Stochastic Variacional Approach:

Takeshi Kodama

Stochastic Variacional Approach:

Takeshi Kodama
and
Tomoi Koide

Variacional Method

VARIACIONAL METHOD

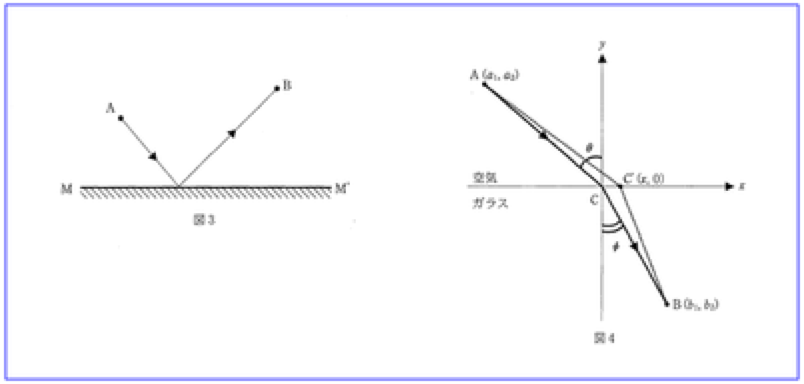
Consider Physical Process as Optimization procedure of a Scalar Quantity

Variacional Method

Consider Physical Process as Optimization procedure of a Scalar Quantity

- Useful for physical insight
- Formal development of the theory
- Approximation Methods

First steps in Physics …



Analogia com ótica com mecânica com conceito de miminizar “Ação”

Pierre-Louis Moreau de Maupertuis
Julho17,1698 –Julho 27,1759

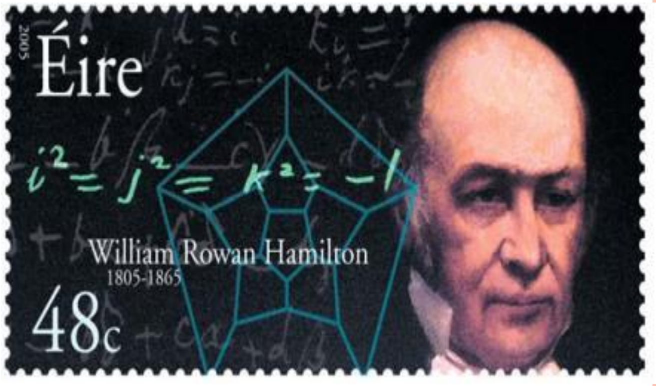
Variational Formlation of Classical Mechanics

$$I[q(t)] = \int dt\, L(q, \frac{dq}{dt}) \longrightarrow \delta I[q(t)] = 0, \quad \forall \delta q(t)$$

