Femtoscopic scales in Pb-Pb, p-Pb and p-p collisions at RHIC and LHC in HydroKinetic Model

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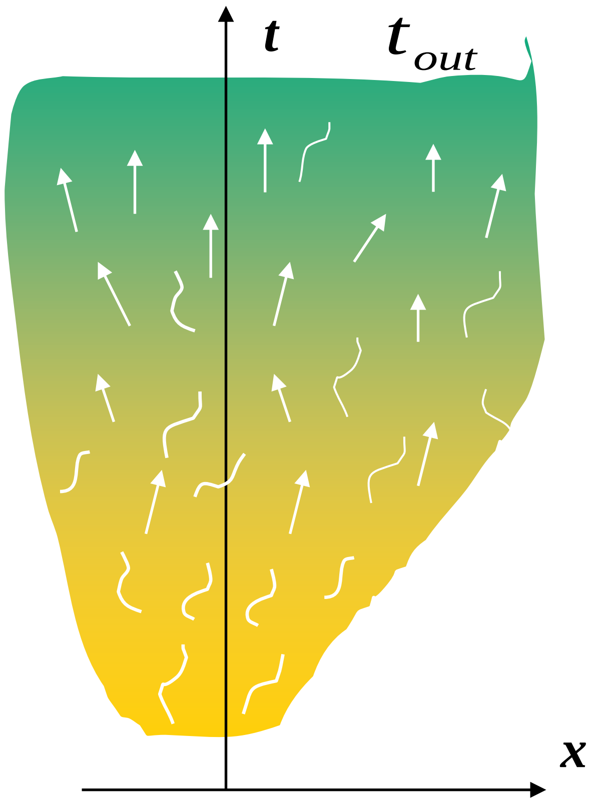
Continuous emission



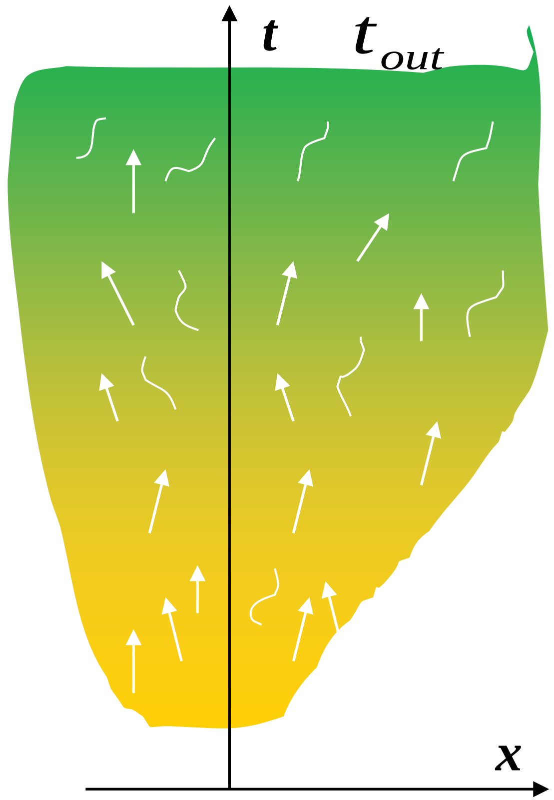
$$f = f_{int} + f_{esc}$$

F. Grassi, Y. Hama, T. Kodama

(1995)

The talk is dedicated
to
the Brazilian physicists

Continuous emission → Hydrokinetics (HKM)



$$f = f_{int} + f_{esc}$$

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(1995)

- HKM is based on solution of Boltzmann + hydrodynamic equations for relativistic finite expanding systems in the relaxation time approximation for emission function;

- provides evaluation of emission function based on escape probabilities with account of deviations (even strong) of distribution functions *f(x,p)* from local equilibrium;

Complete algorithm includes:

- solution of equations of ideal or viscous hydro;
- calculation of non-equilibrium DF and emission function in first approximation;
- switch to UrQMD at space-time hypersurface

Yu.S., Akkelin, Hama: PRL 89 (2002) 052301;
+ Karpenko: PRC 78 (2008) 034906;
Karpenko, Yu.S. : PRC 81 (2010) 054903 ,
PLB 688 (2010) 50;
Karpenko, Yu.S., Werner: PRC 87 (2013) 024914.

Boltzmann equations and escape function

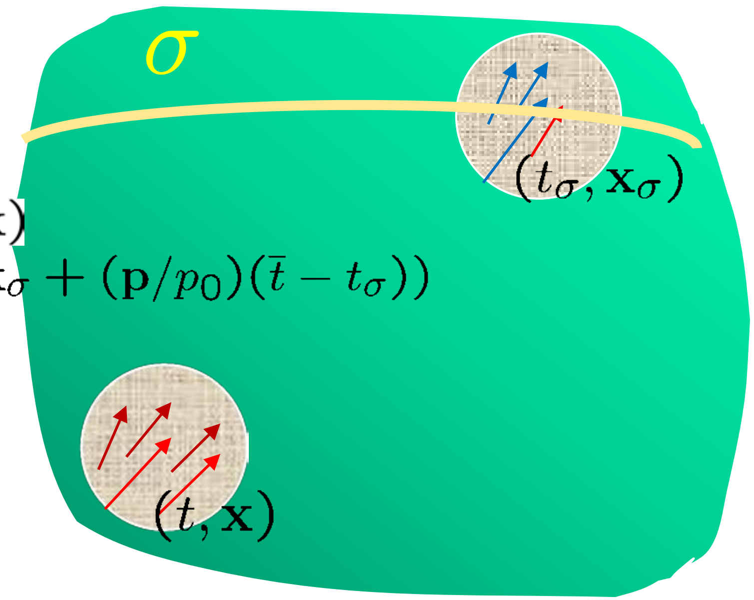
Boltzmann eq. (differential form -1) $\Rightarrow$ $$\frac{p^\mu}{p^0}\frac{\partial f_i(x,p)}{\partial x^\mu} = F_i^{gain}(x,p) - F_i^{loss}(x,p)$$

$F_i^{gain}(x,p)$ and $F_i^{loss}(x,p) = R_i(x,p) f_i(x,p)$ are Gain, Loss terms for i p. species

Probability of particle free propagation (for each component i)

$$\mathcal{P}_\sigma(x,p) = \exp\left(-\int_t^{t_\sigma} d\bar{t}\, R(\bar{x},p)\right)$$

$$x \equiv (t, \mathbf{x})$$
$$\bar{x} \equiv (\bar{t}, \mathbf{x}_\sigma + (\mathbf{p}/p_0)(\bar{t} - t_\sigma))$$



$$f^\sigma_{esc}(x,p) = \mathcal{P}_\sigma(x,p) f(x,p)$$

$$f^\sigma_{esc}(x,p)_{|\sigma} = f(x,p)_{|\sigma}$$

Boltzmann equations and escape function

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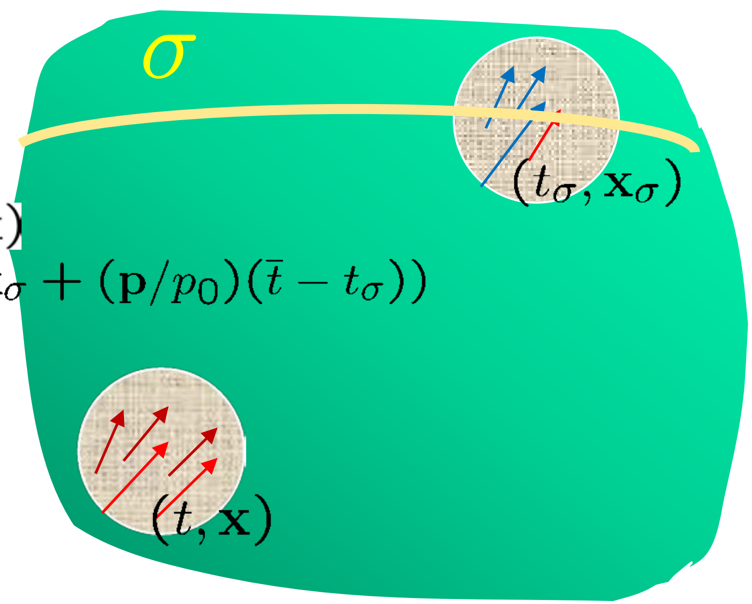
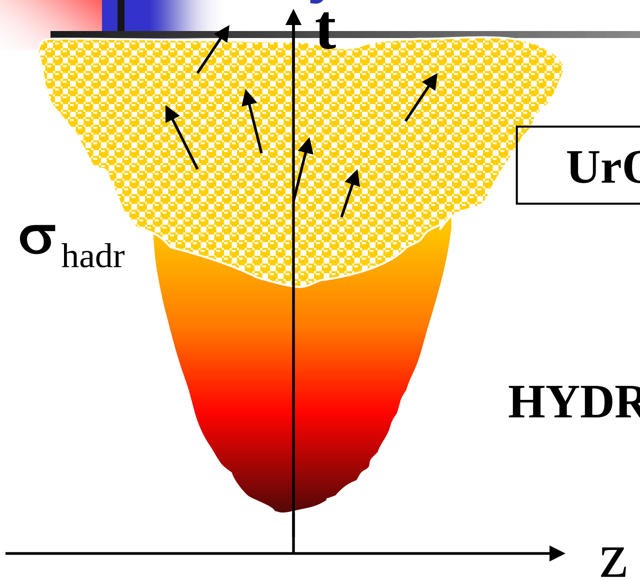


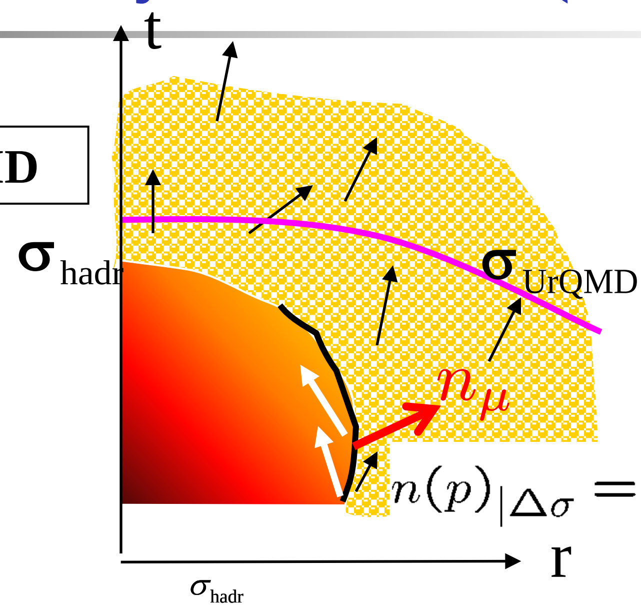
$$f^\sigma_{esc}(x,p) = \mathcal{P}_\sigma(x,p) f(x,p)$$

$$f^\sigma_{esc}(x,p)_{|\sigma} = f(x,p)_{|\sigma}$$

$$\frac{p^\mu}{p^0}\frac{\partial}{\partial x^\mu} f^\sigma_{esc}(x,p) = \mathcal{P}_\sigma(x,p) F^{gain}(x,p)$$ ⟵ **Boltzmann eq. (differential form -2)**

Hydrokinetics vs Hybrid models (HYDRO + UrQMD)





Hybrid model

$$n(p)|_{\Delta\sigma} = \int_{\Delta\sigma} d\sigma n_\mu p^\mu f(x,p) < 0$$

$\sigma_{\text{hadr}} : \tau \equiv \sqrt{t^2 - z^2} = const$

at $r = const$

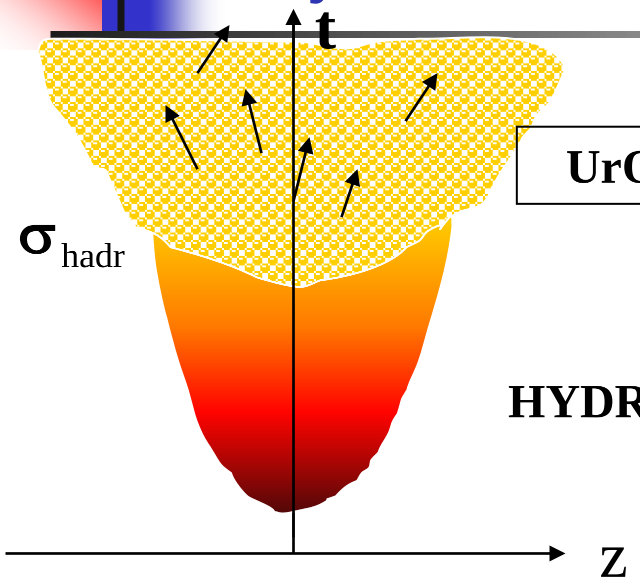
$\sigma_{\text{hadr}} : \tau(r)$ at $z = 0$

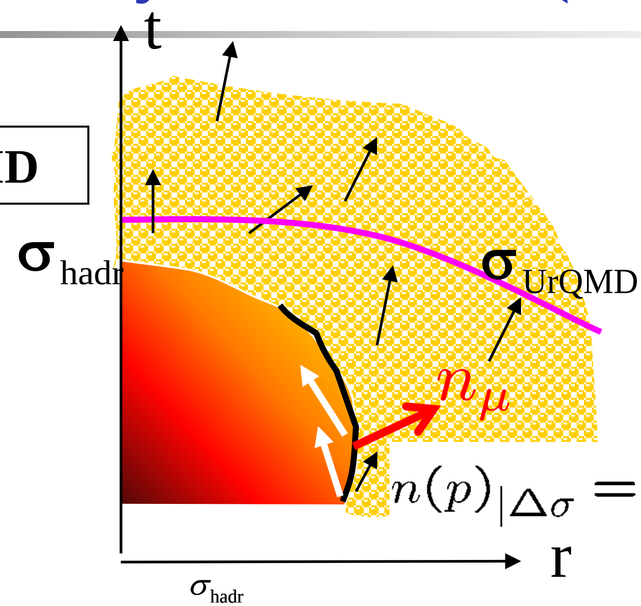
The problems:

- Hadronization hypersurface $\tau(r)$ contains non-space-like sectors (causality problem: Bugaev, PRL 90, 252301, 2003);
- In hybrid models after elimination of negative contribution the at the “bad” sites of σ_{hadr}:

$$T^{\mu\nu}_{hydro}(x) n_\mu(x) \neq T^{\mu\nu}_{UrQMD}(x) n_\mu(x)$$

Hydrokinetics vs Hybrid models (HYDRO + UrQMD)





$\sigma_{hadr}: \tau \equiv \sqrt{t^2 - z^2} = const$

at $r = const$

$\sigma_{hadr}: \tau(r)$ at z = 0

Hybrid model

$$n(p)_{|\Delta\sigma} = \int_{\Delta\sigma} d\sigma n_\mu p^\mu f(x,p) < 0$$

$$\Downarrow$$

$$f^{\sigma}_{esc}(x,p)$$

Hydrokinetics

The problems:

- Hadronization hypersurface $\tau(r)$ contains non-space-like sectors (causality problem: Bugaev, PRL 90, 252301, 2003);
- In hybrid models at the "bad" sites of σ_{hadr}

$$T^{\mu\nu}_{hydro}(x) n_\mu(x) \neq T^{\mu\nu}_{UrQMD}(x) n_\mu(x)$$

- In Hydrokinetics there is no discontinuity:

$$T^{\mu\nu}_{hydro}(x) n_\mu(x) \approx T^{\mu\nu}_{HKM}(x) n_\mu(x)$$

$\Rightarrow$ **The initial conditions for hadronic cascade models should be based on non-local equilibrium distributions**

Escape function dynamics and the medium particlization

BE:

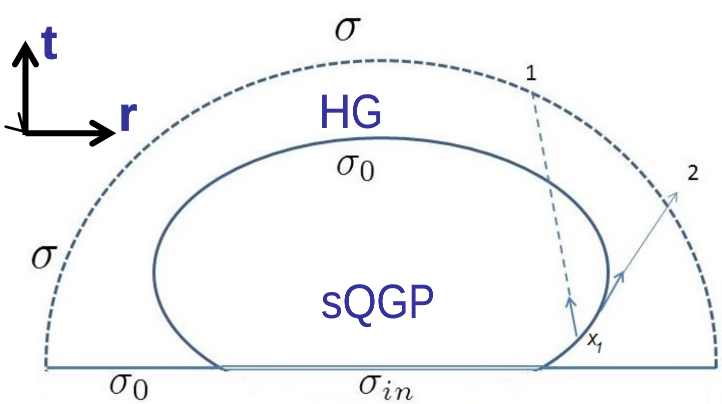
$$\frac{p^i}{p^0}\frac{\partial f(x,p)}{\partial x^i} = F^{gain}(x,p) - F^{loss}(x,p); \quad \frac{p^\mu}{p^0}\frac{\partial}{\partial x^\mu} f^\sigma_{esc}(x,p) = \mathcal{P}_\sigma(x,p) F^{gain}(x,p)$$

One- and two- particle spectra:

$$\sigma = \sigma_{out} \Rightarrow f^\sigma_{esc} = f^\sigma$$

$$p^0 \frac{dN}{d\mathbf{p}} = \int_{\sigma_{out}} d\sigma_\mu(x) p^\mu f(x,p)$$

$$p^0 \frac{dN}{d\mathbf{p}} = \int_{\sigma_{out}} d\sigma_\mu(x) p^\mu f_{esc}(x,p)$$



