al.
[arXiv:1303.6857](https://arxiv.org/abs/1303.6857)



I knew it!

“We rely on a rough dimensional argument adopting $g \sim M_{Z_c^+} \sim 3.9 \text{ GeV}$ ”

L. Maiani et al.
arXiv:1303.6857

1. $\Gamma(Z_c^+ \rightarrow J/\psi \pi^+) \approx 29 \text{ MeV}$
2. $\Gamma(Z_c^+ \rightarrow \psi(2S) \pi^+) \approx 6 \text{ MeV}$
3. $\Gamma(Z_c^+ \rightarrow \eta_c \rho^+) \approx 19 \text{ MeV}$



Not good!



Not good!

“You should do calculations, like my friends, instead of relying in rough dimensional arguments”

Decay width $Z^+ \rightarrow \eta_c \rho^+$

$$\Pi_{\mu\alpha}(p, p', q) = \int d^4x d^4y e^{ip' \cdot x} e^{iqy} \Pi_{\mu\alpha}(x, y)$$

$$\Pi_{\mu\alpha}(x, y) = \langle 0 | T [j_5^{\eta_c}(x) j_\mu^\rho j_\alpha^\dagger(0)] | 0 \rangle$$

Phen. side:

OPE side:

$$C \left(e^{-m_{\eta_c}^2/M^2} - e^{-m_{Z_c}^2/M^2} \right) + D e^{-s_0/M^2} = \frac{Q^2 + m_\rho^2}{Q^2} \frac{m_c \langle \bar{q} g \sigma \cdot G q \rangle}{48\sqrt{2}\pi^2} \int_0^1 d\alpha \frac{e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}}{\alpha(1-\alpha)}$$

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$$C = \frac{g_{Z_c \eta_c \rho}(Q^2) \lambda_{Z_c} m_\rho f_\rho f_{\eta_c} m_{\eta_c}^2}{2m_c m_{Z_c}^2 (m_{Z_c}^2 - m_{\eta_c}^2)}$$

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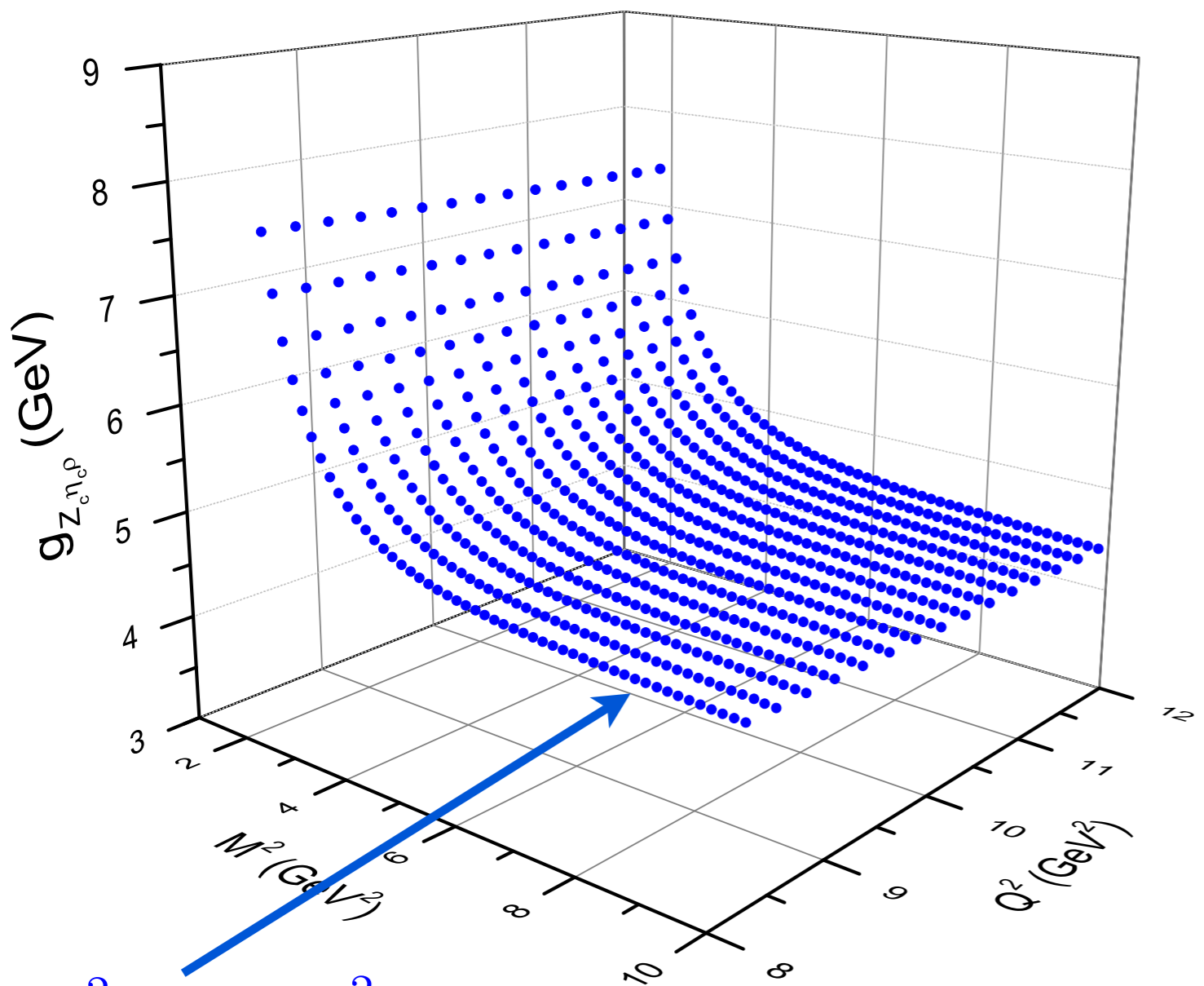
Phen. side:

OPE side:

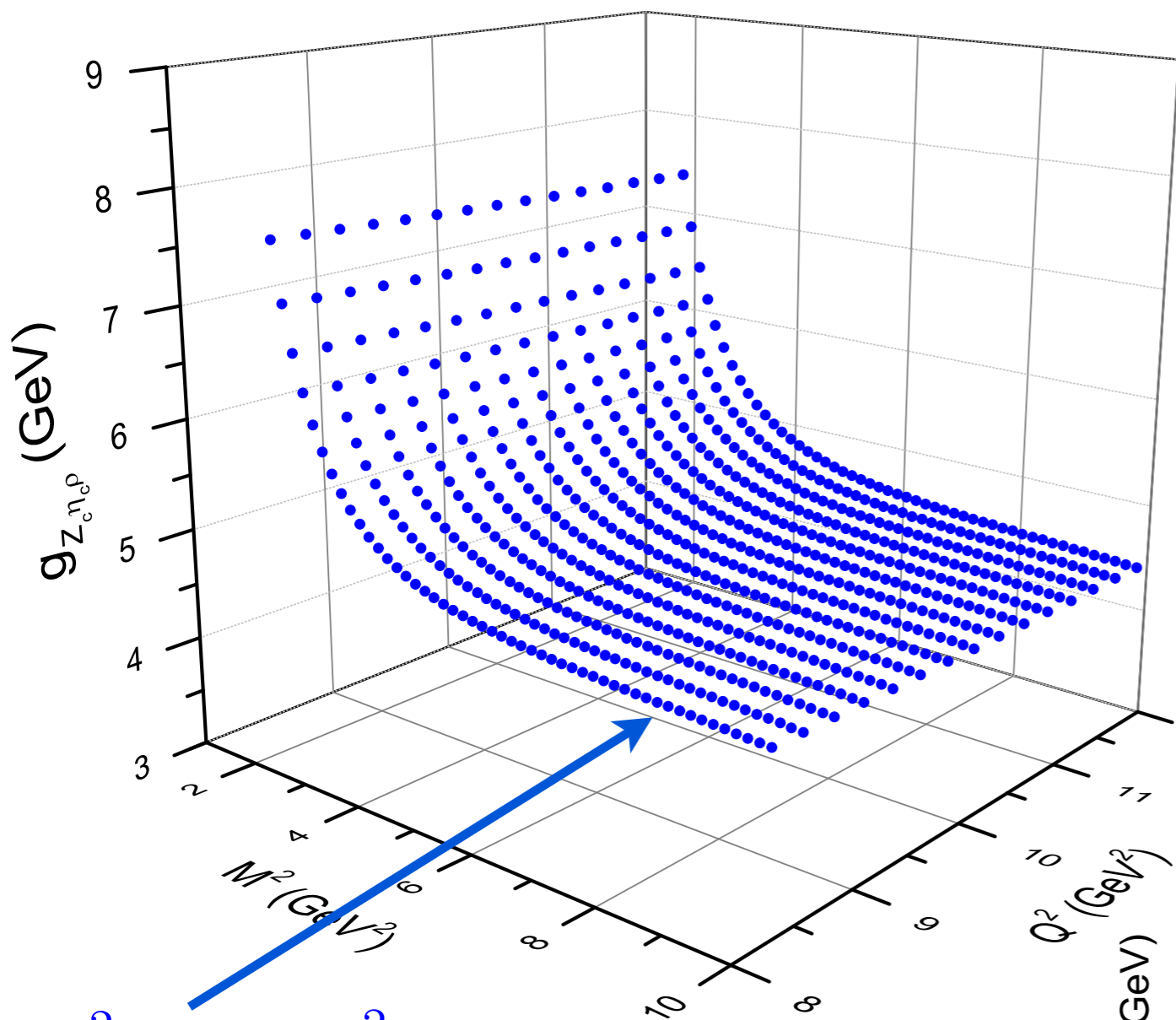
$$C \left(e^{-m_{\eta_c}^2/M^2} - e^{-m_{Z_c}^2/M^2} \right) + D e^{-s_0/M^2} = \frac{Q^2 + m_\rho^2}{Q^2} \frac{m_c \langle \bar{q} g \sigma \cdot G q \rangle}{48\sqrt{2}\pi^2} \int_0^1 d\alpha \frac{e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}}{\alpha(1-\alpha)}$$

form factor

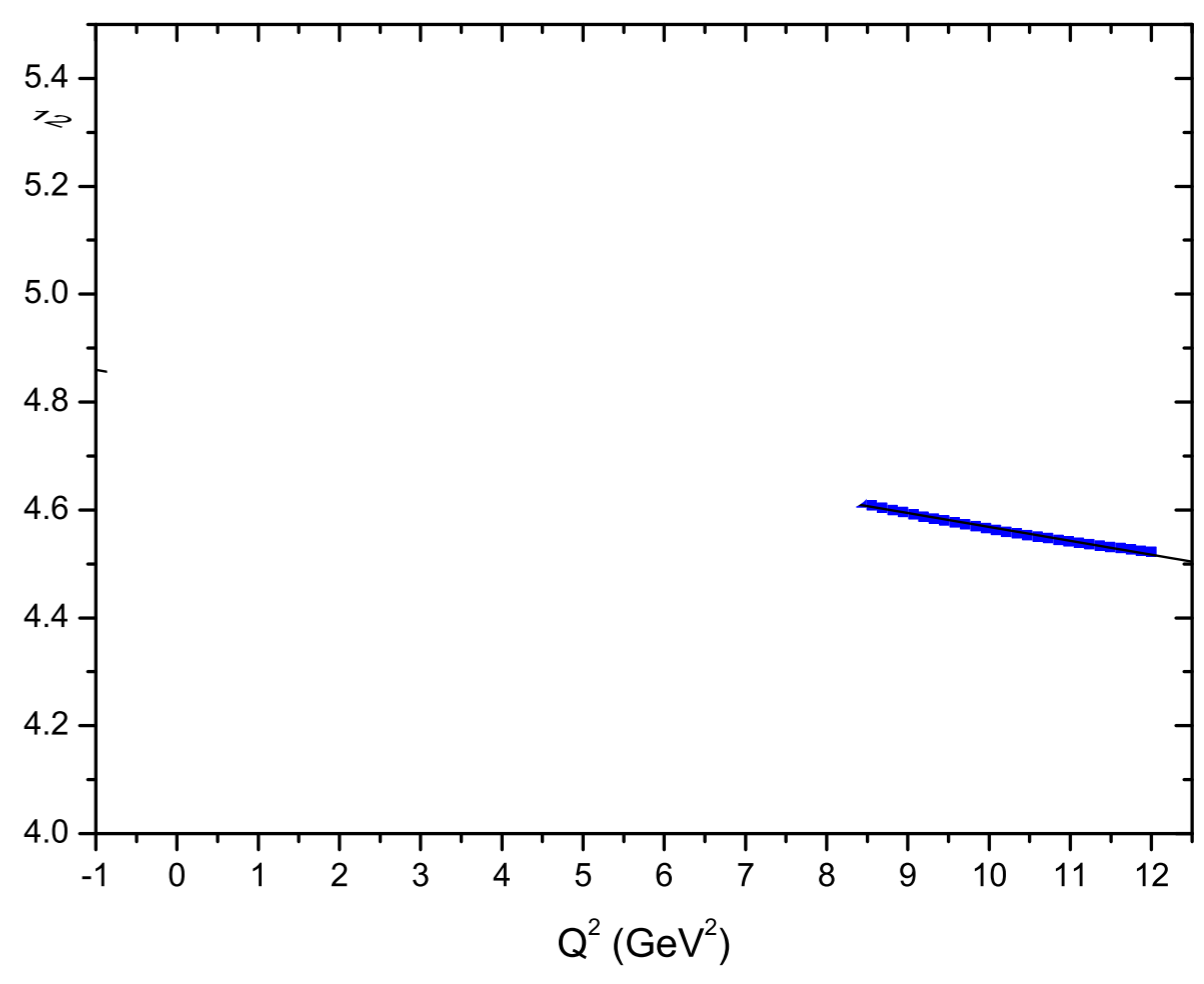
$$C = \frac{g_{Z_c \eta_c \rho}(Q^2) \lambda_{Z_c} m_\rho f_\rho f_{\eta_c} m_{\eta_c}^2}{2m_c m_{Z_c}^2 (m_{Z_c}^2 - m_{\eta_c}^2)}$$