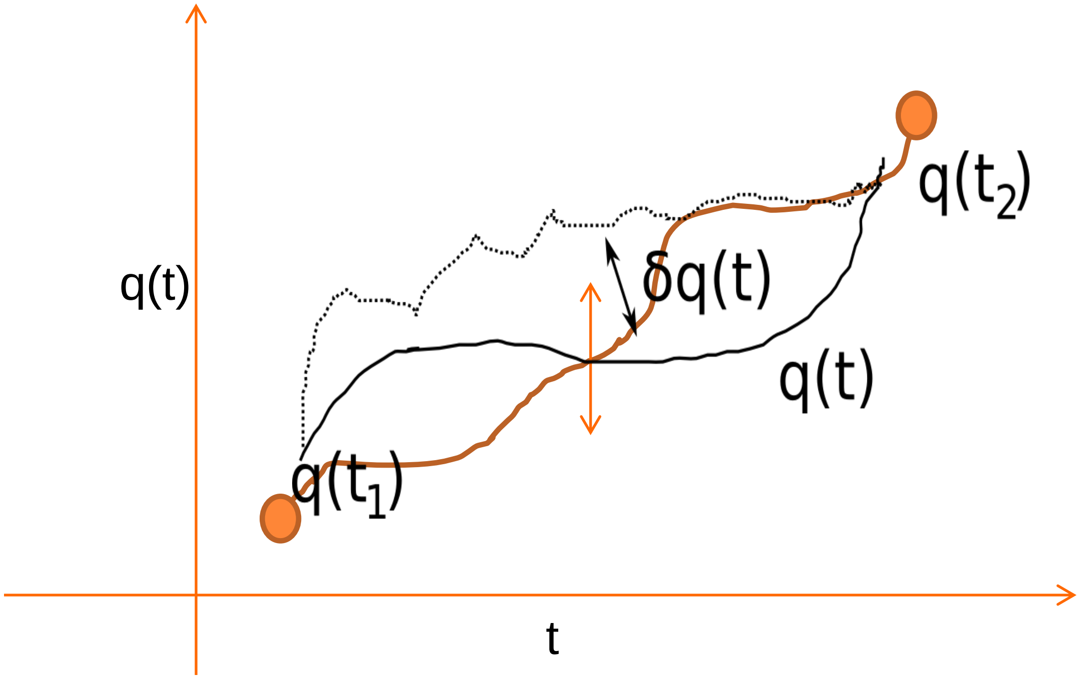
ou

$$I[q(t), p(t)] = \int dt \left\{ p \frac{dq}{dt} - H(q, p) \right\}$$

$$\longrightarrow \delta I[q(t), p(t)] = 0, \quad \forall \delta q(t), \delta p(t)$$