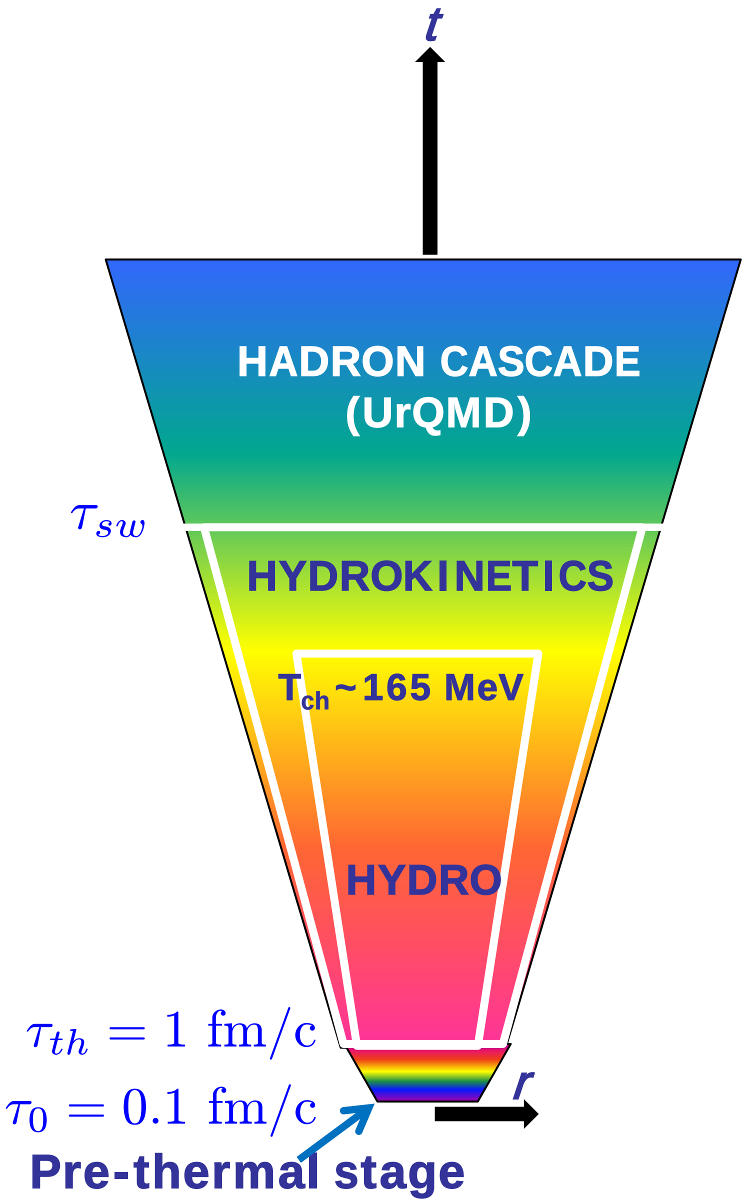
Gauss theorem:

Emission f-on $S(x,p)$

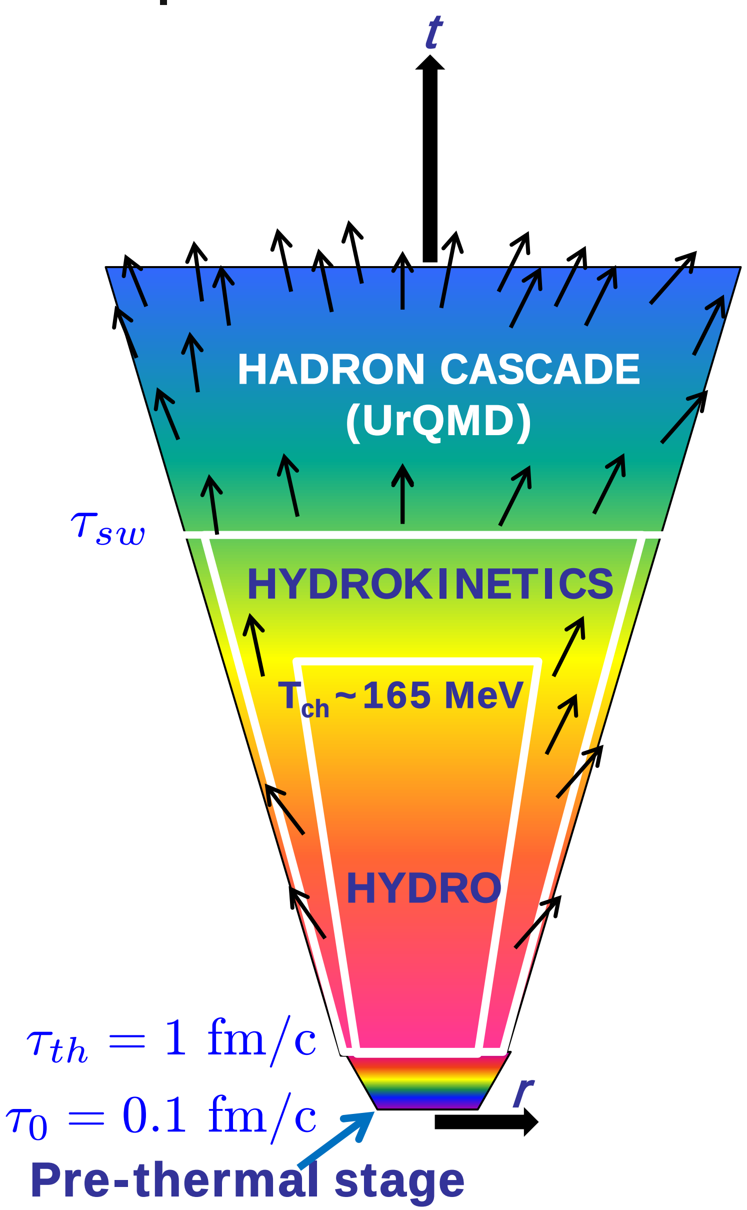
$$p^0 \frac{dN}{d\mathbf{p}} = p_0 \int_{\sigma_0} d^3\sigma_\mu(x_0) p^\mu f^\sigma_{esc}(x_0,p) + p_0 \int_{\sigma_0}^{\sigma} d^4x \mathcal{P}_\sigma(x,p) F^{gain}(x,p),$$

$$p^0 \frac{dN}{d\mathbf{p}} = p^0 \int_{\sigma_0} d^3\sigma_\mu(x_0) p^\mu f(x_0,p) + p^0 \int_{\sigma_0}^{\sigma} d^4x \left[F^{gain}(x,p) - F^{loss}(x,p)\right]$$

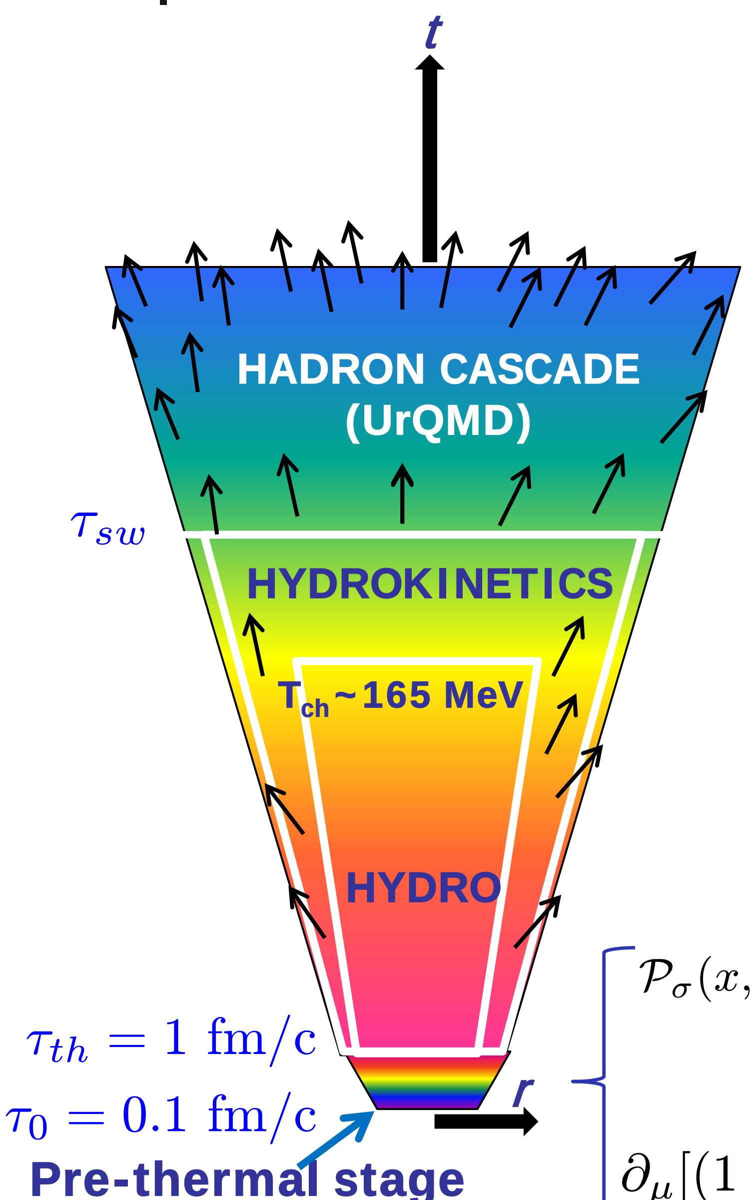
Hybrid hydrokinetic model (HKM)



Hybrid hydrokinetic model (HKM)



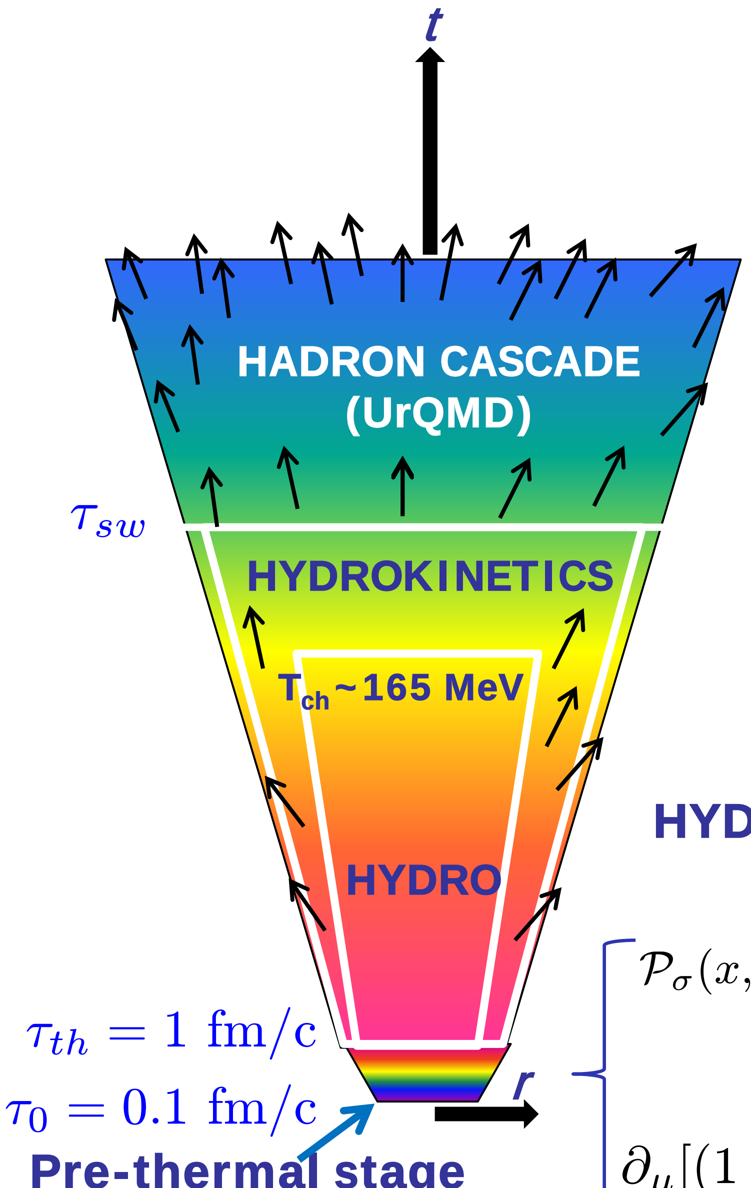
Hybrid hydrokinetic model (HKM)



$$\begin{cases} \mathcal{P}_\sigma(x,p) = \exp\left(-\int_t^{t_\sigma} \frac{d\bar{t}}{\tau_{rel}(\overline{x},p)}\right) \qquad \overline{x} \equiv (\bar{t}, \mathbf{x}_\sigma + (\mathbf{p}/p_0)(\bar{t} - t_\sigma)) \\ \partial_\mu[(1 - \mathcal{P}_{\sigma_{th}}(x) T^{\mu\nu}_{\mathtt{hyd}}(x)] = -T^{\mu\nu}_{\mathtt{pQCD}}(x)\partial_\mu \mathcal{P}_{\sigma_{th}}(x) \end{cases}$$

Akkelin, Yu.S. PRC (2010)

Hybrid hydrokinetic model (HKM)



HYDRO $\partial_\nu T^{\mu\nu} = 0; \quad p(\epsilon),\ \mu_B$ EoS from LatQCD

$$\mathcal{P}_\sigma(x,p) = \exp\left(-\int_t^{t_\sigma} \frac{d\bar{t}}{\tau_{rel}(\bar{x},p)}\right) \qquad \bar{x} \equiv (\bar{t}, \mathbf{x}_\sigma + (\mathbf{p}/p_0)(\bar{t} - t_\sigma))$$

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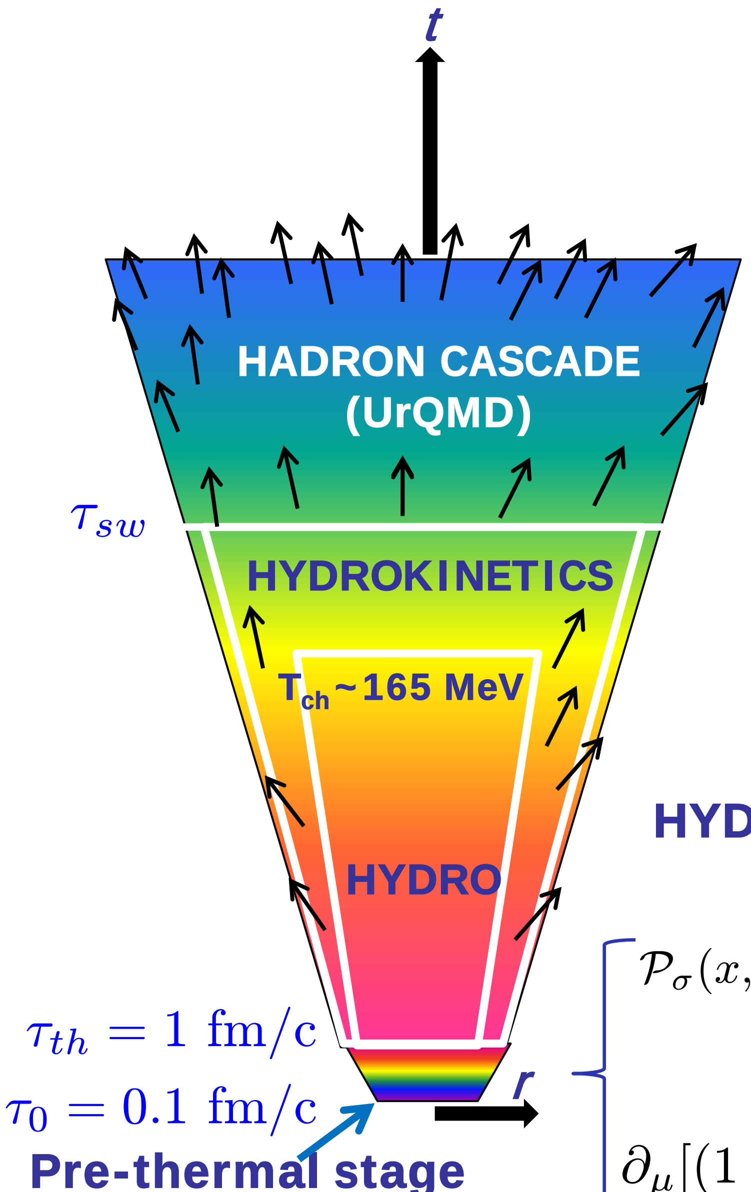
Hybrid hydrokinetic model (HKM)



HYDROKINETICS

$$f^{\sigma}_{esc}(x,p) = \mathcal{P}_{\sigma}(x,p) f(x,p)$$

$$\frac{p^{\mu}}{p^{0}} \frac{\partial}{\partial x^{\mu}} f^{\sigma}_{esc}(x,p) = \mathcal{P}_{\sigma}(x,p) F^{gain}(x,p)$$

$$\partial_{\nu} T^{\mu\nu} = 0; \quad p(\epsilon, n_i),\ n_i$$

$$\partial_{\mu}(n_i(x) u^{\mu}(x)) = -\Gamma_i n_i + \sum_j b_{ij} \Gamma_j n_j(x)$$