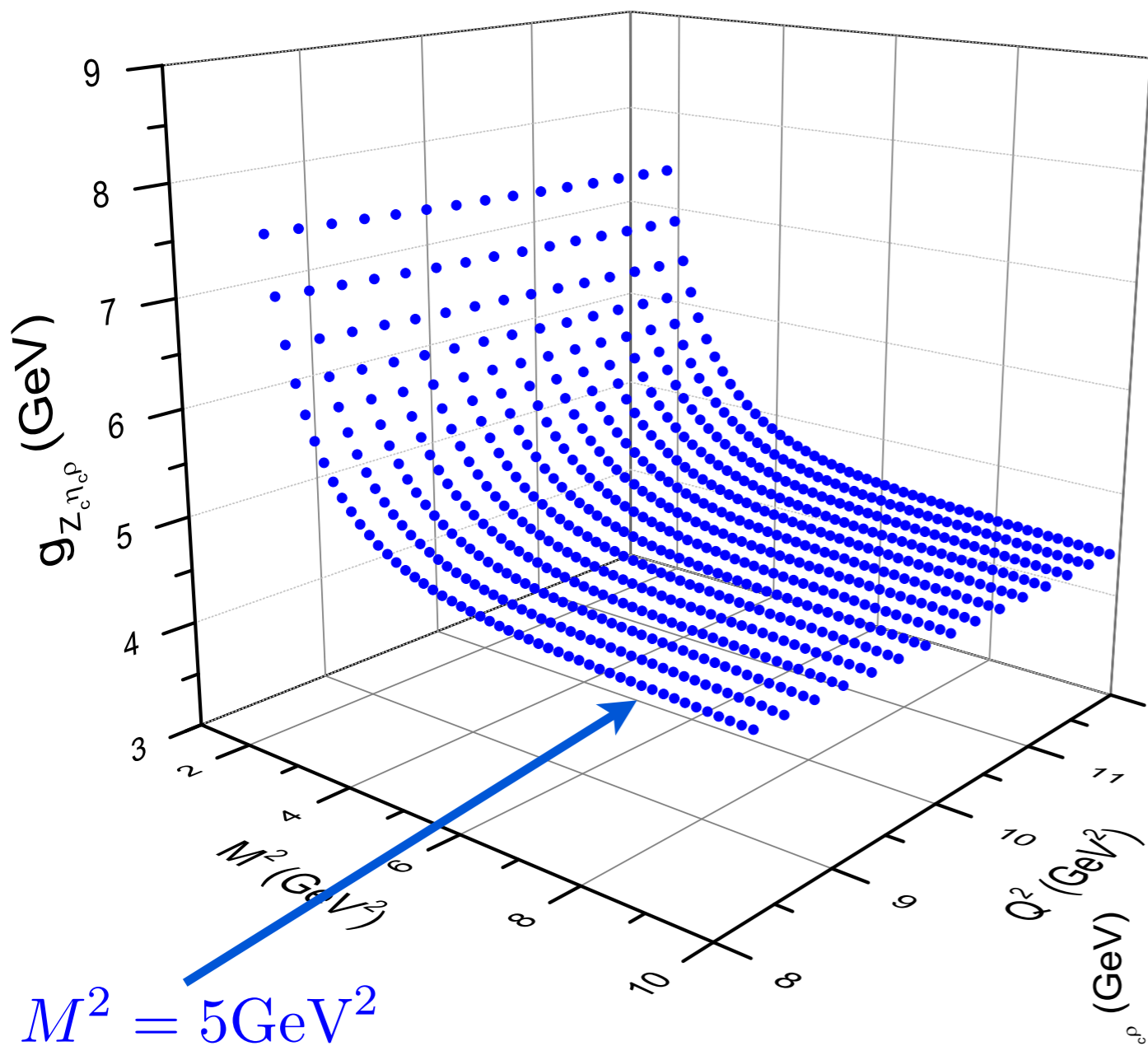


$M^2 = 5 \text{ GeV}^2$



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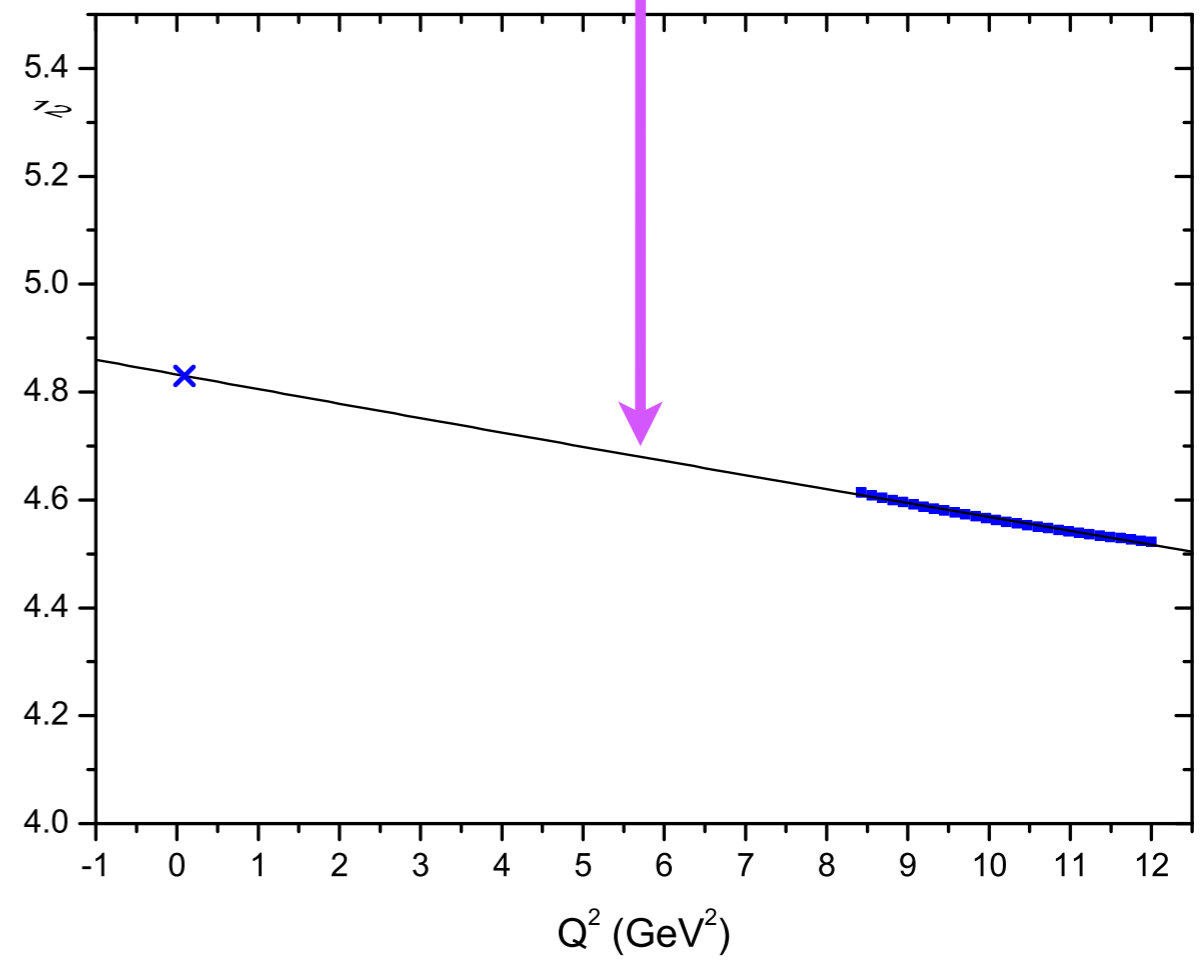