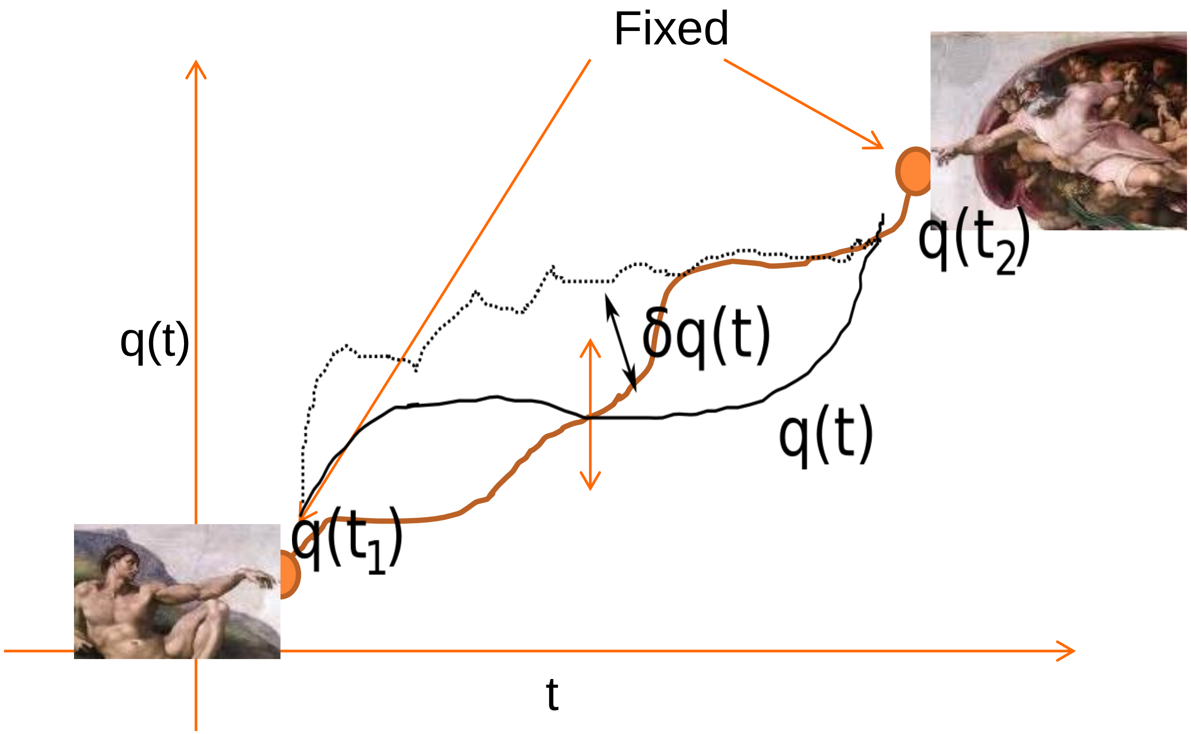


$$\delta I[q(t)] = 0, \quad \forall \delta q(t)$$



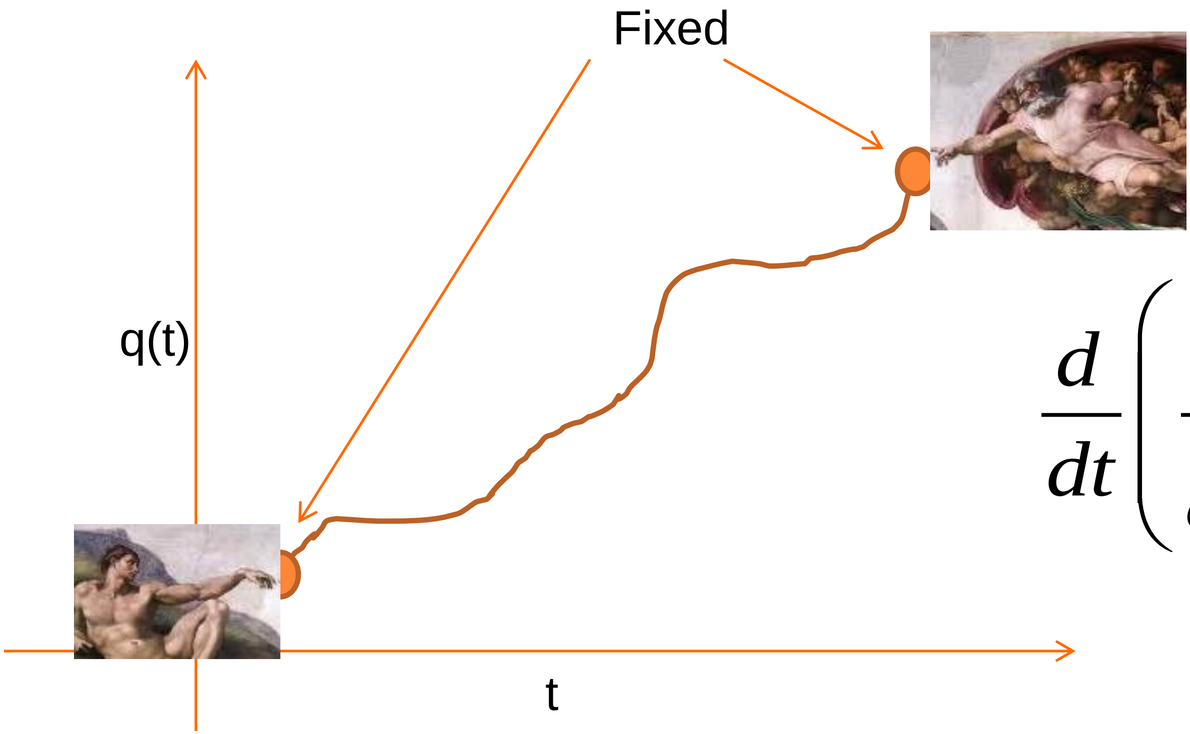
IMPORTANT !

$$\delta I[q(t)] = 0, \quad \forall \delta q(t)$$



IMPORTANT !

$$\delta I[q(t)] = 0, \quad \forall \delta q(t)$$



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

Although the variational approach assumes the future information is fixed but the resultant equation reduces to the problem of initial condition !!

Another Aspects of Variational Approach

Symmetry and Conservation Laws

Amalie Emmy Noether

March, 23, 1882 - abril, 14, 1935

Another Aspects of Variational Approach

Symmetry and Conservation Laws

Action is scalar !

Amalie Emmy Noether

March, 23, 1882 - abril, 14, 1935

Another Aspects of Variational Approach

Symmetry and Conservation Laws

In Quantum Mechanics, this role of Variational Approach is replaced by the representation of operators in Hilbert space of physical states.

$$I_{QM}\left[\psi\right] = \int dt \left\langle \psi\left(t\right) \middle| i\hbar\partial_t - H \middle| \psi\left(t\right) \right\rangle$$

Once the variational approach is established for a problem.....

$$I_{True} = I_{True}\left[\left\{\vec{q}\left(t\right)\right\}\right],$$

$$\delta I_{True} = 0,$$

Use as Approximation method

$$\vec{q}\left(t\right) \cong \sum_{i=1}^{N} C_i\left(t\right)\vec{n}_{\ddot{i}}$$

$$I_{True} \Rightarrow I_{App}\left[\left\{C_i\left(t\right), i = 1,.., N\right\}\right],$$

$$\delta I_{App} = 0,$$

$$for \quad \left\{C_i\left(t\right), i = 1,.., N\right\}$$

Once the variational approach is established for a problem…..