EoS of hadron gas of 359 hadrons

HYDRO $\partial_{\nu} T^{\mu\nu} = 0; \quad p(\epsilon),\ \mu_B$

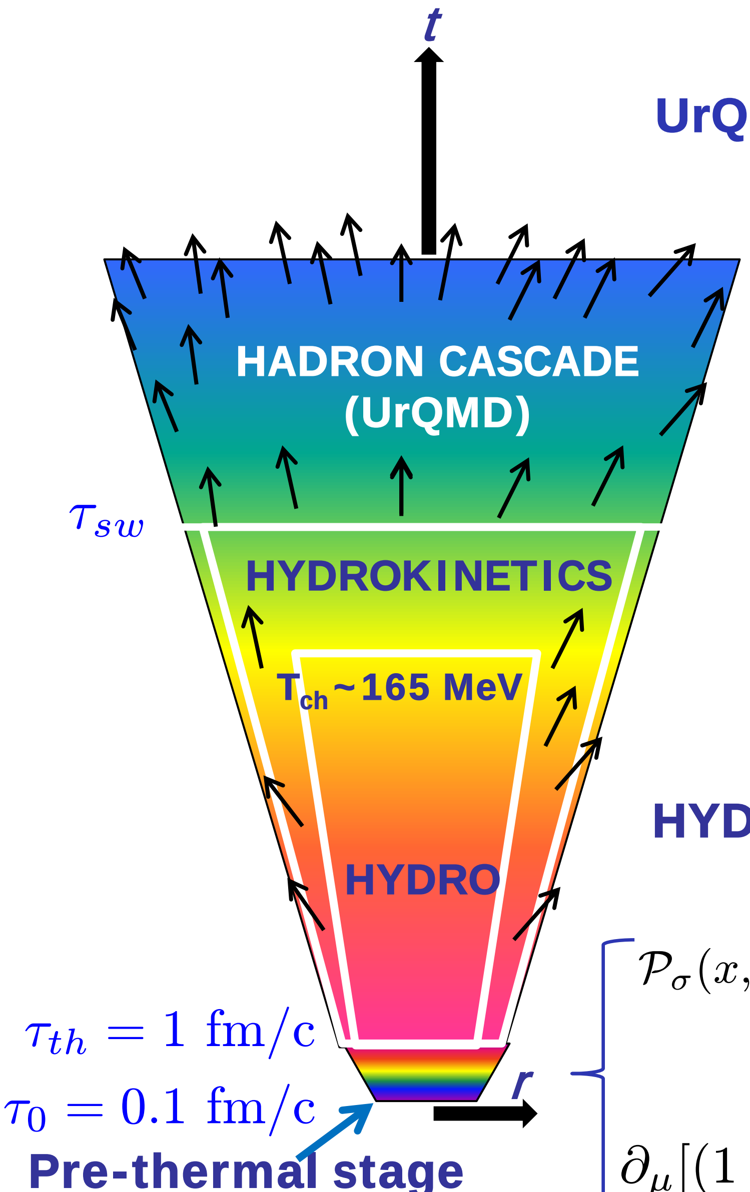
EoS from LatQCD

$$\mathcal{P}_{\sigma}(x,p) = \exp\left(-\int_{t}^{t_{\sigma}} \frac{d\bar{t}}{\tau_{rel}(\bar{x},p)}\right) \qquad \bar{x} \equiv (\bar{t}, \mathbf{x}_{\sigma} + (\mathbf{p}/p_0)(\bar{t} - t_{\sigma}))$$

$$\partial_{\mu}[(1 - \mathcal{P}_{\sigma_{th}}(x) T^{\mu\nu}_{\mathtt{hyd}}(x)] = -T^{\mu\nu}_{\mathtt{pQCD}}(x) \partial_{\mu} \mathcal{P}_{\sigma_{th}}(x)$$

Akkelin, Yu.S. PRC (2010)

Hybrid hydrokinetic model (HKM)



UrQMD — Ultra-relativistic Quantum Molecular Dynamics — Bass et al, Frankfurt Group 2000-2012

HYDROKINETICS

$$f^{\sigma}_{esc}(x,p) = \mathcal{P}_{\sigma}(x,p) f(x,p)$$

$$\frac{p^{\mu}}{p^{0}} \frac{\partial}{\partial x^{\mu}} f^{\sigma}_{esc}(x,p) = \mathcal{P}_{\sigma}(x,p) F^{gain}(x,p)$$

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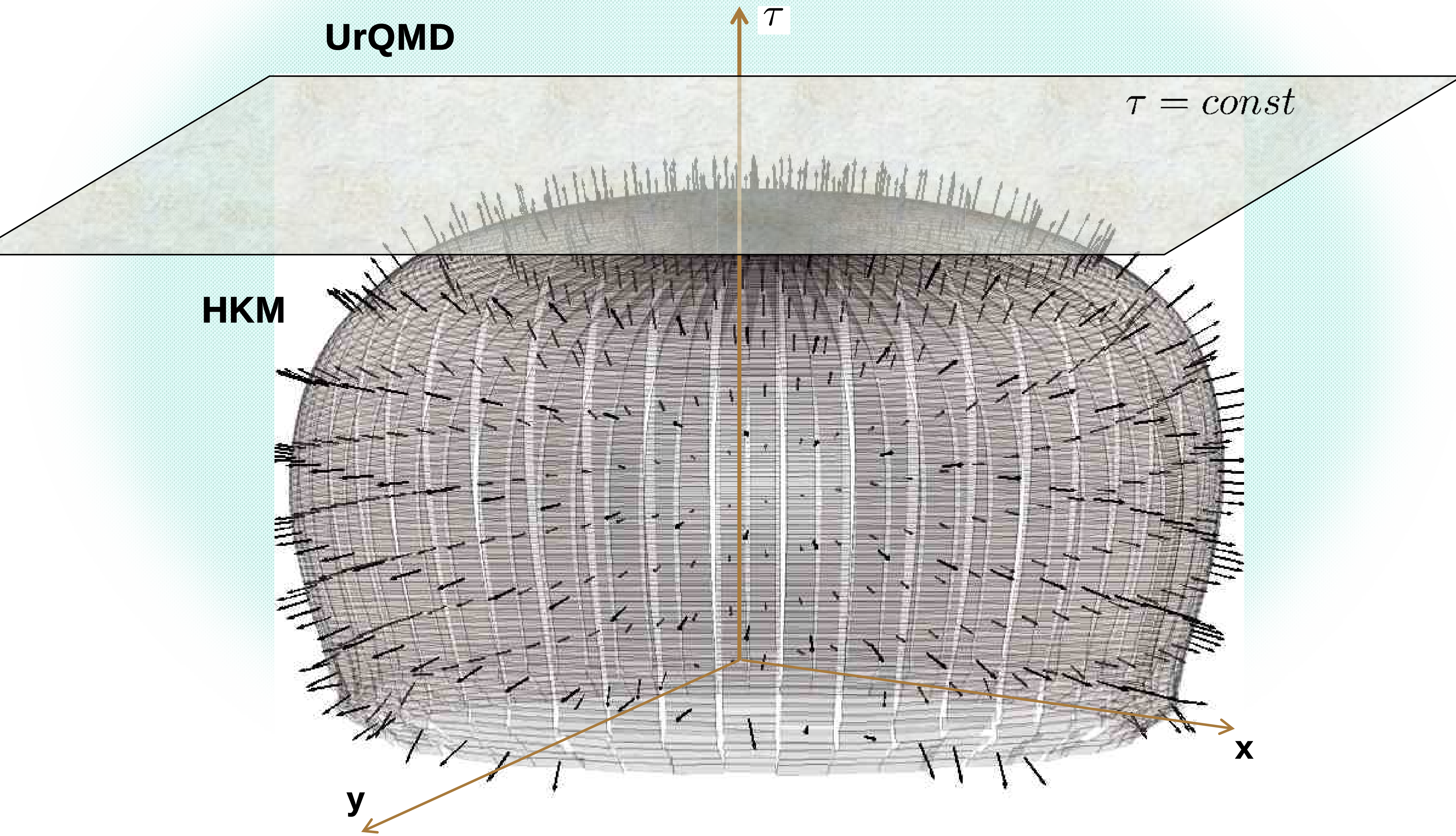
$$\partial_{\mu}[(1 - \mathcal{P}_{\sigma_{th}}(x) T^{\mu\nu}_{\mathtt{hyd}}(x)] = -T^{\mu\nu}_{\mathtt{pQCD}}(x) \partial_{\mu} \mathcal{P}_{\sigma_{th}}(x)$$

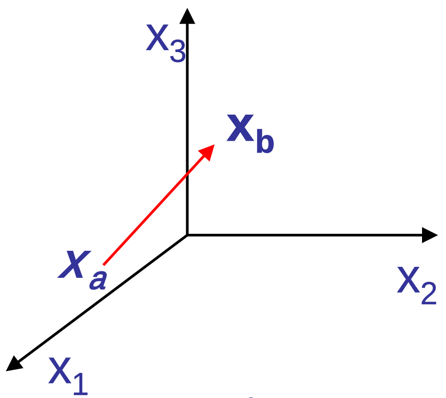
Akkelin, Yu.S. PRC (2010)

Hybrid HKM (hHKM): matching of HKM and UrQMD at the space-like hypersurface $\tau = \tau(x = y = 0, T = T_{ch})$

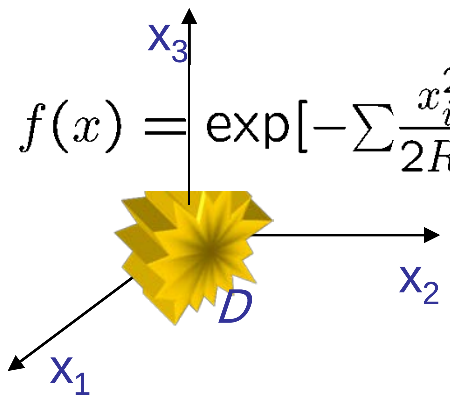


Interferometry microscope (Kopylov / Podgoretcky – 1971)

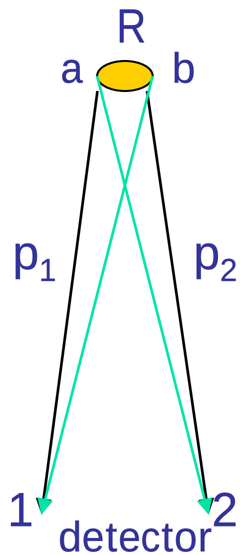
The idea of the correlation femtoscopy is based on quantum statistic for emitted identical particles.



$\boxed{t=0}$



$$f(x) = \exp[-\sum \frac{x_i^2}{2R_i^2}]$$



Momentum representation

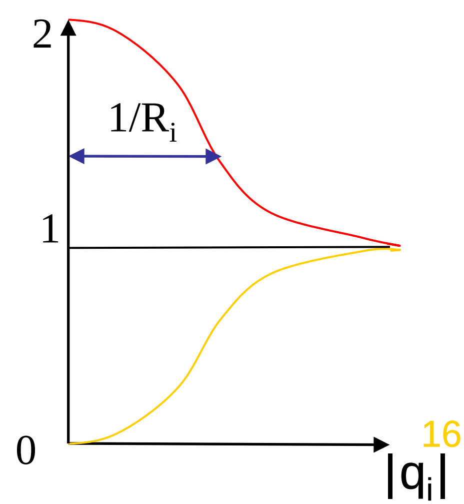
$$\Psi_{x_a,x_b}(p_1,p_2) = \frac{1}{\sqrt{2}}\left[e^{-i\mathbf{p}_1\cdot\mathbf{x}_a}e^{-i\mathbf{p}_2\cdot\mathbf{x}_b} \pm e^{-i\mathbf{p}_2\cdot\mathbf{x}_a}e^{-i\mathbf{p}_1\cdot\mathbf{x}_b}\right]$$

Probabilities:

$$W_{x_a,x_b}(p_1,p_2) = \left|\Psi_{x_a,x_b}(p_1,p_2)\right|^2 = 1 \pm \cos\left[\overbrace{(\mathbf{p}_1 - \mathbf{p}_2)}^{\mathbf{q}} \cdot (\mathbf{x}_a - \mathbf{x}_b)\right]$$

$$W_D(p_1,p_2) = \int d^3x_a d^3x_b f(x_a) f(x_b) W_{x_a,x_b}(p_1,p_2) =$$

$$1 \pm \left|\int d^3x f(x) e^{i\mathbf{q}\mathbf{x}}\right|^2 = 1 \pm \exp\left[-\sum q_i^2 R_i^2\right]$$



THE DEVELOPMENT OF THE FEMTOSCOPY

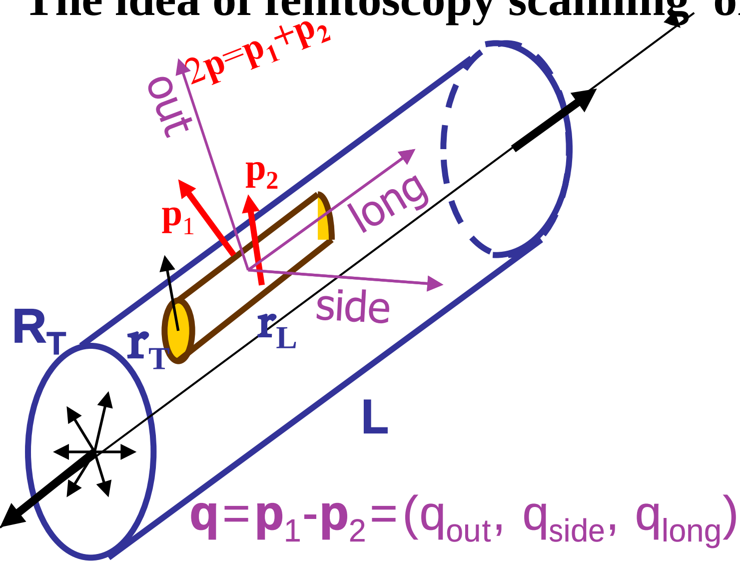
Even ultra small systems can have an internal structure. Then the distribution function *f(x,p)* and emission function of such an objects are inhomogeneous and, typically, correlations between the momentum *p* of emitted particle and its position x appear.

S.Pratt, Phys.Rev.D33(1986)1314. A.N.Makhlin,Yu.M.Sinyukov, Sov.J.Nucl.Phys. 46 (1987) 345.
A.N.Makhlin,Yu.M.Sinyukov Z.Phys.C39(1988) 69. Y.Hama, S.S.Padula, Phys.Rev. D37 (1988) 3237.

In this case and in general the interferometry microscope measure the homogeneity lengths in the systems [Yu. S. 1993-1995].

$$\frac{\left|f(p,x_0+\bar{\lambda})-f(p,x_0)\right|}{f(p,x_0)}=\frac{1}{2} \qquad \left.\frac{\partial f(p,x)}{\partial x_i}\right|_{x_0(p)}=0 \qquad \Longrightarrow \qquad \lambda_i^{\,2}=\frac{f(x_0,p)}{\left|f''_{x_i}(x_0,p)\right|}$$

The idea of femtoscopy scanning of a source over momentum Averchenkov/Makhlin/Yu.S. (1987)



Interferomerty radii:

$$R_L(p_T)\approx\lambda_L=\tau\sqrt{\frac{T_{f.o.}}{m_T}}/cosh(y), m_T=\sqrt{m^2+p_T^2}$$

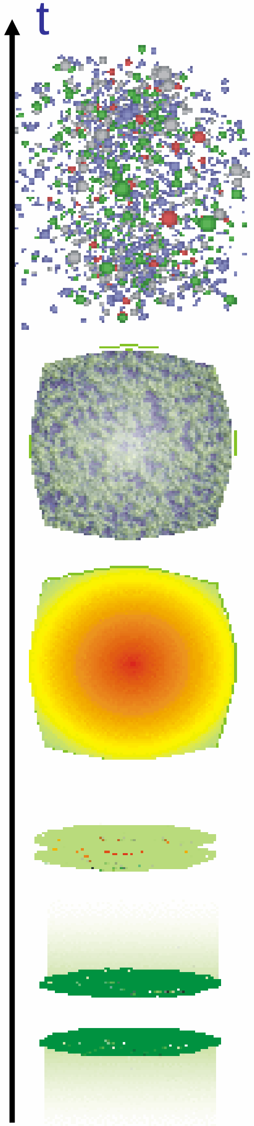
$$R_S\approx\lambda_T=R_T/\sqrt{1+Im_T/T_{f.o.}},\ I\propto<v_T^2>$$