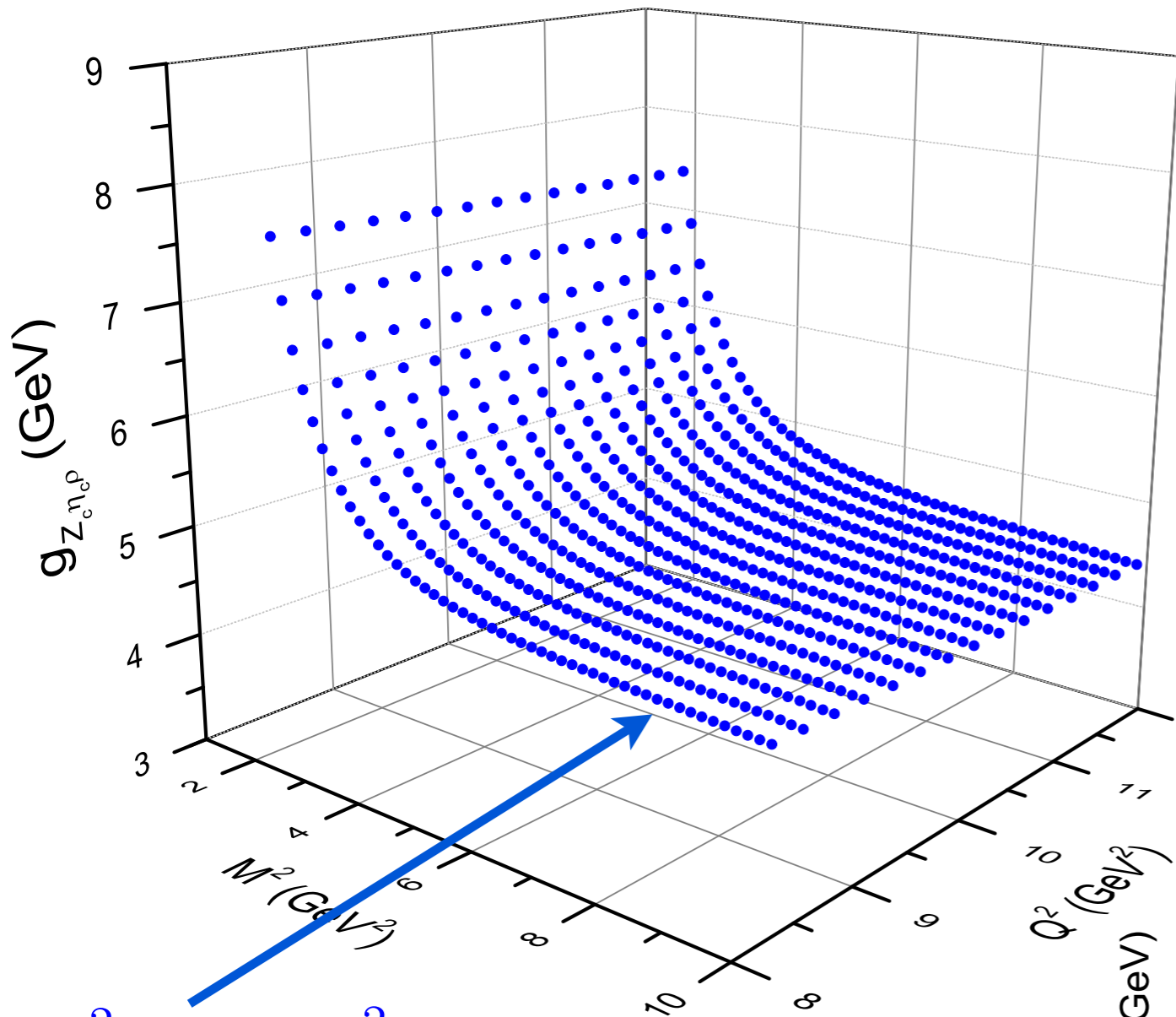


$M^2 = 5 \text{ GeV}^2$

$$g_{Z_c \eta_c \rho}(Q^2) = g_1 e^{-g_2 Q^2}$$

$g_1 = 4.83 \text{ GeV}, g_2 = (5.6 \times 10^{-3}) \text{ GeV}^{-2}$



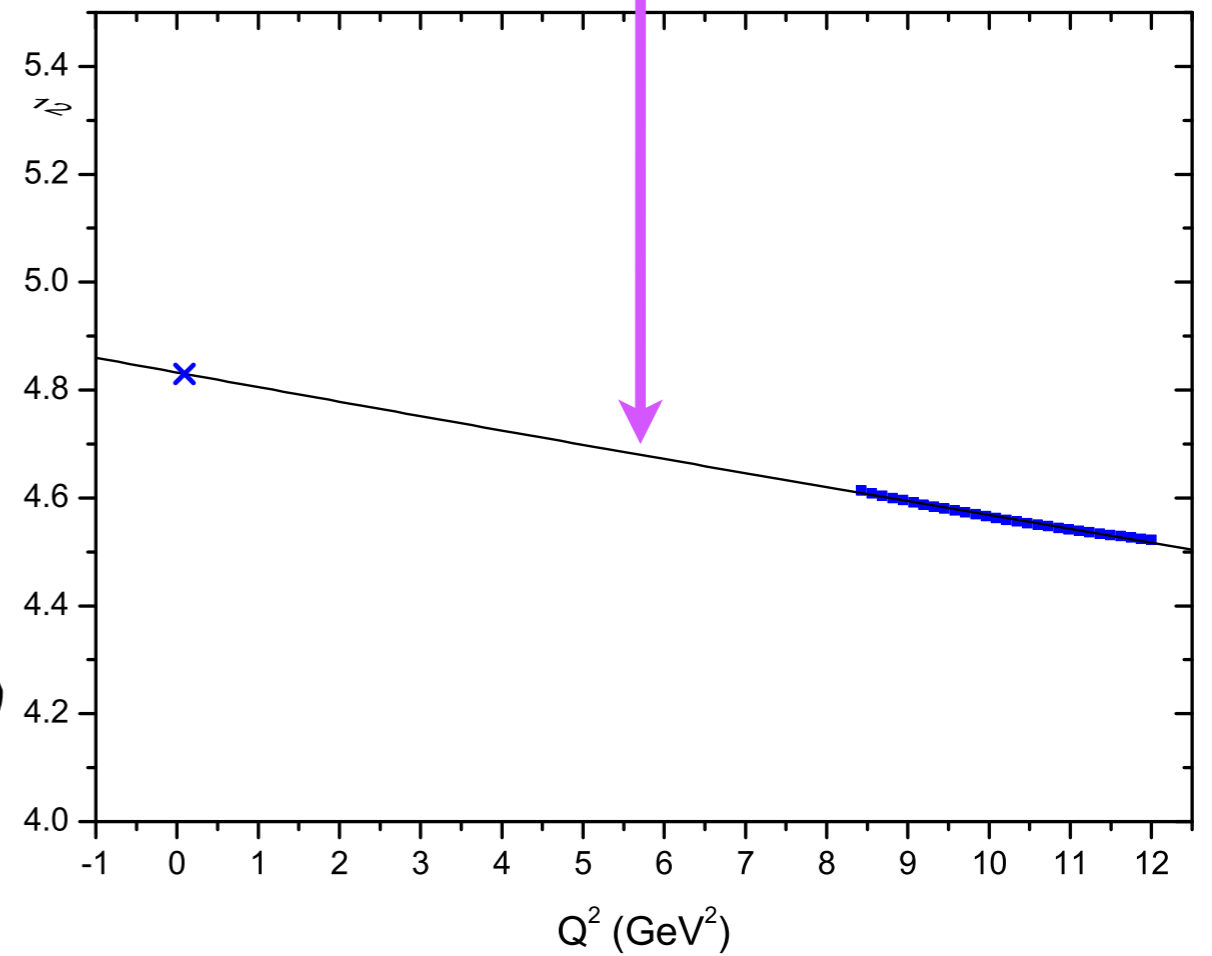