$$I_{True} = I_{True}\left[\left\{\vec{q}(t)\right\}\right],$$

$$\delta I_{True} = 0,$$

Use as Approximation method or Model Construction

$$\vec{q}(t) \cong \sum_{i=1}^{N} C_i(t)\vec{n}_{\ddot{\imath}}$$

$$I_{True} \Rightarrow I_{App}\left[\left\{C_i(t), i = 1,.., N\right\}\right],$$

$$\delta I_{App} = 0,$$

$$for \quad \left\{C_i(t), i = 1,.., N\right\}$$

$$\vec{q}(t) \Rightarrow \xi_M$$

$$I_{True} \Rightarrow I_{Model}\left[\xi_M\right],$$

$$\delta I_{Model} = 0,$$

$$for \quad \xi_M$$

Examples in QM

$$I_{QM}[\psi] = \int dt \left\langle \psi(t) \middle| i\hbar\partial_t - H \middle| \psi(t) \right\rangle$$

- Hartree-Fock Approx.
- QMD Model
- …

Examples in Hydro

$$I_{Hydro}[n,\vec{v},\lambda] = \int d^4x \left[-\varepsilon(n) + \lambda(\partial_\mu j^\mu) \right]$$

- Fluid dynamics as Coarse Grained Effective theory (Hirano san's talk, P. Mota et al, see also P Mota, et al, The European Physical Journal A 48 (11), 1-12 For hydrodynamical description to be valid, we need only a- local isotropy, b- strong correlation between the energy density and density)
- …

As we know…

Variational Principle in Classical Mechanics Can be understood as Stationary Path in Feynman’s Path Integral Representation of Quantum Mechanis.

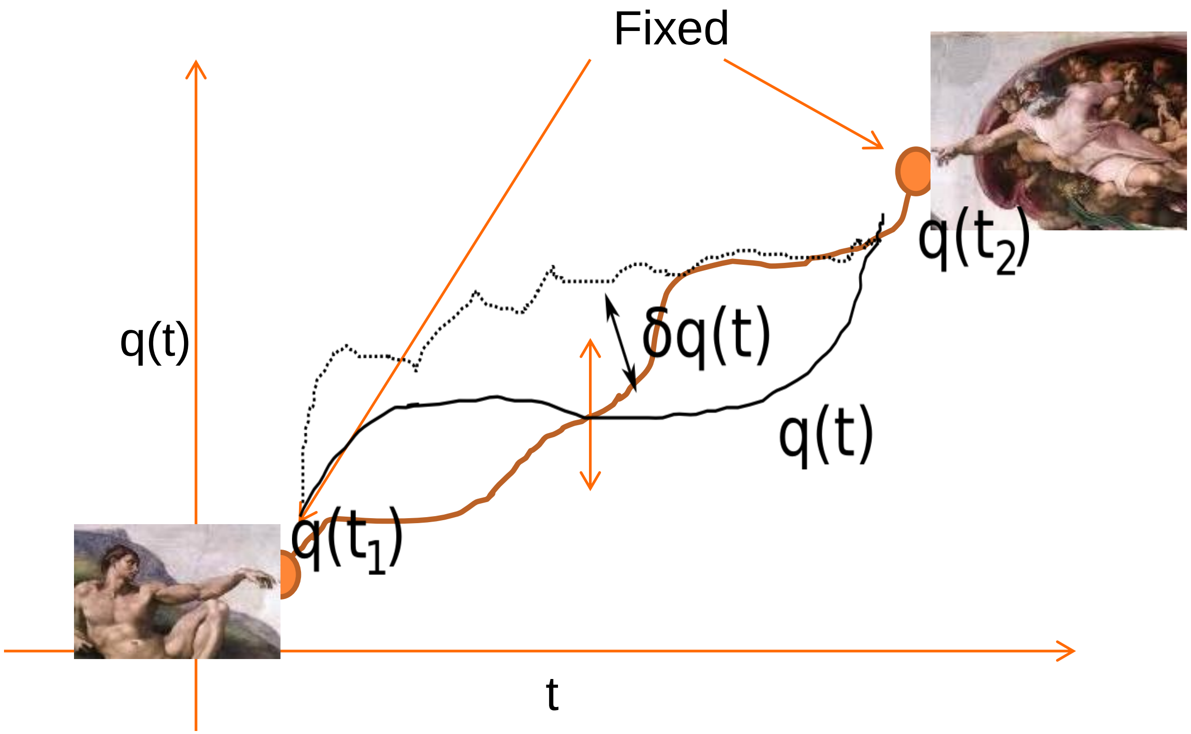
As we don’t know…

Inversely, Quantum Mechanics can be derived from the Classical Mechanics in terms of Variational Principle...?