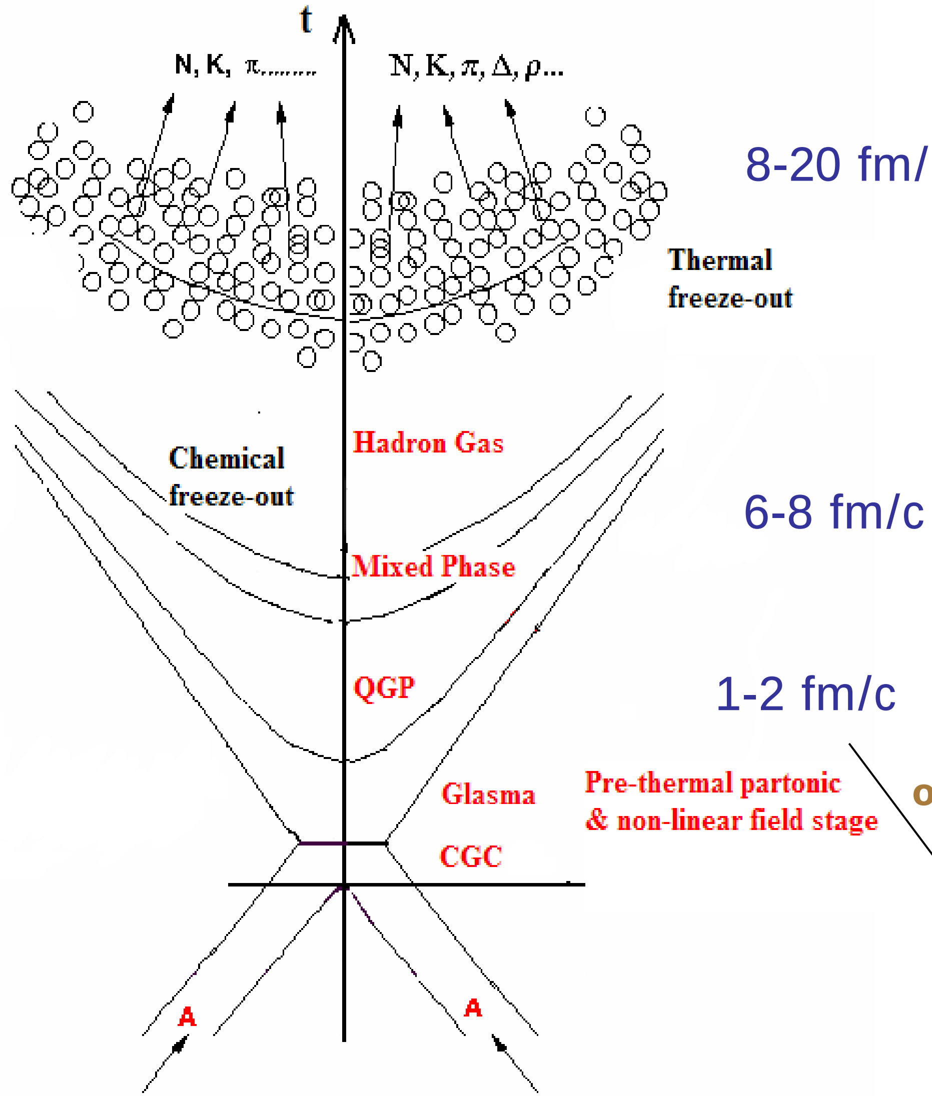
$$R_o^2\approx\lambda_T^2+v^2\langle\triangle t^2\rangle_p-2v\langle\triangle x_o\triangle t\rangle_p, v=\frac{p_{out}}{p_0}$$

$$C(p,q)=\frac{d^6N/d^3p_1d^3p_2}{d^3N/d^3p_1d^3N/d^3p_2}\approx 1+e^{R_L^2(p)q_L^2+R_s^2(p)q_s^2+R_O^2(p)q_O^2}$$

QGP $\Longrightarrow$ $R_{out}/R_{side}>>1$ $Exp: R_{out}/R_{side}\approx 1$ RHIC HBT PUZZLE

Expecting Stages of Evolution in Ultrarelativistic A+A collisions





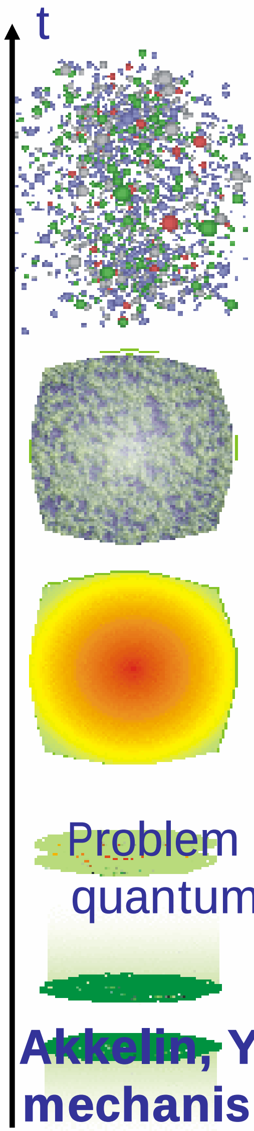
8-20 fm/c

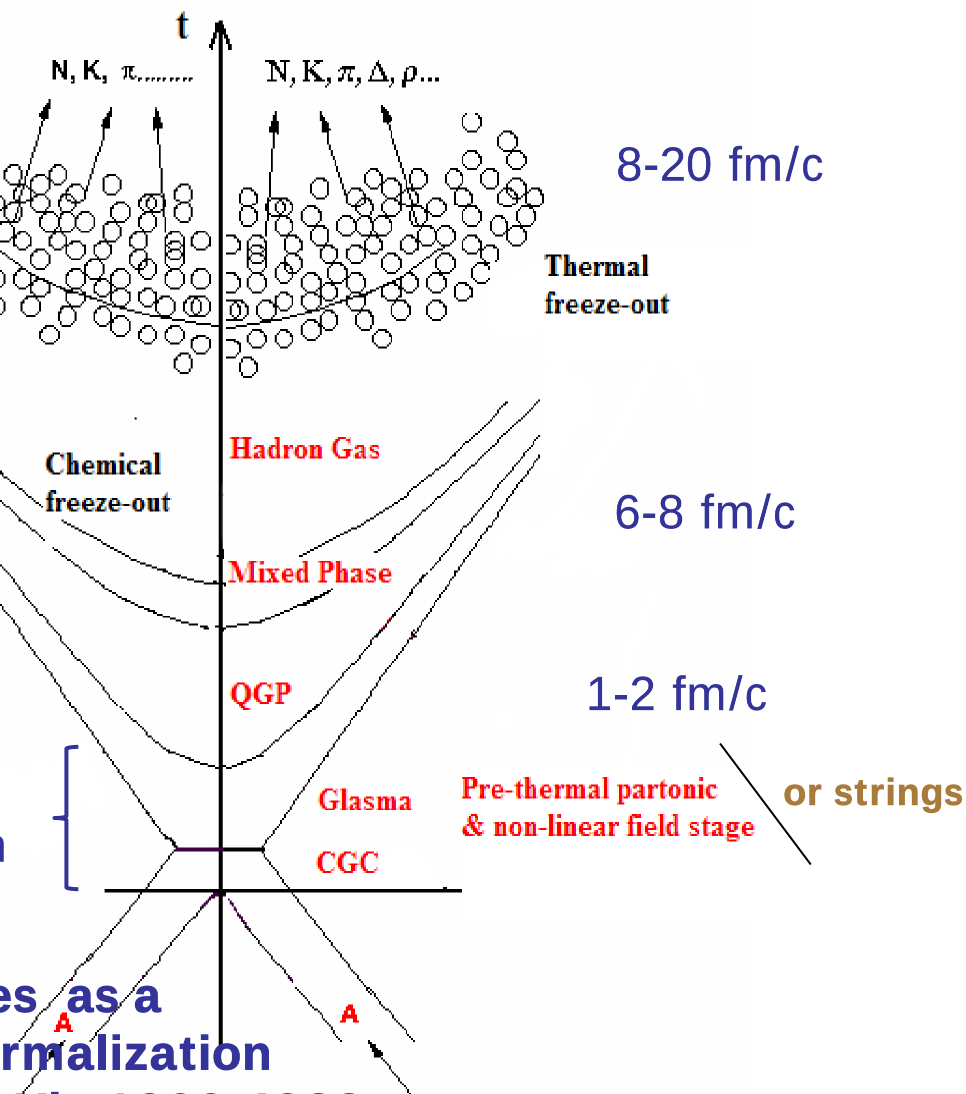
6-8 fm/c

1-2 fm/c

or strings

Experimentally discovered thermalization as a great theoretical problem



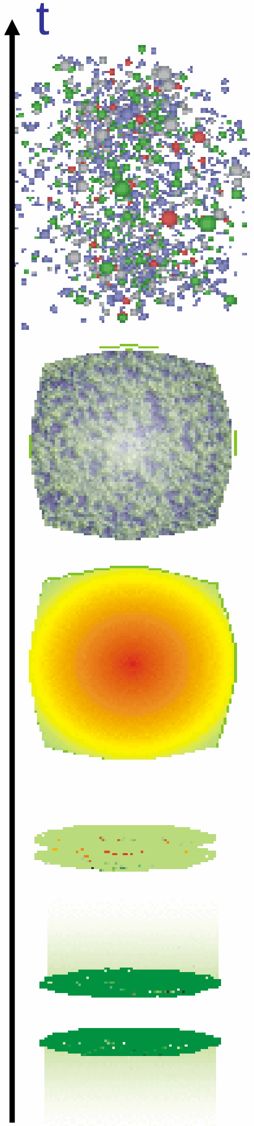


Problem of thermalization for isolated quantum systems with unitary evolution

⇓

Akkelin, Yu.S. “Entanglement of scales as a mechanism for decoherence and thermalization in relativistic heavy ion collisions” ArXiv 1309.4388

Observations and initial and final time scales

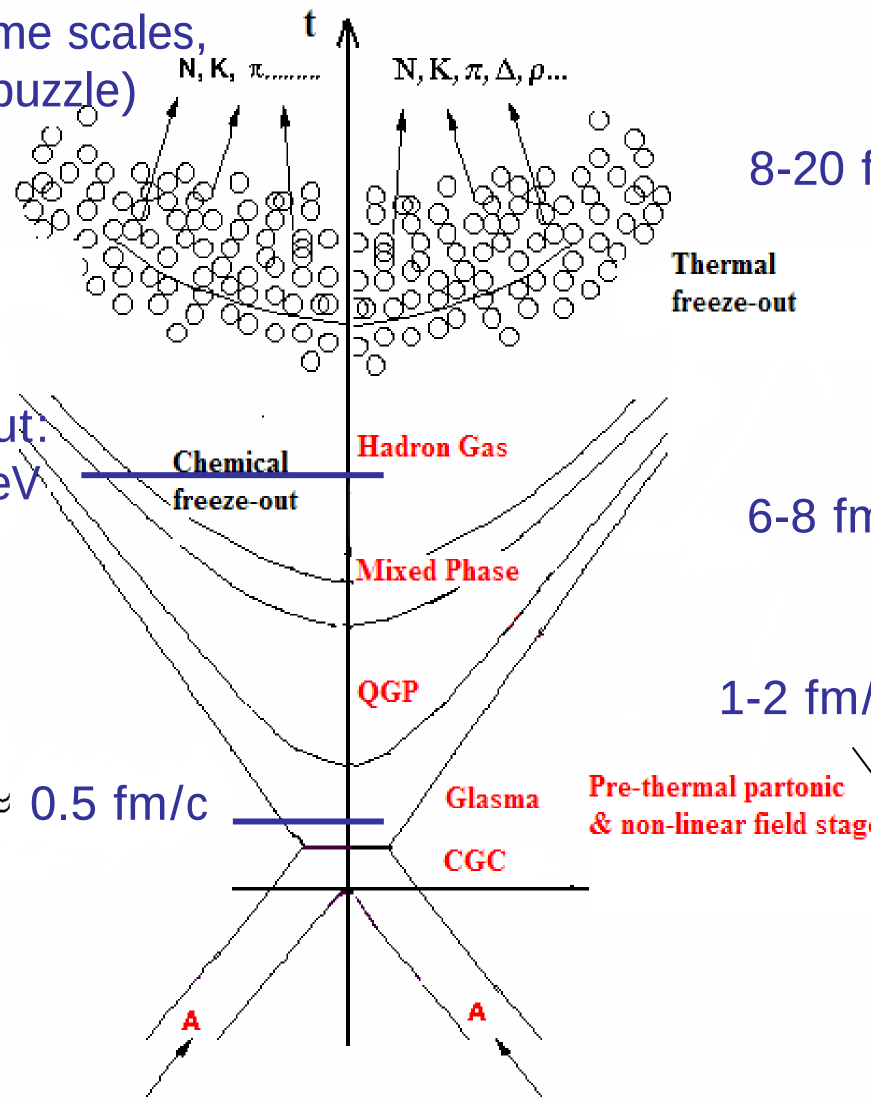


Relatively small space-time scales,
$R_{out} / R_{side} \sim 1$ (HBT puzzle)

Early thermal freeze-out:
Tth ≈ Tch ≈ 150 MeV

Elliptic flows

Early thermalization at ≈ 0.5 fm/c



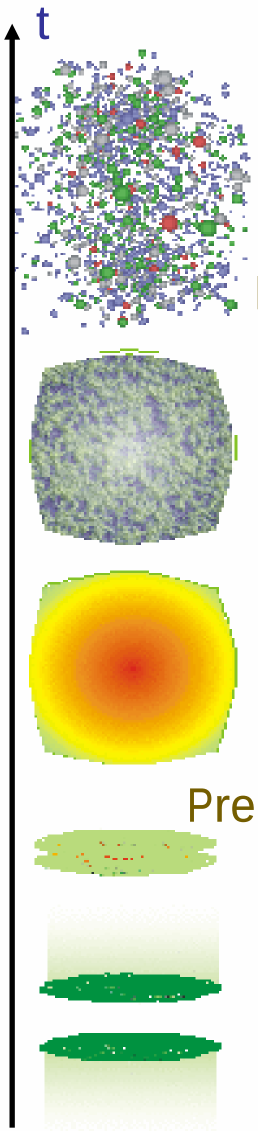
8-20 fm/c

6-8 fm/c

1-2 fm/c

or strings

Corrections to the time scales



Relatively small space-time scales,
$R_{out} / R_{side} \sim 1$ (HBT puzzle)

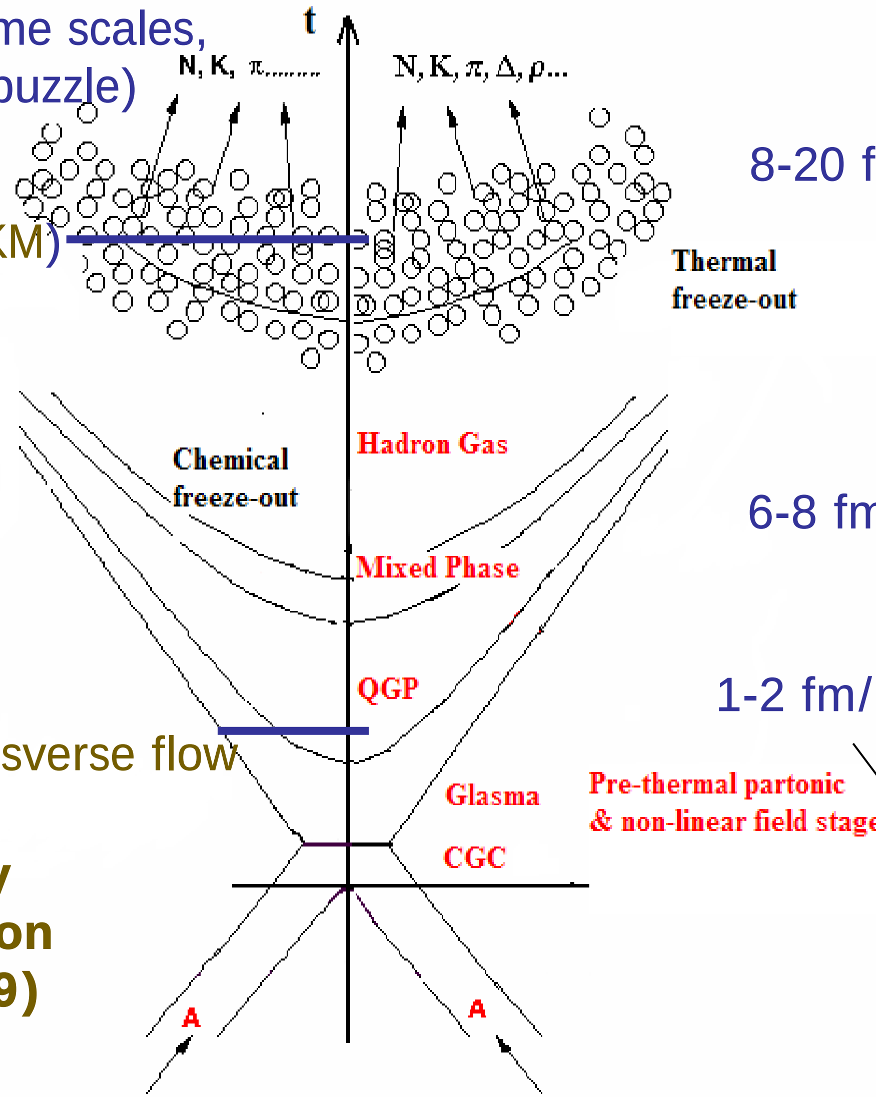
HydroKinetic Model (HKM)

No need for early freeze-out

Elliptic flows

Pre-thermal anisotropic transverse flow

No need for very early thermalization (Yu.S. 2006 -2009)