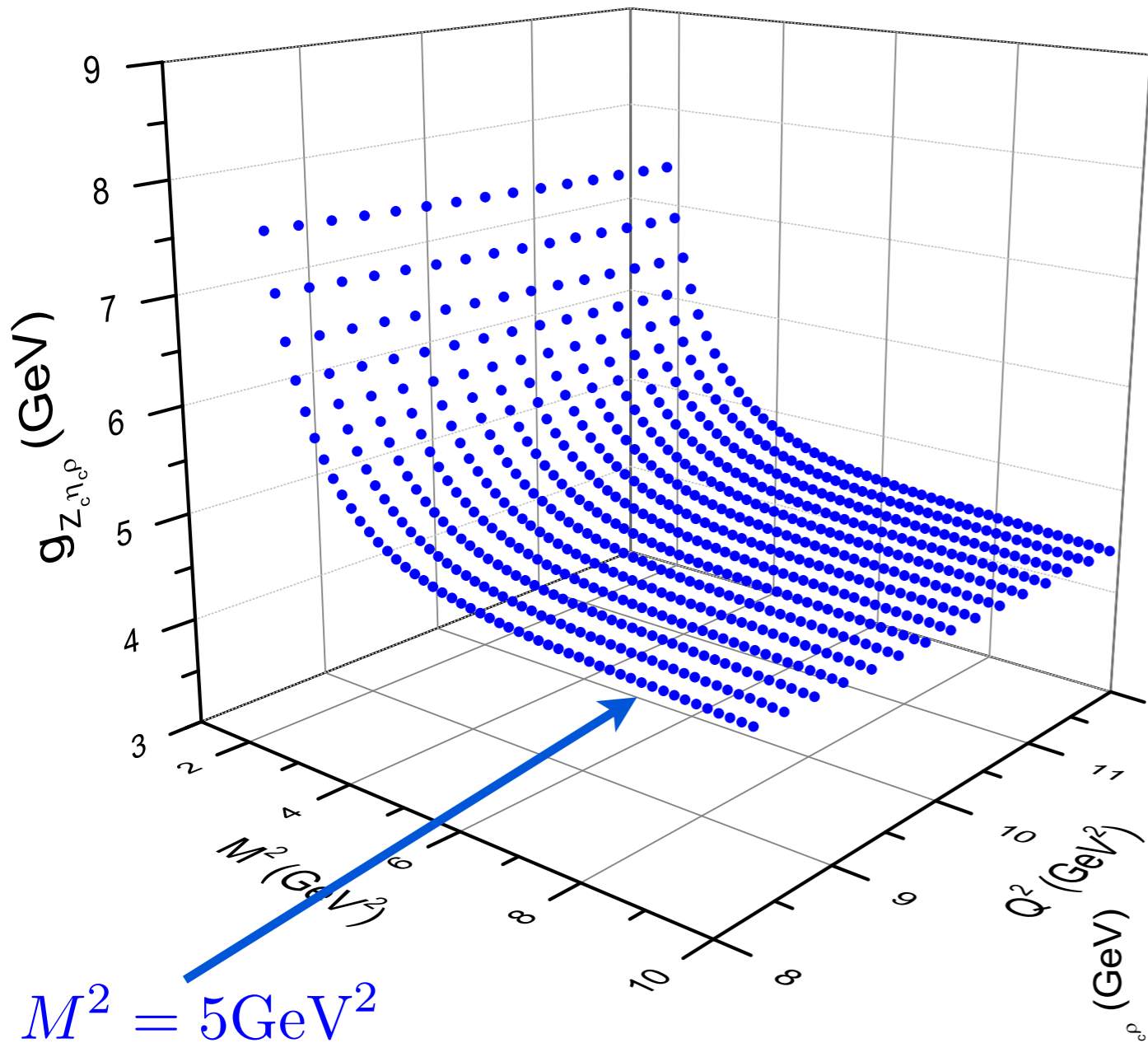
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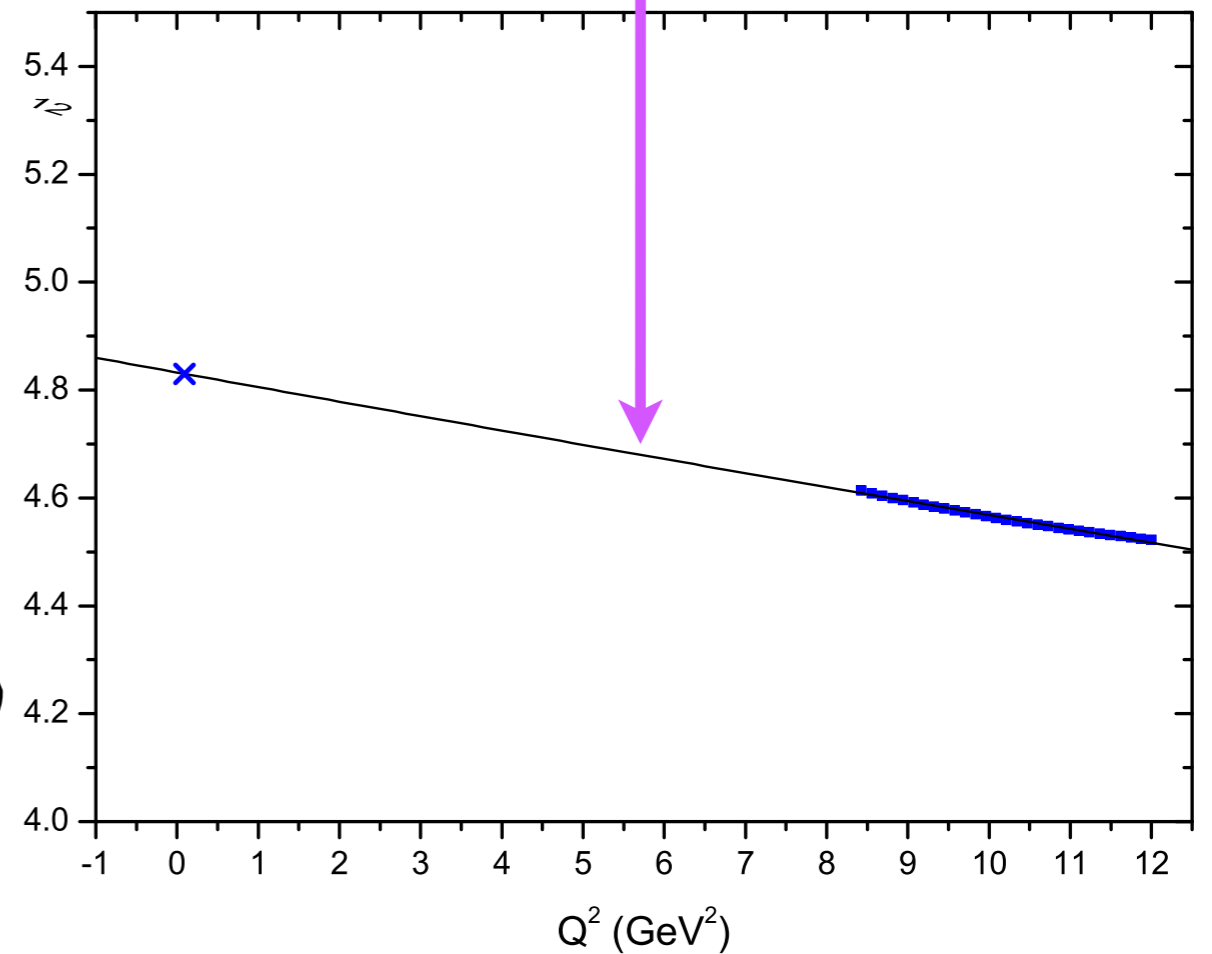
$$g_{Z_c \eta_c \rho} = g_{Z_c \eta_c \rho}(-m_\rho^2) = (4.85 \pm 0.81) \text{ GeV}$$