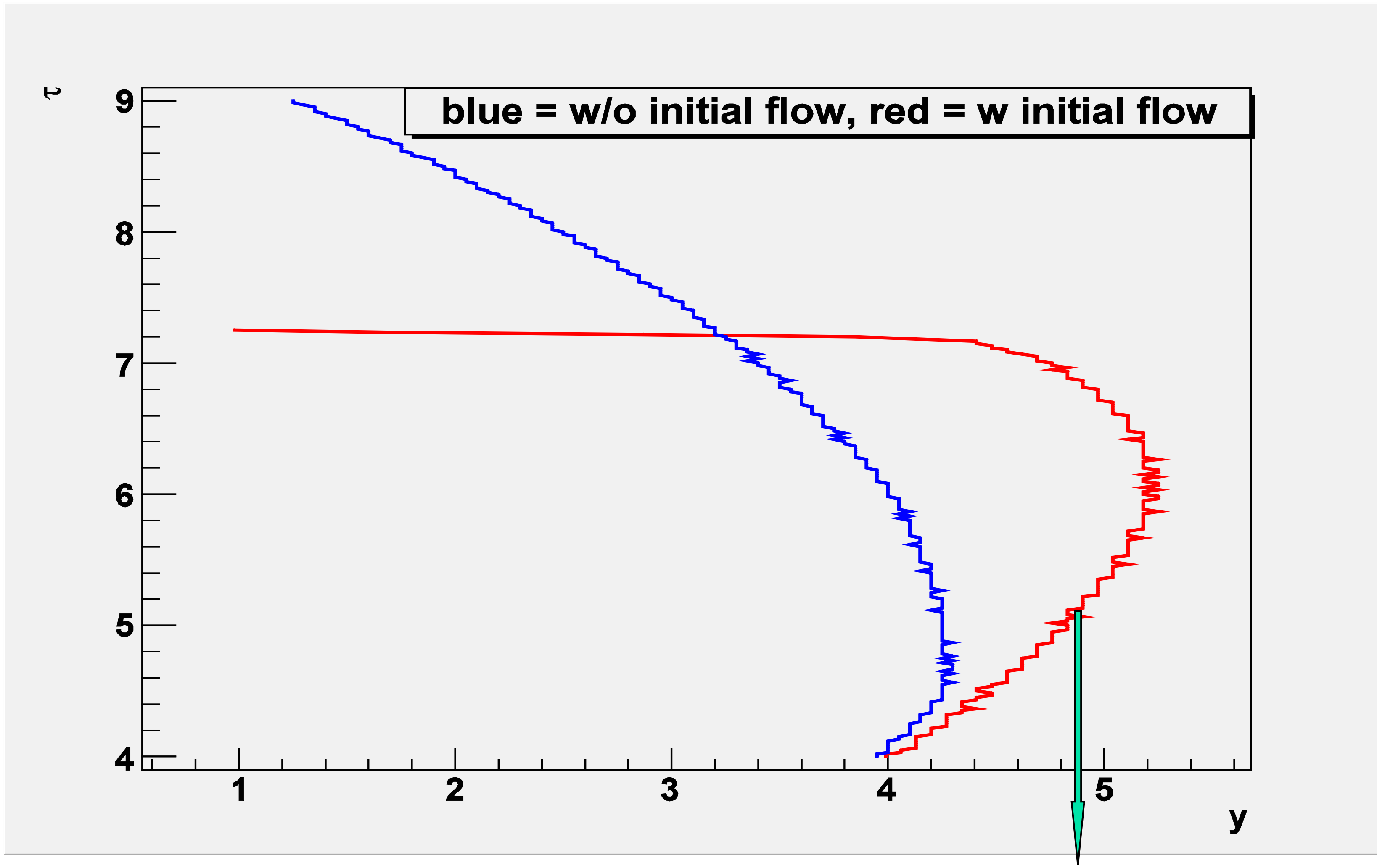
8-20 fm/c

6-8 fm/c

1-2 fm/c

or strings

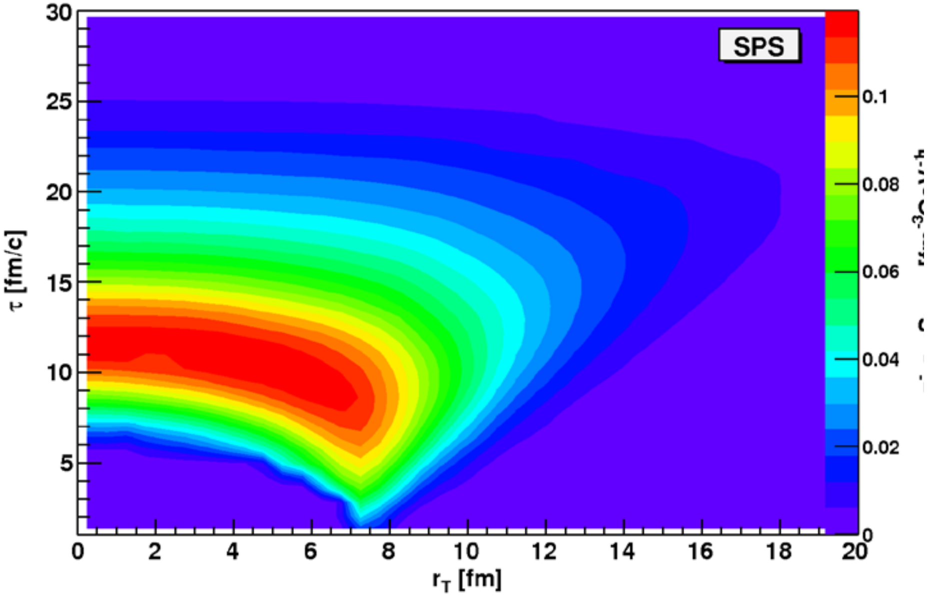
Ro/Rs ratio and prethermal flows

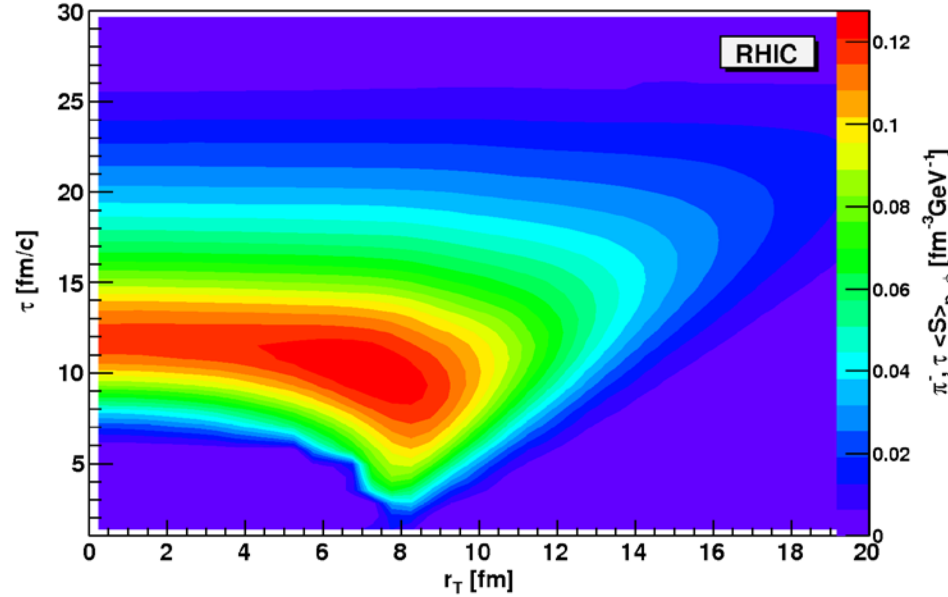


$$R_{out}^2 \approx R_{side}^2 + v^2\langle \triangle t^2\rangle_p - 2v\langle \triangle x_{out}\triangle t\rangle_p, v = \frac{p_T}{p^0}$$

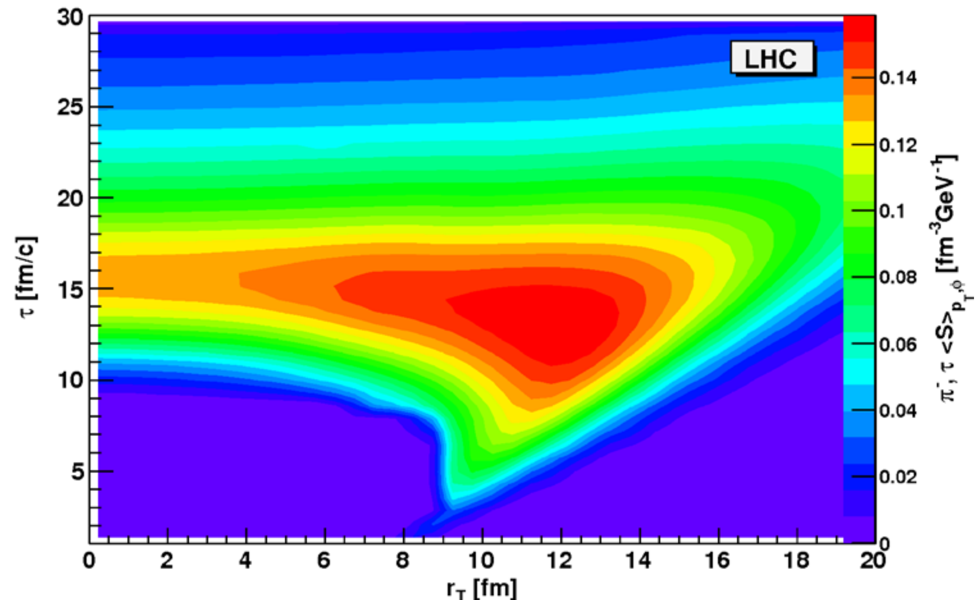
Yu.S. Act. Phys. Pol. 37 (2006) 3343

Emission functions for top SPS, RHIC and LHC energies





$$R_{out}^2 \approx R_{side}^2 + v^2\langle\triangle t^2\rangle_p - 2v\langle\triangle x_{out}\triangle t\rangle_p, v = \frac{p_T}{p^0}$$



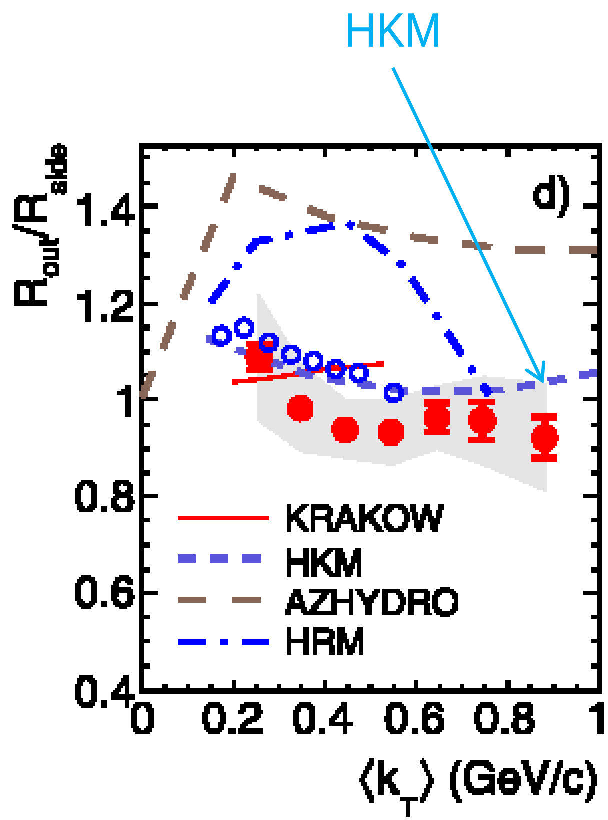
HKM prediction: solution of the HBT Puzzle

Two-pion Bose–Einstein correlations in central Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV ☆ ALICE Collaboration Physics Letters B 696 (2011) 328-

ALICE

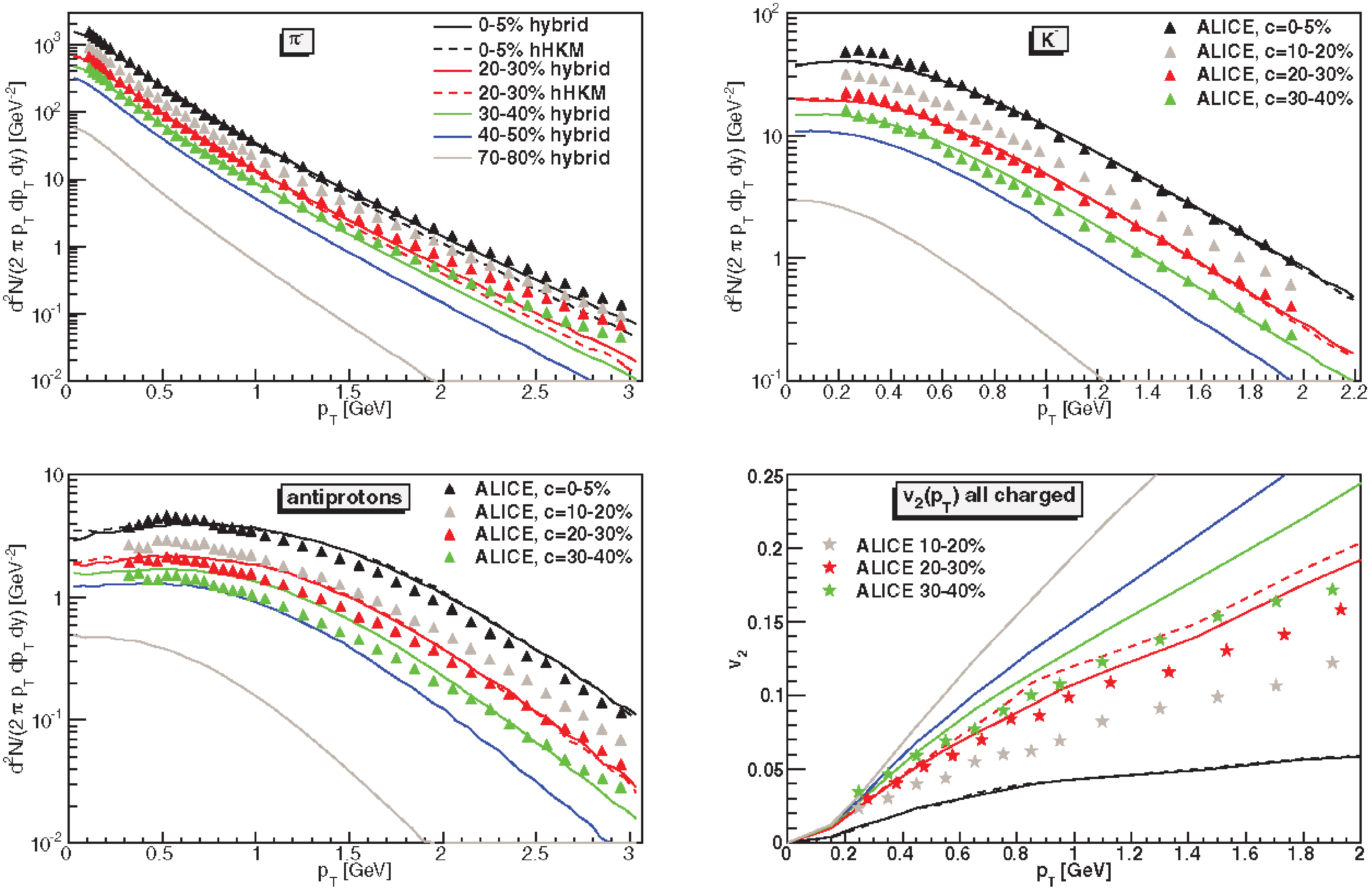
Quotations:

Available model predictions are compared to the experimental data in Figs. 2-d and 3. Calculations from three models incorporating a hydrodynamic approach, AZHYDRO [45], KRAKOW [46,47], and HKM [48,49], and from the hadronic-kinematics-based model HRM [50,51] are shown. An in-depth discussion is beyond the scope of this Letter but we notice that, while the increase of the radii between RHIC and the LHC is roughly reproduced by all four calculations, only two of them (KRAKOW and HKM) are able to describe the experimental R_{out}/R_{side} ratio.



[48] I.A. Karpenko, Y.M. Sinyukov, Phys. Lett. B 688 (2010) 50.
[49] N. Armesto, et al. (Eds.), J. Phys. G 35 (2008) 054001.

HKM results for hadron spectra and v_2 for different centralities

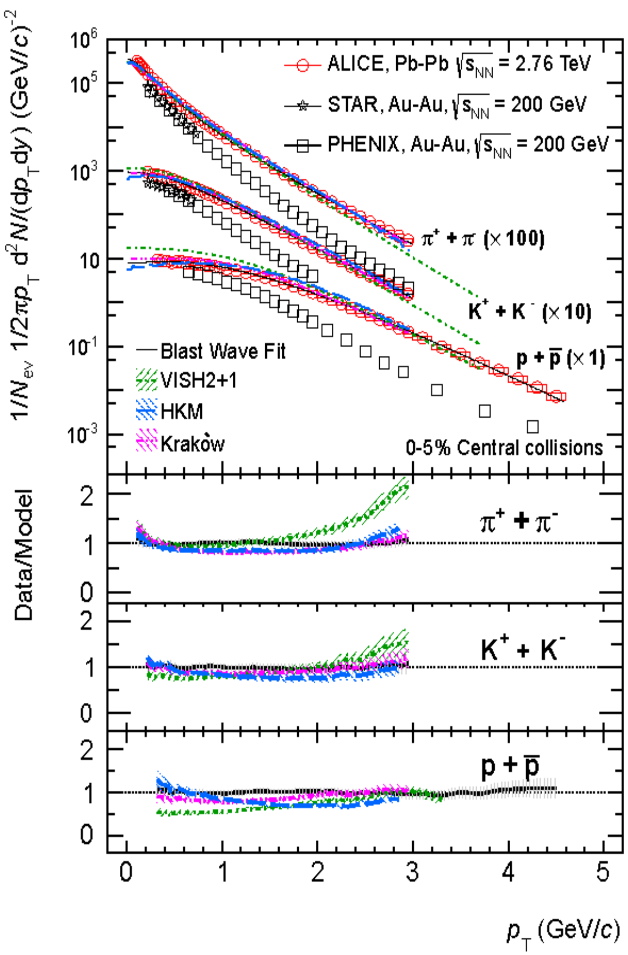


Predictions for particle yield at LHC in central collisions

Pion, Kaon, and Proton Production in Central Pb–Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

The ALICE Collaboration* **Phys. Rev. Lett. 109, 252301 (2012)**

ALICE



Quotations

This interpretation is supported by the comparison with HKM [39, 40], a similar model in which, after the hydrodynamic phase, particles are injected into a hadronic cascade model (UrQMD [41, 42]), which further transports them until final decoupling. The hadronic phase builds additional radial flow, mostly due to elastic interactions, and affects particle ratios due to inelastic interactions. HKM yields a better description of the data. At the LHC, hadronic final state interactions, and in particular antibaryon-baryon annihilation, may therefore be an important ingredient for the description of particle yields [43, 40], contradicting the scenario of negligible abundance-changing processes in the hadronic phase. The third model shown in Fig. 1 (Kraków [44, 45]) introduces non-equilibrium corrections due to viscosity at the transition from the hydrodynamic description to particles, which change the effective T_{ch}, leading to a good agreement with the data. In the region $p_T \lesssim 3$ GeV/c (Kraków) and $p_T \lesssim 1.5$ GeV/c (HKM) the last two models reproduce the experimental data within $\sim$20%, supporting a hydrodynamic interpretation of the transverse momentum spectra at the LHC. These models also describe correctly other features of the space-time evolution of the system, as measured by ALICE with charged pion correlations [46].

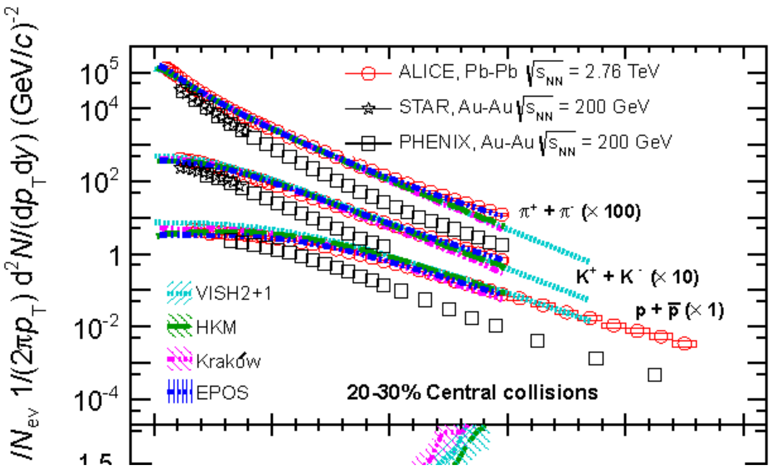
[39] Y. Karpenko and Y. Sinyukov, J.Phys. **G38**, 124059 (2011), nucl-th/1107.3745.
[40] Y. Karpenko, Y. Sinyukov, and K. Werner, (2012), nucl-th/1204.5351.

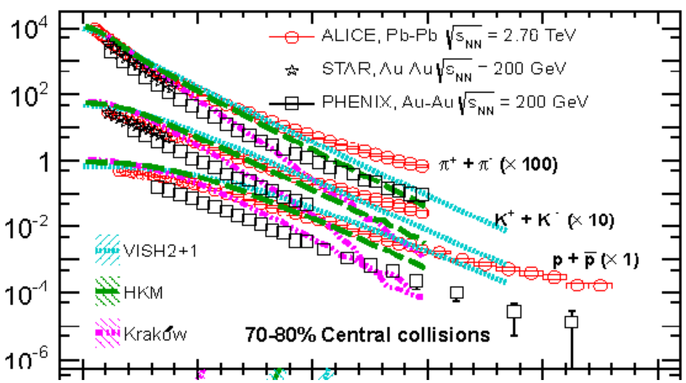
Predictions for particle spectra at LHC in non-central collisions

Centrality Dependence of π, K, p in Pb–Pb at $\sqrt{s_{NN}} = 2.76$ TeV

ALICE Collaboration

arXiv:1303.0737v1 [hep-ex]





Quotations:

The difference between VISH2+1 and the data are possibly due to the lack of an explicit description of the hadronic phase in the model. This idea is supported by the comparison with HKM [47, 50]. HKM is a model similar to VISH2+1, in which after the hydrodynamic phase particles are injected into a hadronic cascade model (UrQMD), which further transports them until final decoupling. The hadronic phase builds up additional radial flow and affects particle ratios due to the hadronic interactions. As can be seen, this model yields a better description of the data. The protons at low p_T, and hence their total number, are rather well reproduced, even if the slope is significantly smaller than in the data. Antibaryon-baryon annihilation is an important ingredient for the description of particle yields in this model [47, 50].

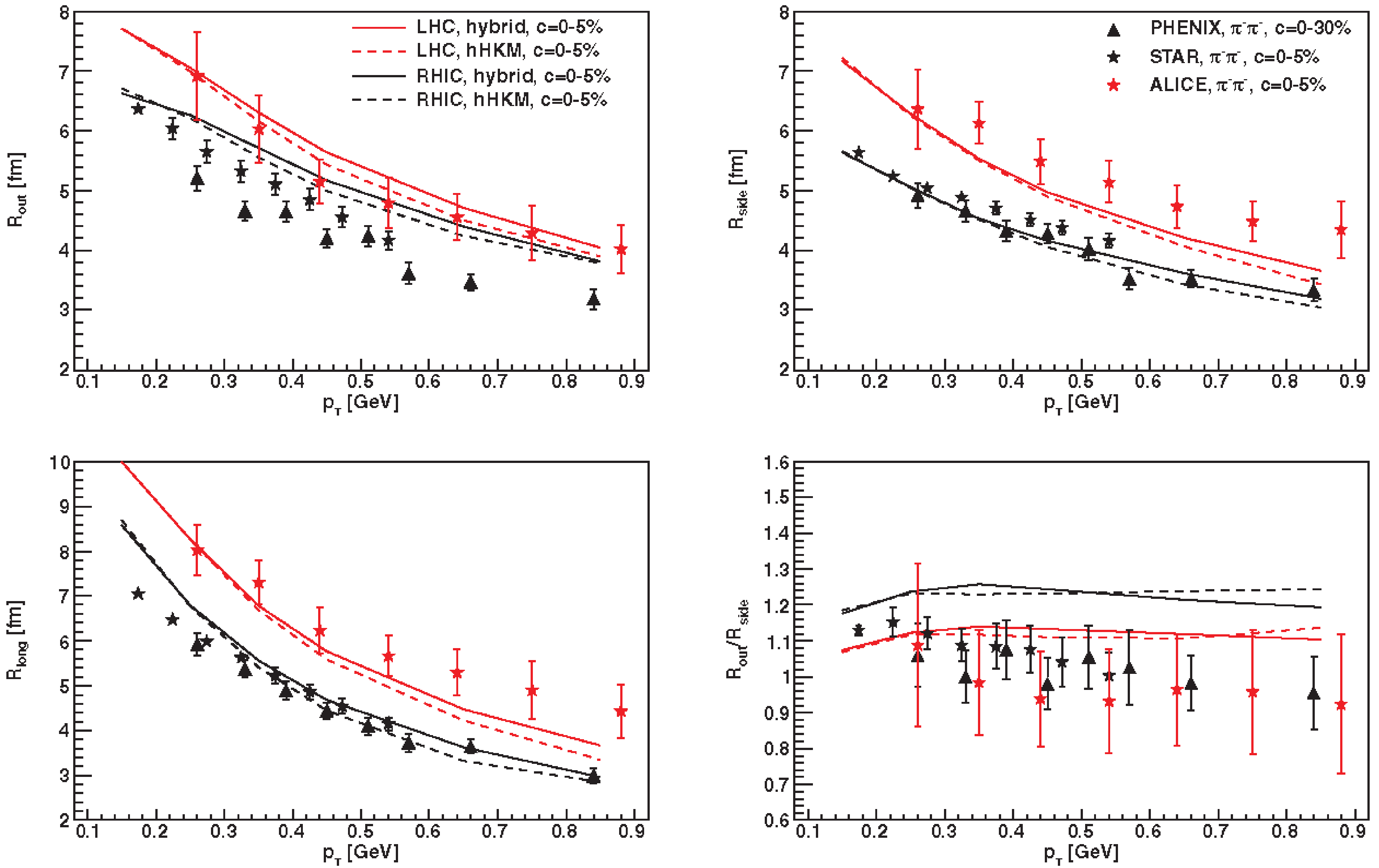
....

Phys. Rev. C 87, 024914 (2013)

[47] Y. Karpenko, Y. Sinyukov, and K. Werner, (2012), arXiv:1204.5351 [nucl-th]

[50] Y. Karpenko and Y. Sinyukov, J.Phys.G **G38**, 124059 (2011).

Interferometry radii for central RHIC and LHC events



HKM predictions for kaon femtoscopy

Freeze-out Dynamics via Charged Kaon Femtoscopy in $\sqrt{s_{NN}}$=200 GeV Central Au+Au Collisions

STAR Collaboration

arXiv:1302.3168 [nucl-ex]

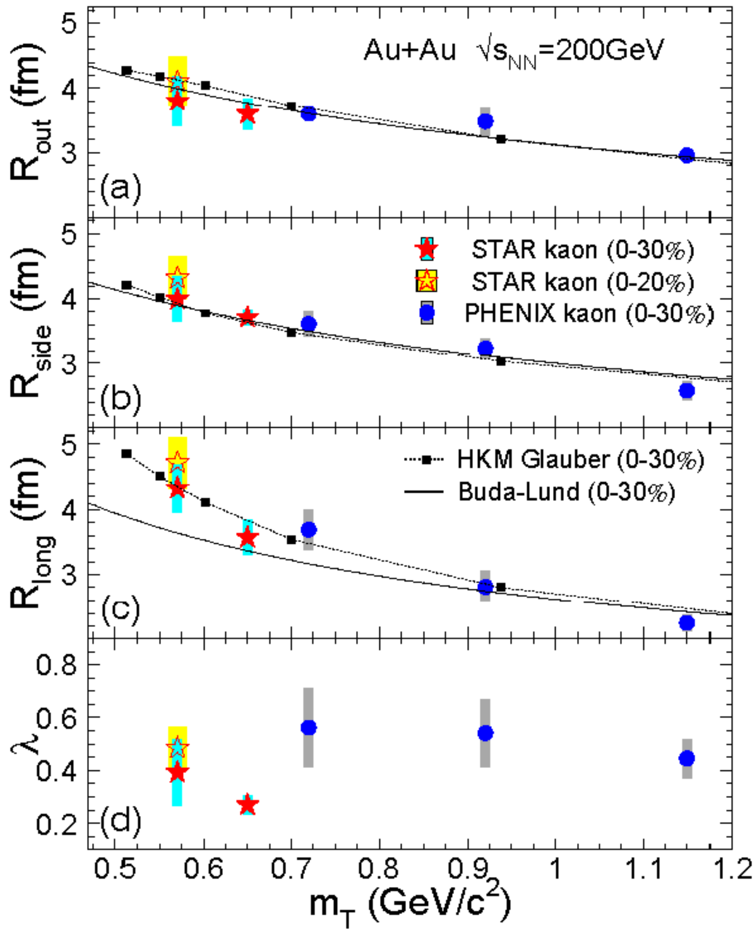


FIG. 4. Transverse mass dependence of Gaussian radii (a) R_{out}, (b) R_{side} and (c) R_{long} for mid-rapidity kaon pairs from the 30% most central Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV. STAR data are shown as solid stars; PHENIX data [10] as solid circles (error bars include both statistical and systematic uncertainties). Hydro-kinetic model [23] with initial Glauber condition and Buda-Lund model [22] calculations are shown by solid squares and solid curves, respectively.

Quotations:

Our measurement at $0.2 \leq k_T \leq 0.36$ GeV/c clearly favours the HKM model as more representative of the expansion dynamics of the fireball.

....

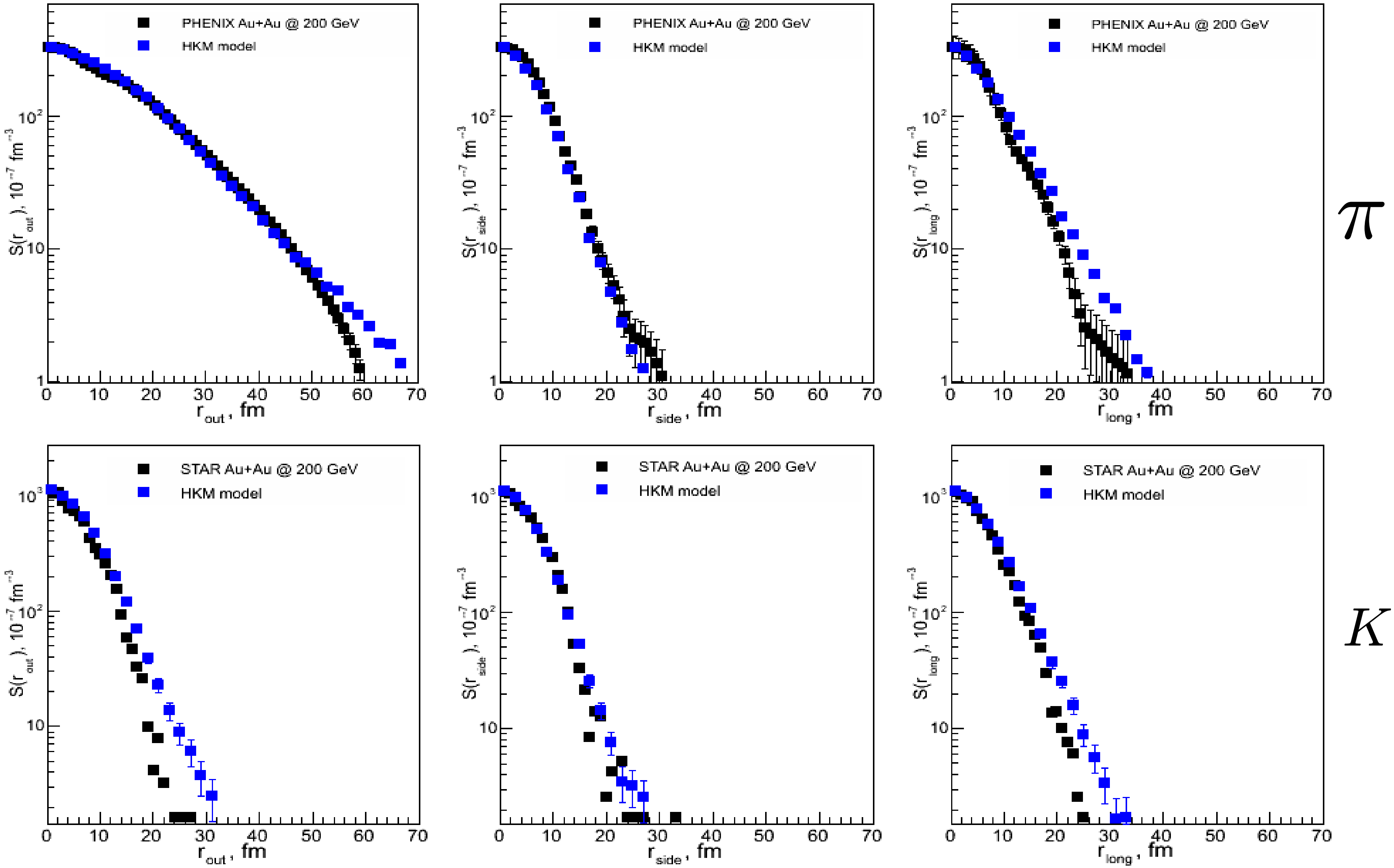
In the outward and sideward directions, this decrease is adequately described by m_T–scaling. However, in the longitudinal direction, the scaling is broken. The results are in favor of the hydro-kinetic predictions [23] over pure hydrodynamical model calculations.

....

[23] I. A. Karpenko and Y. M. Sinyukov, Phys. Rev. C **81** (2010) 054903.