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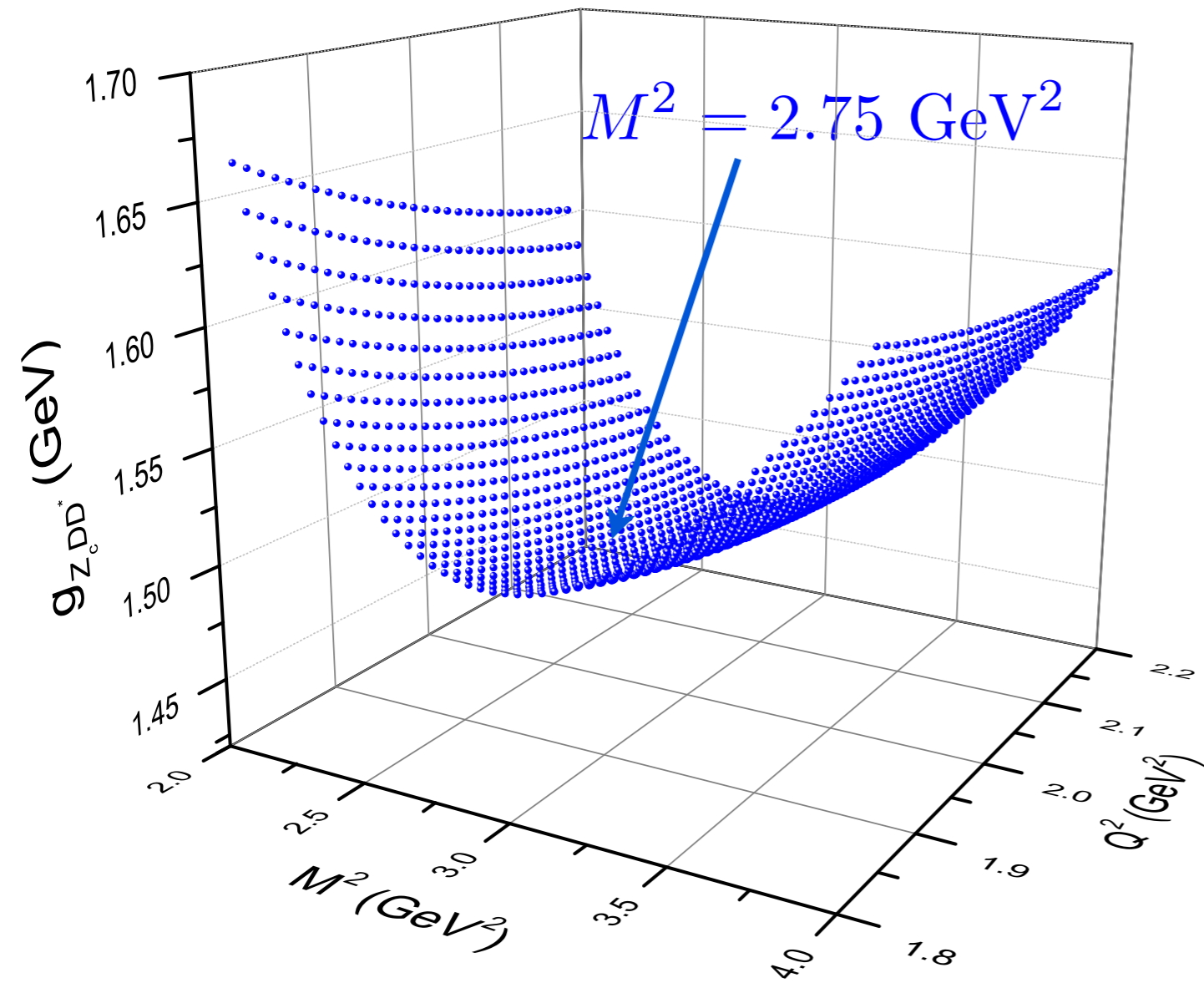
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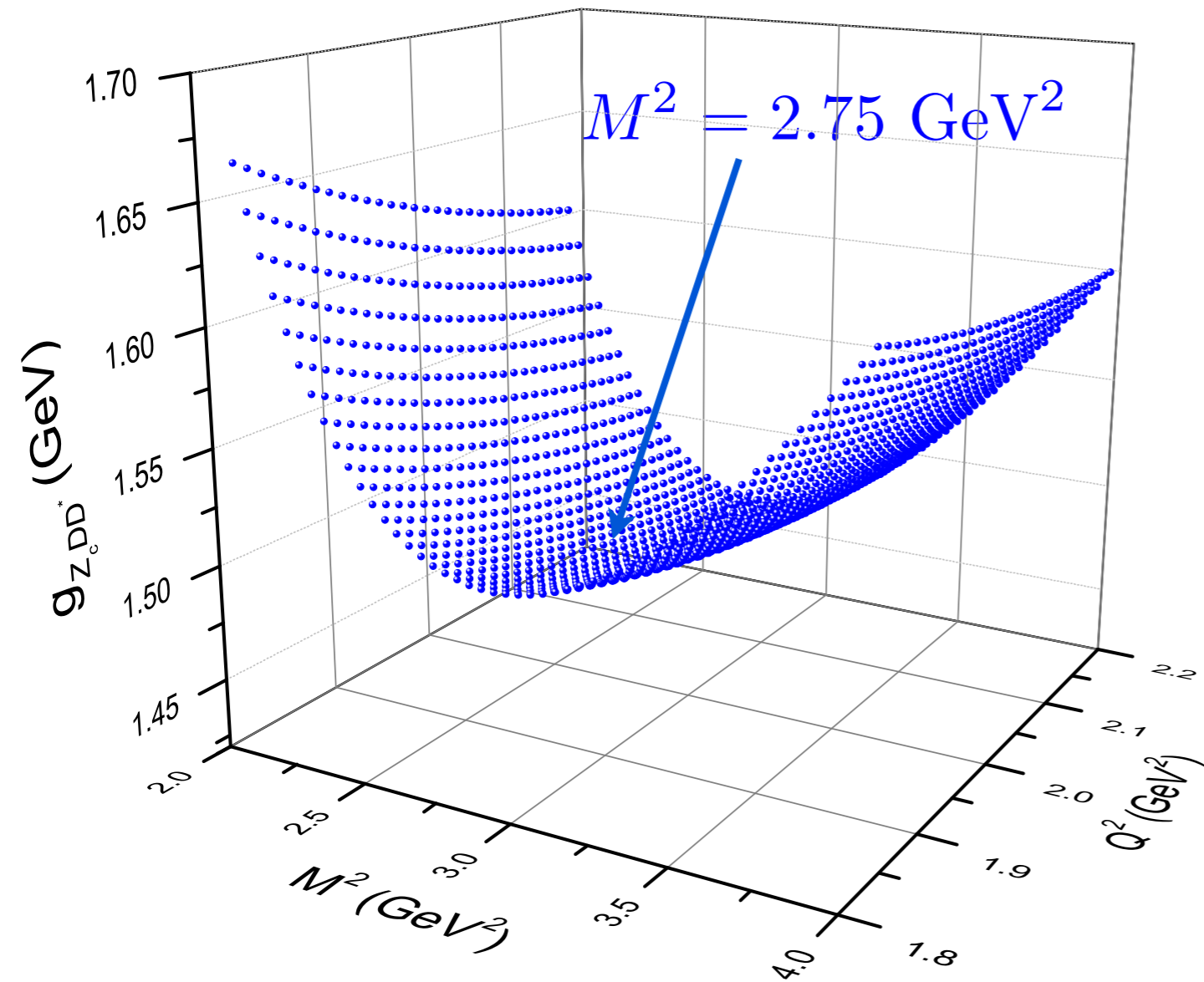
$$g_{Z_c \eta_c \rho} = g_{Z_c \eta_c \rho}(-m_\rho^2) = (4.85 \pm 0.81) \text{ GeV}$$

$$\Gamma(Z_c^+(3900) \rightarrow \eta_c \rho^+) = (27.5 \pm 8.5) \text{ MeV.}$$

Decay width $Z^+ \rightarrow D^* D^+$



Decay width $Z^+ \rightarrow D^* D^+$

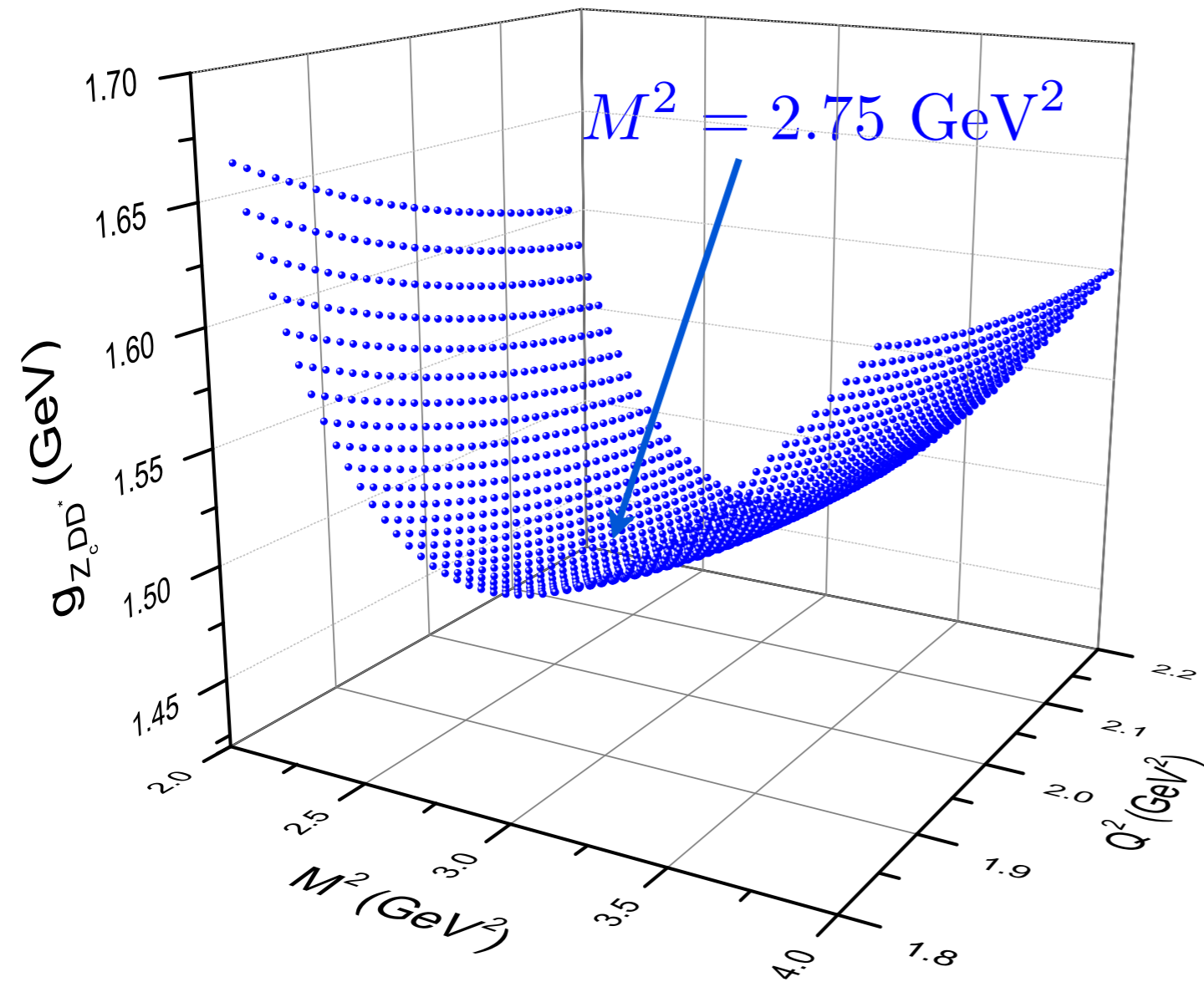


$$g_{Z_c D^* D}(Q^2) = 1.7e^{-0.08Q^2}$$



$$g_{Z_c D^* D}(Q^2 = -m_D^2) = (2.5 \pm 0.3) \text{ GeV}$$

Decay width $Z^+ \rightarrow D^* D^+$



$$g_{Z_c D^* D}(Q^2) = 1.7e^{-0.08Q^2}$$



$$g_{Z_c D^* D}(Q^2 = -m_D^2) = (2.5 \pm 0.3) \text{ GeV}$$

$$\Gamma(Z_c^+ \rightarrow D^+ \bar{D}^{*0}) = (3.2 \pm 0.7) \text{ MeV}$$

Vertex	coupling constant (GeV)	decay width (MeV)
$Z_c^+(3900) J/\psi \pi^+$	3.89 ± 0.56	29.1 ± 8.2
$Z_c^+(3900) \eta_c \rho^+$	4.85 ± 0.81	27.5 ± 8.5
$Z_c^+(3900) D^+ \bar{D}^{*0}$	2.5 ± 0.3	3.2 ± 0.7
$Z_c^+(3900) \bar{D}^0 D^{*+}$	2.5 ± 0.3	3.2 ± 0.7