Source function for pions (top) and kaons (bottom) at RHIC

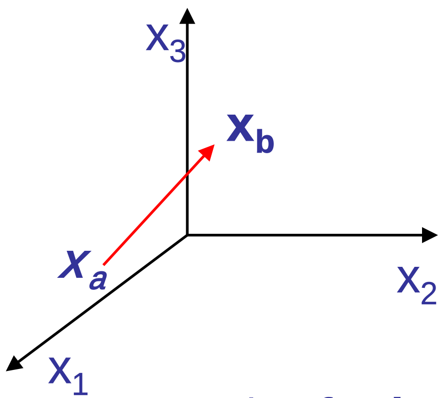
Shapoval, Yu.S arXiv:1308.6272



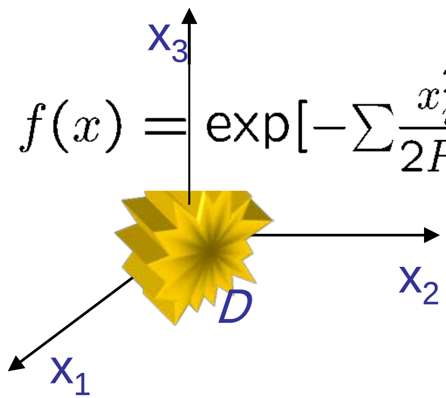
Correlation femtoscopy of small systems

Yu.S, Shapoval: PRD 87, 094024 (2013)

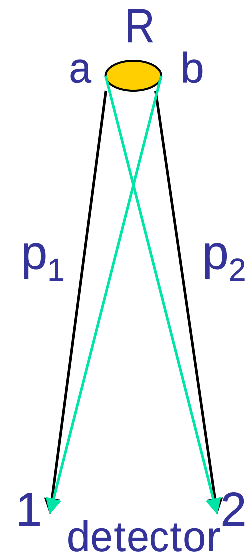
The idea of the correlation femtoscopy is based on quantum statistic for emitted identical particles.



t=0



$$f(x) = \exp[-\sum \frac{x_i^2}{2R_i^2}]$$



Momentum representation **for bosons**

$$\Psi_{x_a,x_b}(p_1,p_2) = \frac{1}{\sqrt{2}}\left[e^{-i\mathbf{p}_1\cdot\mathbf{x}_a}e^{-i\mathbf{p}_2\cdot\mathbf{x}_b} + e^{-i\mathbf{p}_2\cdot\mathbf{x}_a}e^{-i\mathbf{p}_1\cdot\mathbf{x}_b}\right]$$

Probabilities: $W_{x_a,x_b}(p_1,p_2) = \left|\Psi_{x_a,x_b}(p_1,p_2)\right|^2 = 1+\cos\left[\overbrace{(\mathbf{p}_1-\mathbf{p}_2)}^{\mathbf{q}}\cdot(\mathbf{x}_a-\mathbf{x}_b)\right]$

Model of independent emitters

$$W_D(p_1,p_2) = \int_D d^3x_1 d^3x_2 f(x_1)f(x_2)W_{x_1,x_2}(p_1,p_2) = 1 + \left|\int d^3x f(x)e^{i\mathbf{q}\mathbf{x}}\right|^2 = 1 + \exp\left[-\sum q_i^2 R_i^2\right]$$

$$f(x) = \frac{1}{2}[\delta^3(x-x_a) + \delta^3(x-x_b)] \Longrightarrow W_{\{x_a,x_b\}} = 1 + \cos^2\left(\frac{1}{2}q\Delta x\right)$$

Probabilities of one- and two- identical bosons emitted **independently** from **distinguishable/orthogonal** quantum states with points of emission $\mathbf{x_1}$ and $\mathbf{x_2}$ $(\mathbf{x_1} - \mathbf{x_2} = \Delta \mathbf{x})$

$$\rho_1\colon A_1(p) = e^{ipx_1} e^{-ip^0\tau} \text{ and } \rho_2\colon A_2(p) = e^{ipx_2} e^{-ip^0\tau}; \quad \rho_1 + \rho_2 = 1$$

$$\rho_{11} \quad : \quad A_{11}(p_1,p_2) = e^{ip_1x_1} e^{ip_2x_1}; \; \rho_{22}\colon A_{22}(p_1,p_2) = e^{ip_1x_2} e^{ip_2x_2} \times e^{-i(p_1^0+p_2^0)\tau}$$

$$\rho_{12} \quad : \quad A_{12}(p_1,p_2) = \frac{1}{\sqrt{2}} \left(e^{ip_1x_1} e^{ip_2x_2} + e^{ip_1x_2} e^{ip_2x_1} \right); \; \rho_{11} + \rho_{22} + \rho_{12} = 1$$

If $\rho_{ii} = \rho_i^2$ and $\rho_1 = \rho_2 = 1/2$ then $\rho_{11} = \rho_{22} = 1/4$ and $\rho_{12} = 1/2$. Then

HBT (Glauber, Feynman, 1965)

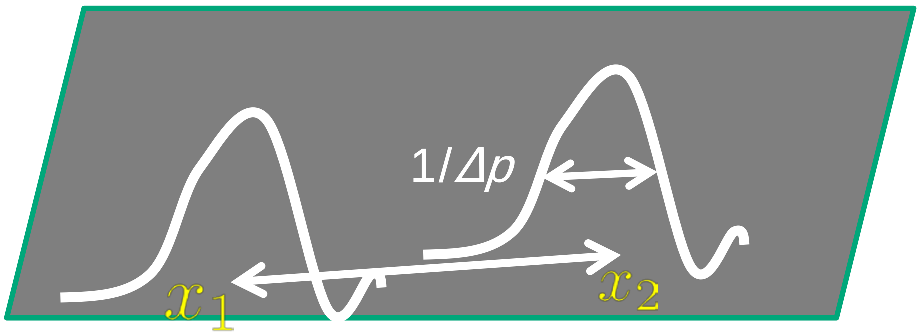
$$W_{x_1,x_2}(p_1,p_2) = \underbrace{\sum}_{i \le j = 1,2} \rho_{ij} \left| A_{ij} \right|^2 = 1 + \frac{1}{2} \cos\left(q \Delta x \right) = \underbrace{\sum}_{i,j=1,2} \rho_i \rho_j \left| A_{ij} \right|^2 = C(p_1,p_2)$$

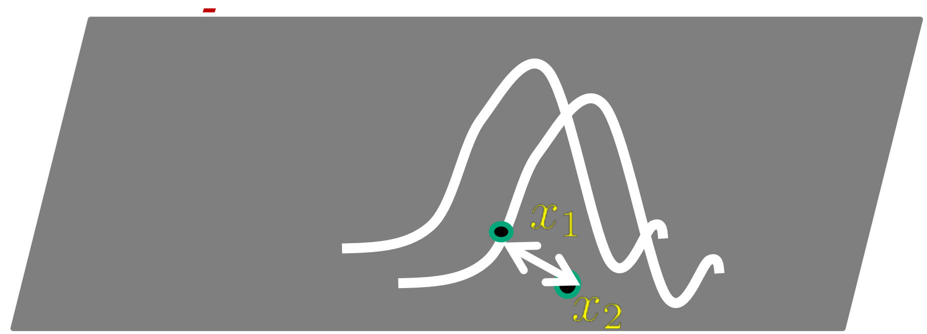
$$\neq \underbrace{\sum}_{i,j=1,2} \rho_i \rho_j (1 + \cos\left(q(x_i - x_j) \right)) = 1 + \cos^2 \left(\frac{1}{2} q \Delta x \right)$$

Double counting!

Uncertainty principle and distinguishability of emitters

The distance between the centers of emitters $\Delta x = x_1 - x_2$ is larger than their sizes related to the widths of the emitted wave packets $1/\Delta p$.





Distinguishable emitters $\Delta x \gg 1/\Delta p$

The states are orthogonal

Wave function for emitters

$$\psi_x(\mathbf{p}) = e^{i\mathbf{px}} e^{i\varphi(x)}\, \tilde{f}(\mathbf{p}) e^{-ip^0\tau}$$

Spectrum:

$$f(\vec{p}) = \tilde{f}^2(\vec{p}) = const$$

$$f(\vec{p}) = \tilde{f}^2(\vec{p}) = \frac{1}{(2\pi p_0^2)^{3/2}} e^{-\frac{\vec{p}^2}{2p_0^2}}$$

Indistinguishable emitters $\Delta x \ll 1/\Delta p$

The states are almost the same

Criterion : overlapping of the wave packages:

$$I_{\mathbf{xx'}} = \left| \int d^3\mathbf{r}\, \psi_x(\tau, \mathbf{r}) \psi^*_{x'}(\tau, \mathbf{r}) \right| /N$$

Phase correlations $\left\langle e^{i\phi(x)} e^{-i\phi(x')} \right\rangle =$

$$I_{\mathbf{xx'}} = \delta^3(\mathbf{x} - \mathbf{x'}) \qquad t = t'$$

$$I_{\mathbf{xx'}} = e^{-\frac{p_0^2(\mathbf{x}-\mathbf{x'})^2}{2}}$$

The uncertainty principle for momenta measurement.

The measurement of the particle momenta *p* has accuracy depending on the duration of the measurement Δt: $\Delta p \sim 1/\Delta t$ [Landau, Lifshitz, v. IV]. So one can measure the time of particle emission without noticeable violation of the momentum spectra with accuracy not better than $1/\Delta p$

$$\delta(t_1 - t_2) \Longrightarrow G^{t}_{t_1 t_2} \to e^{-\frac{\Delta p^2 (t_1 - t_2)^2}{2}}$$

The milestones

Femtoscopy for independent distinguishable emitters (standard model)

Femtoscopy for partially indistinguishable and so partially non-independent emitters (for small systems where the uncertainty principle is important)

$$\left\langle e^{i(\phi(x_1)-\phi(x_2))} \right\rangle = \delta^4(x_1 - x_2) \Longrightarrow G_{x_1 x_2} = I_{\mathbf{x}_1 \mathbf{x}_2} e^{-\frac{\Delta p^2 (t_1 - t_2)^2}{2}}$$

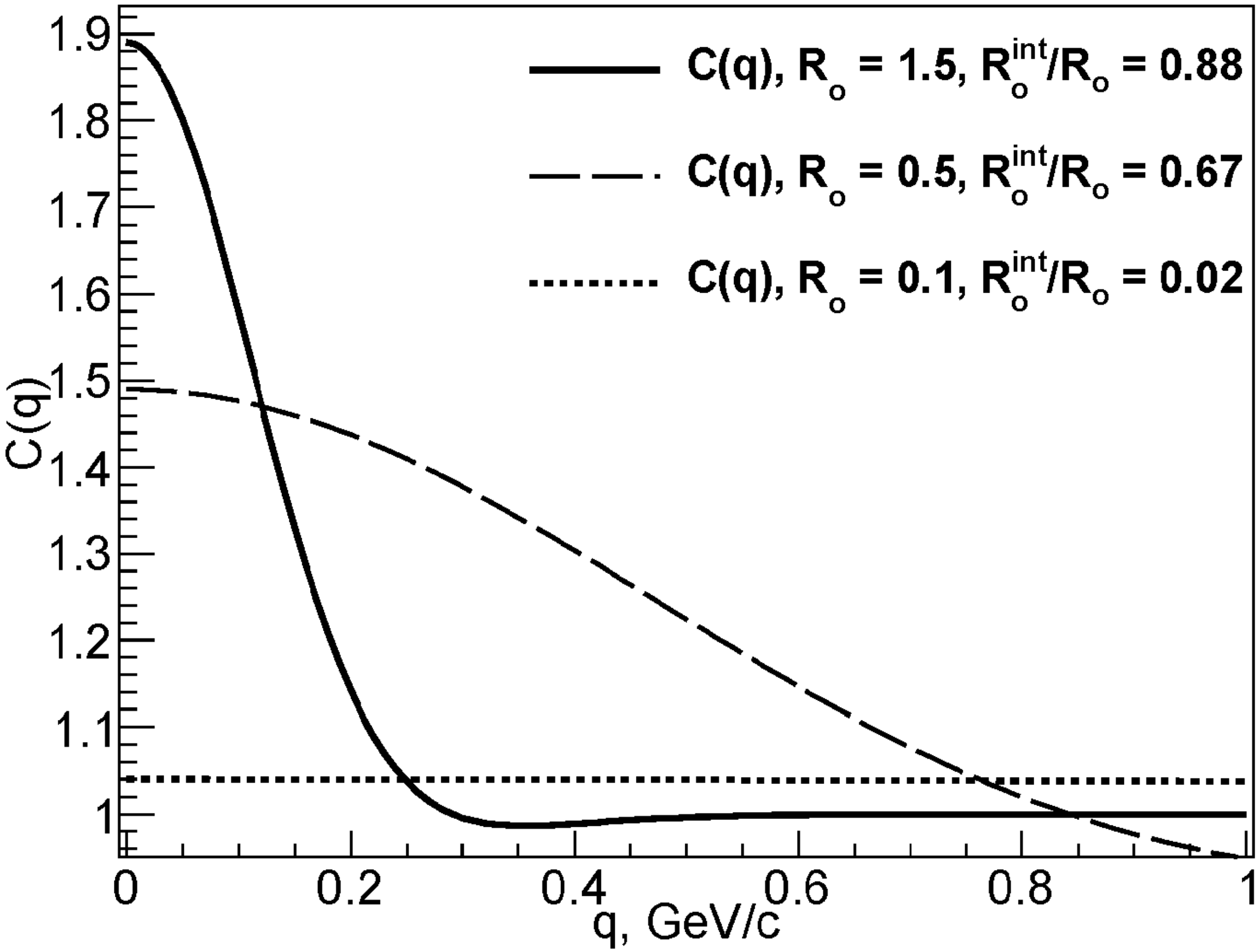
Overlap integral

The 4-points phase correlator for maximally possible chaotic and independent emitters permitted by uncertainty principle is decomposed into the sum of products of the two-point correlators G_{ij} and contains also term that eliminate the double accounting.

$$\langle e^{i(\varphi(x_1)+\varphi(x_2)-\varphi(x'_1)-\varphi(x'_2))} \rangle = G_{x_1 x'_1} G_{x_2 x'_2} + G_{x_1 x'_2} G_{x_2 x'_1} - G_{x_1 x'_2} G_{x_2 x'_1} G_{x_1 x_2}$$

Double counting

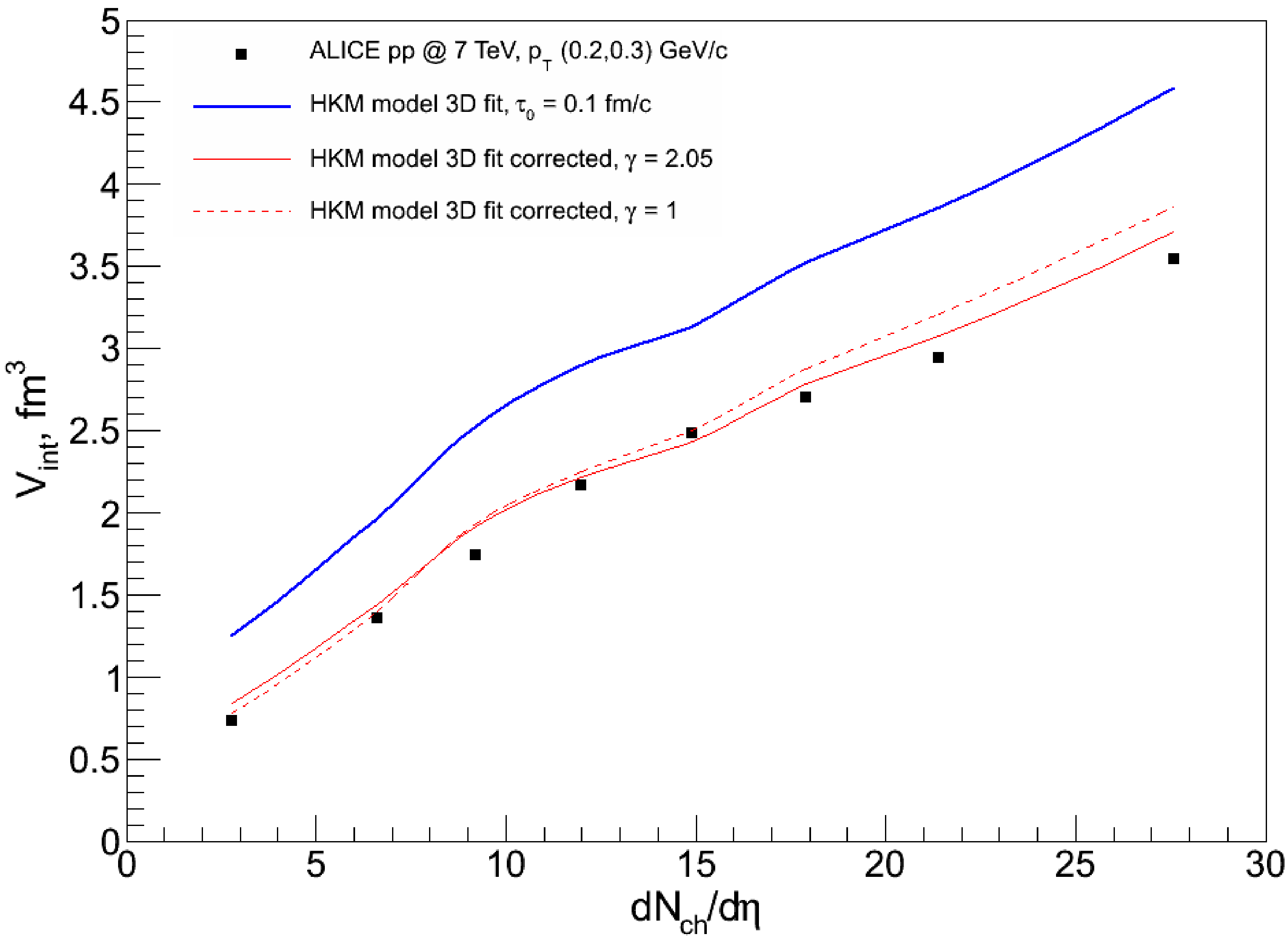
The Bose-Einstein correlation function for small systems