$$\Gamma_{Z_c^+} = (63 \pm 18) \text{ MeV}$$

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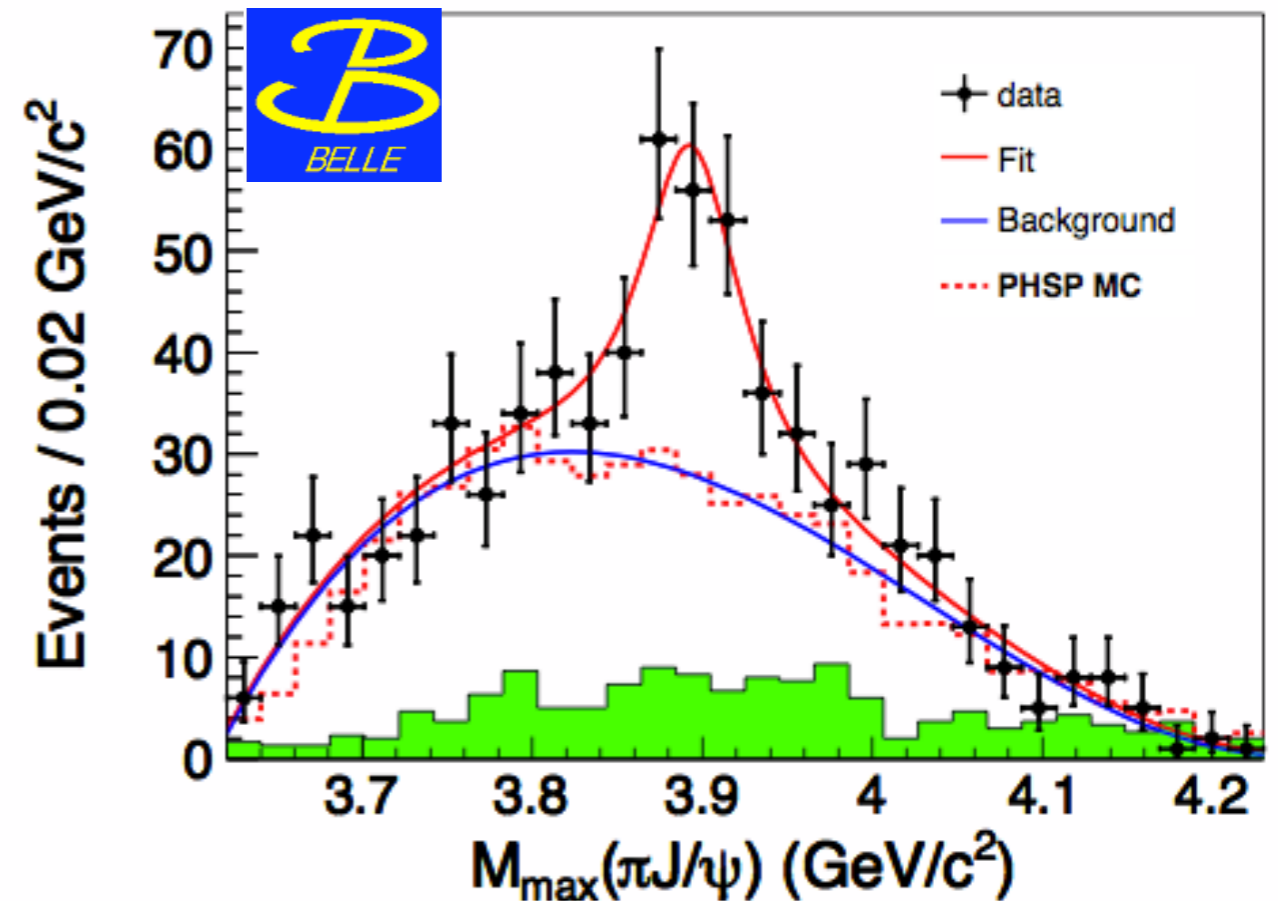
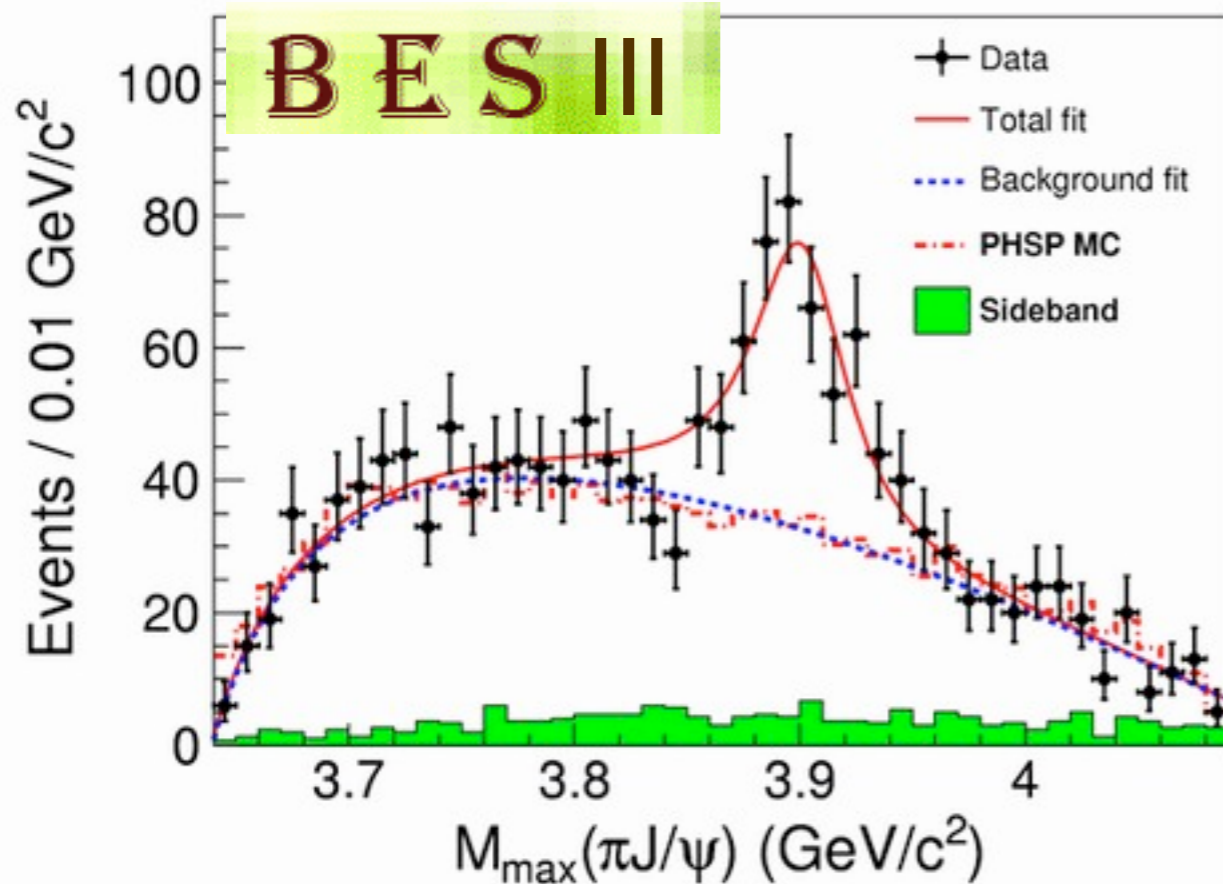
$$\Gamma_{Z_c^+} = (63 \pm 18) \text{ MeV}$$

$$\Gamma_{Z_c^+}^{BESS} = (46 \pm 22) \text{ MeV}$$

$$\Gamma_{Z_c^+}^{BELLE} = (63 \pm 35) \text{ MeV}$$

Very good agreement

Conclusions



$Z_c^+(3900) \rightarrow J^P = 1^+$ tetraquark state
 \rightarrow charged partner of the $X(3872)$

$$\Gamma_{Z_c^+} = (63 \pm 18) \text{ MeV}$$







Conclusions II

- Lots of charmonia in the last 10 years, new bottomonia start to appear: a new spectroscopy?
 - Emerging consensus that $X(3872)$ is a mixed charmonium-molecular state.
- Discovery of $Y(4260)$, $Y(4360)$ and $Y(4660)$ represent an overpopulation of the 1^- charmonium states.
 - Absence of open charm production in the Y decay is inconsistent with $c\bar{c}$ interpretation
- Z^+ states, need confirmation, but only molecule or tetraquark interpretations are possible