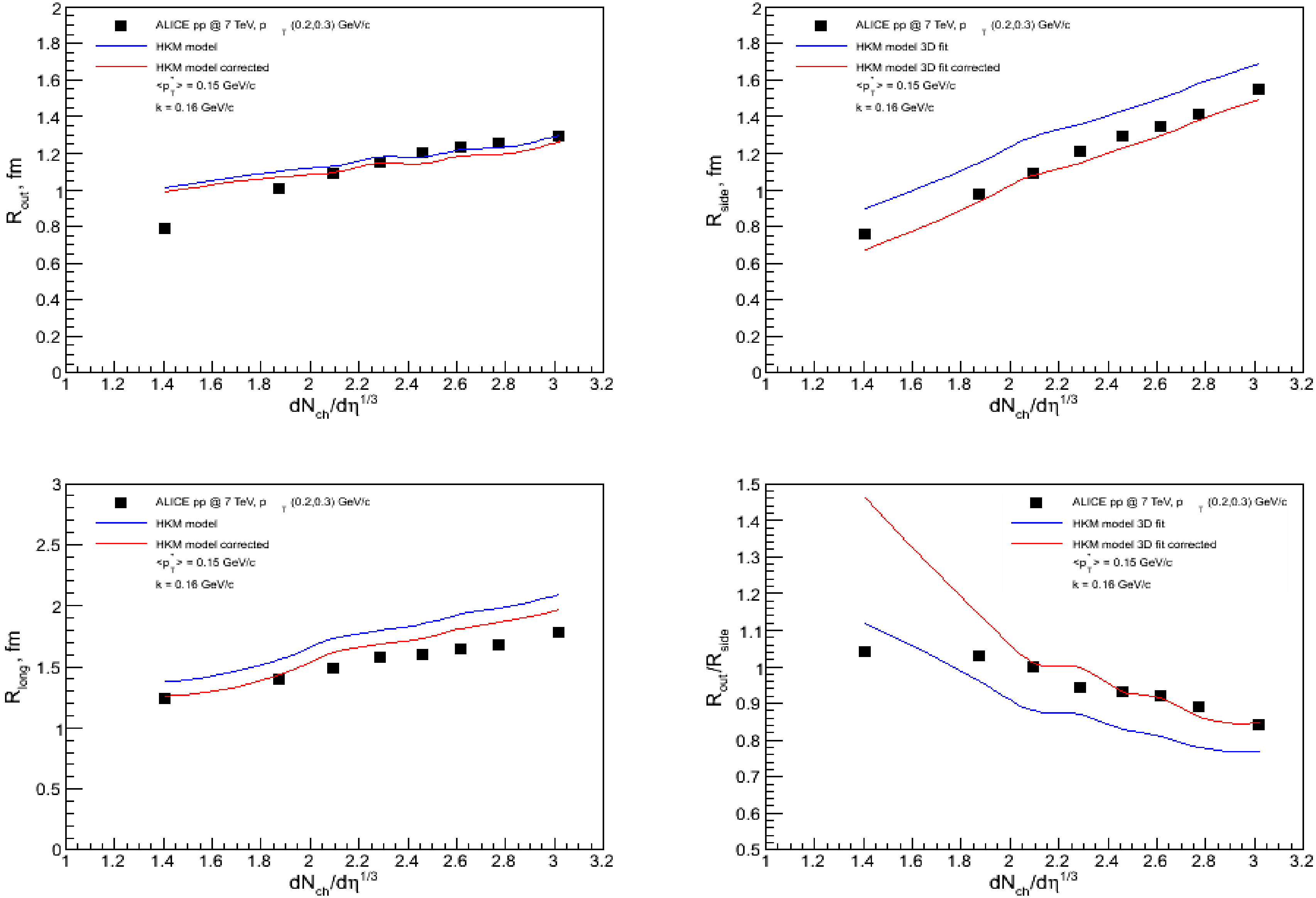
The behavior of the two-particle Bose-Einstein correlation function (*side*-projection) where the uncertainty principle and correction for double accounting are utilized. The momentum dispersion k=m=0.14 GeV, p_T=0, T=R.

p+p -interferometry volume in HKM vs multiplicity in view of the uncertainty principle

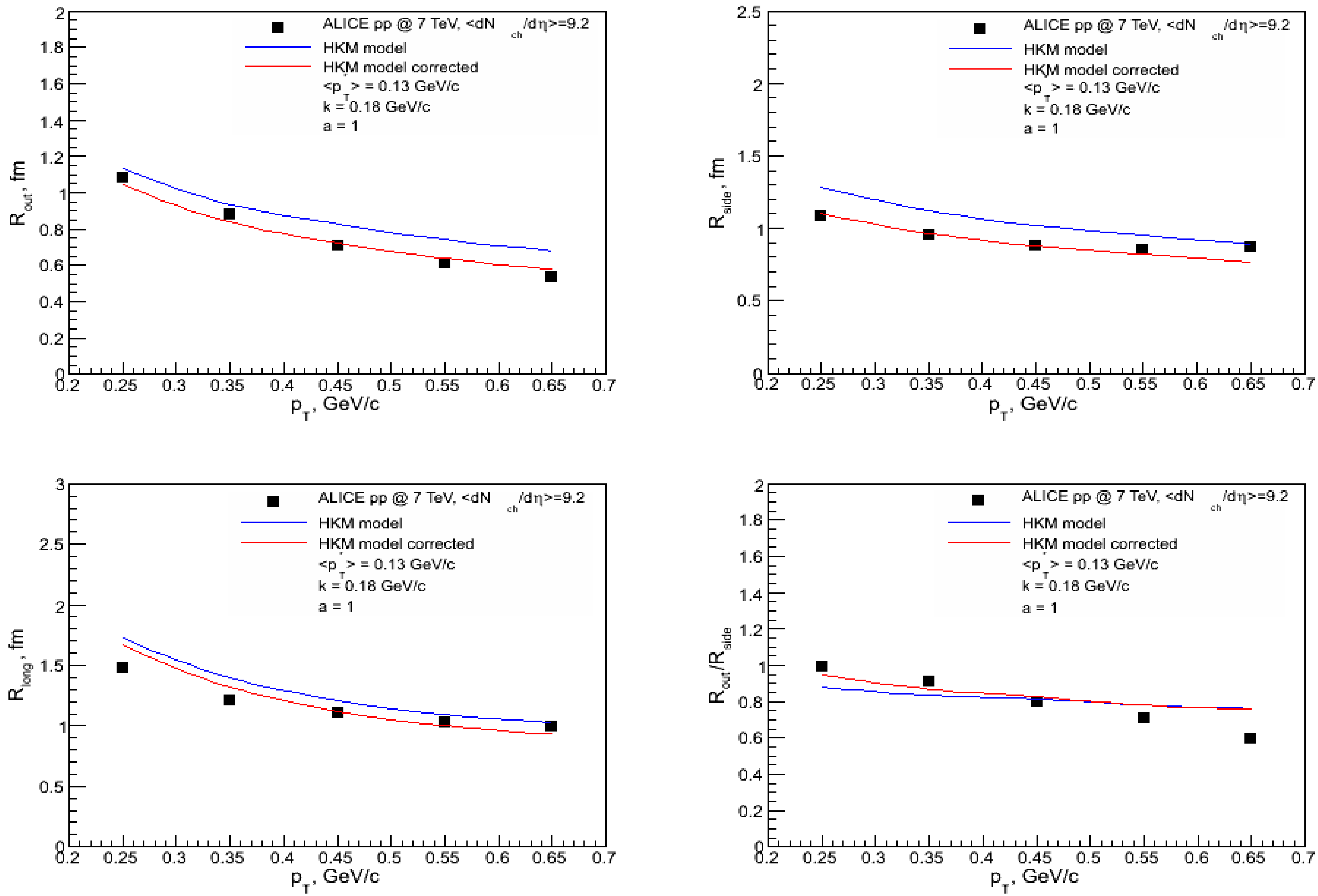
Shapoval, Braun-Munzinger, Karpenko, Yu.S. PLB 725 (2013) 139



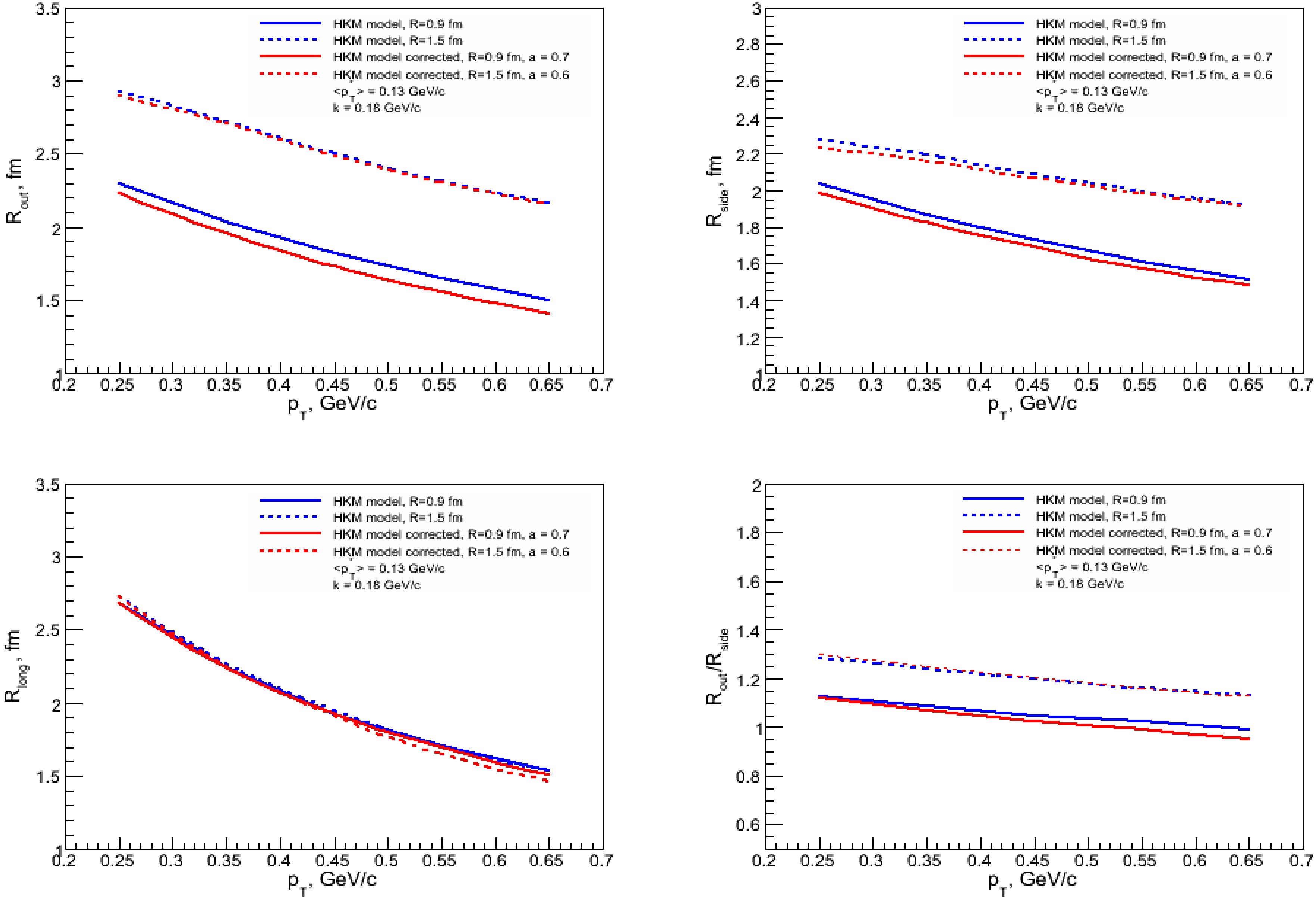
p+p -interferometry radii in HKM vs multiplicity in view of the uncertainty principle



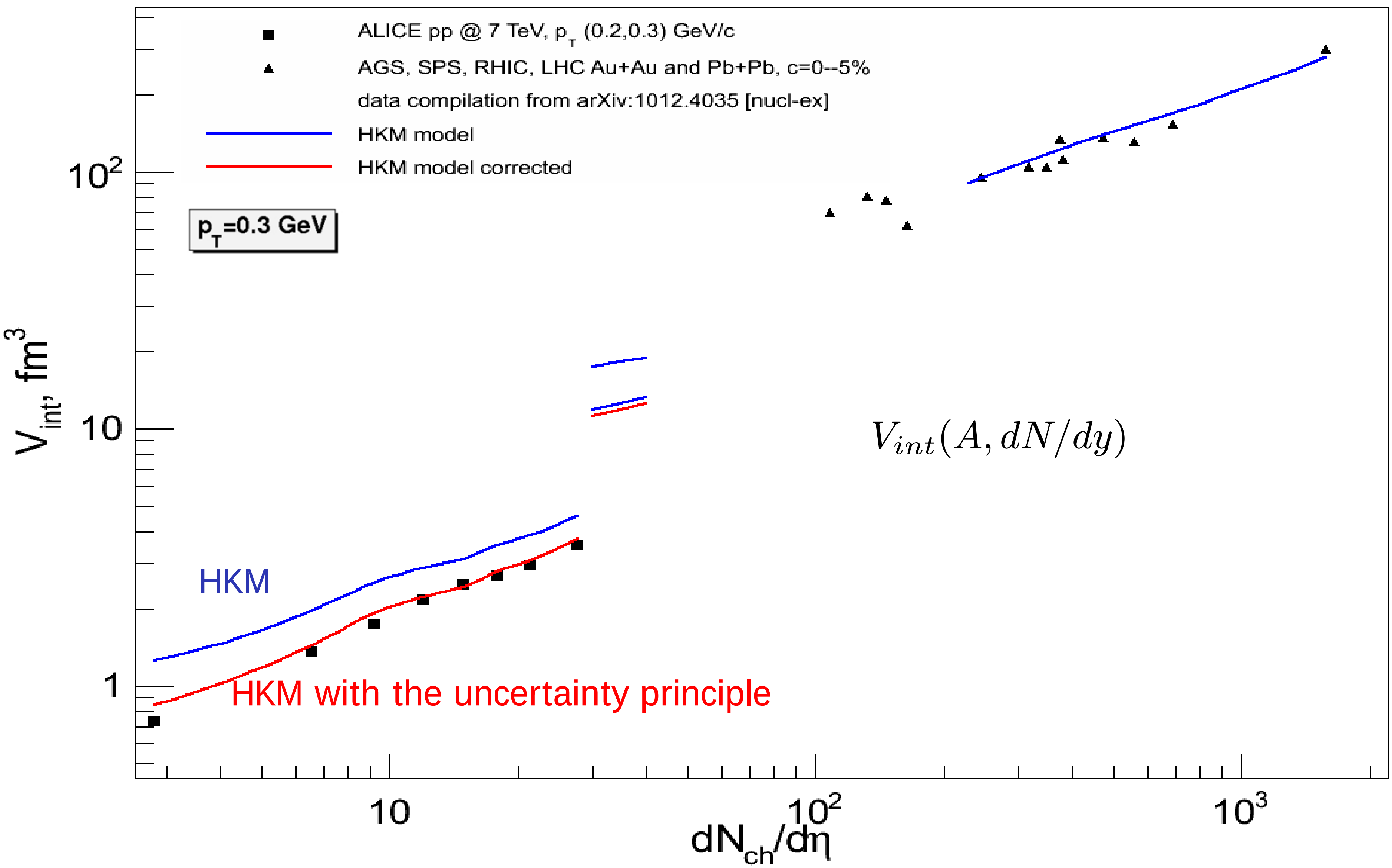
p+p -interferometry radii in HKM vs transv. momentum in view of the uncertainty principle



p+Pb -interferometry radii in HKM vs transverse momentum at $dN_{ch}/d\eta = 35$ (prediction, PLB 725 (2013) 139)



Interferometry volume in pp, pPb, AA systems in HKM in view of the uncertainty principle



Conclusion

- HKM is the useful tool to solve the problem of particlization at non-space-like parts of hypersurfaces of hadronisation/chemical freeze-out. In combination with hadronic cascade models (like UrQMD), that are entered into game on the space-like hypersurf. with non-equilibrium input from HKM, it become reliable model of A+A collisions.

- The main mechanisms that leads to the paradoxical behavior of the interferometry scales, are discovered within HKM and conformed experimentally by ALICE LHC in 2011.
- In particular, decrease of R_{out}/R_{side} ratio with growing energy and saturation of the ratio at large energies happens due to a magnification of positive $r-\tau$ correlations between space and time positions of emitted pions and a developing of pre-thermal collective transverse flows.

- Unexpected result is good description of the femtoscopy scale in HKM with account for uncertainty principle.
- Non-thermal (UrQMD) stage at the late times play an important role at LHC.

- The comparison of V_{int} vs $dN/d\eta$ for pp, pPb and AA collisions conforms probably the result of Akkelin, Yu.S. : PRC 70 . 064901 (2004); PRC 73 034908 (2006) that the interferometry volume depends not only on multiplicity but also on initial size of colliding systems.

Obrigado pela sua atenção!

Thank you for your attention!

Basic idea of hydrokinetics

Boltzmann Eq. (integral form)

$$f^{\sigma}(x,p) = f^{\sigma}_{esc}(x_0,p) + \int\limits_{t_{\sigma_0}(\mathbf{x_0})}^{t_\sigma} dt' \mathcal{P}_\sigma(x',p) G(x',p)$$

where

$$x_0 \equiv (t_{\sigma_0}(\mathbf{x}_0), \mathbf{x}_0 = \mathbf{x}_\sigma + (\mathbf{p}/p_0)(t_{\sigma_0}(\mathbf{x}_0) - t_\sigma))$$

$$x' \equiv (t', \mathbf{x}_\sigma + (\mathbf{p}/p_0)(t' - t_\sigma))$$

Method of solution (relaxation time approx. for emission function)

$$J(x,p) \approx R_{l.eq.}(x,p) + J^{decay}(x,p),$$

$$G(x,p) \approx R_{l.eq.}(x,p) f_{l.eq.}(x,p) + G^{decay}(x,p)$$

$$f_i(x,p) \approx f_{i,l.eq.}(x,p), \quad \int d^3p \otimes, \quad \sum_i \int d^3p \frac{p^\nu}{p^0} \otimes$$

$$\frac{p^\mu}{p^0}\partial_\mu f_i(x,p) = (f_i^{l.eq.}(x,p) - f_i(x,p)) R_i(x,p) + G_i^{decay}(x,p) - L_i^{decay}(x,p)$$

$$\partial_\mu (n_i(x) u^\mu(x)) = -\Gamma_i n_i + \sum_j b_{ij} \Gamma_j n_j(x) \qquad \partial_\mu T^{\mu\nu}(x) = 0$$

360 Eqs