As we don’t know…

Inversely, Quantum Mechanics can be derived from the Classical Mechanics in terms of Variational Principle...?

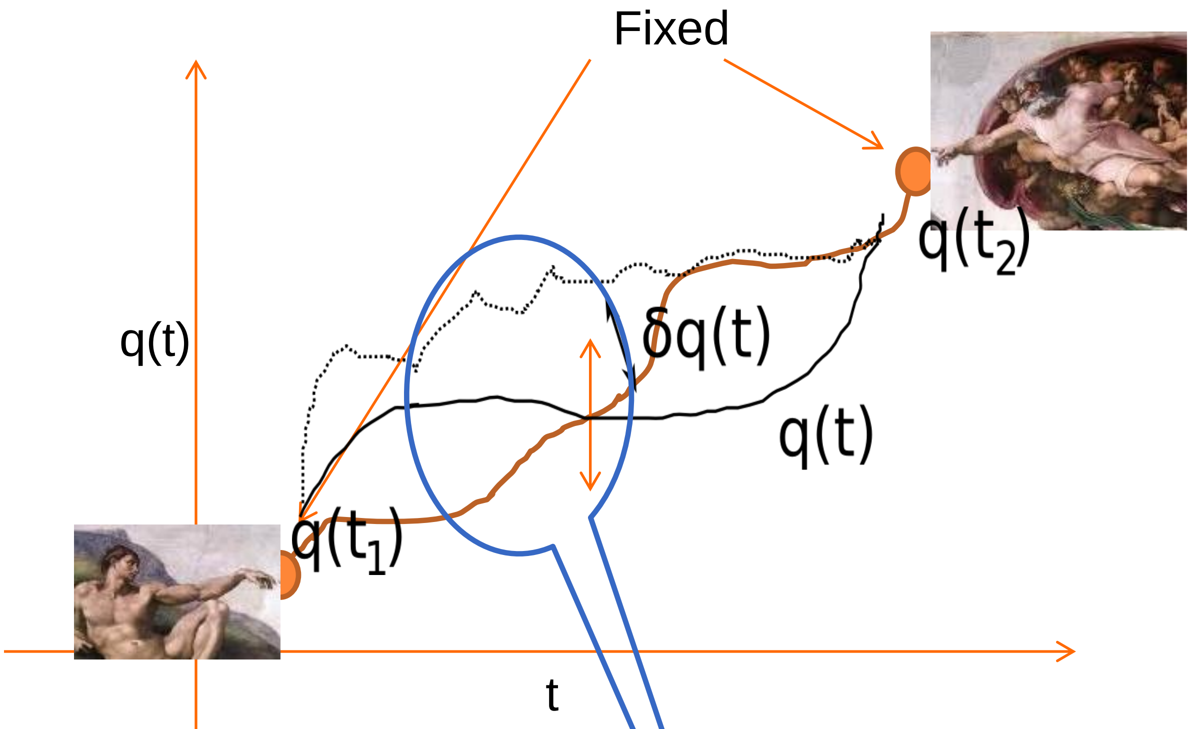
How to Do?



Instead of choosing one optimal path,

$$\delta I[q(t)] = 0, \quad \forall \delta q(t)$$

How to Do?



Instead of choosing one optimal path,

$$\delta I[q(t)] = 0, \quad \forall \delta q(t)$$

Determine the optimal distribution of paths under the influence of random noise….

NOISE??? WHAT??

Essential Difference from the ideal case

ESSENTIAL DIFFERENCE FROM THE IDEAL CASE IS THE PRESENCE OF A NOISE

How to deal with the presence of noise?

- Again, Hirano san’s talk
- Also Giorgio’s talk

STOCHASTIC PROCESS

The effect of microscopic degrees of freedom can be treated as noise, with Stochastic Differential Equation (SDE)

Classical case

$$\frac{d}{dt}X(t) = V(t)$$

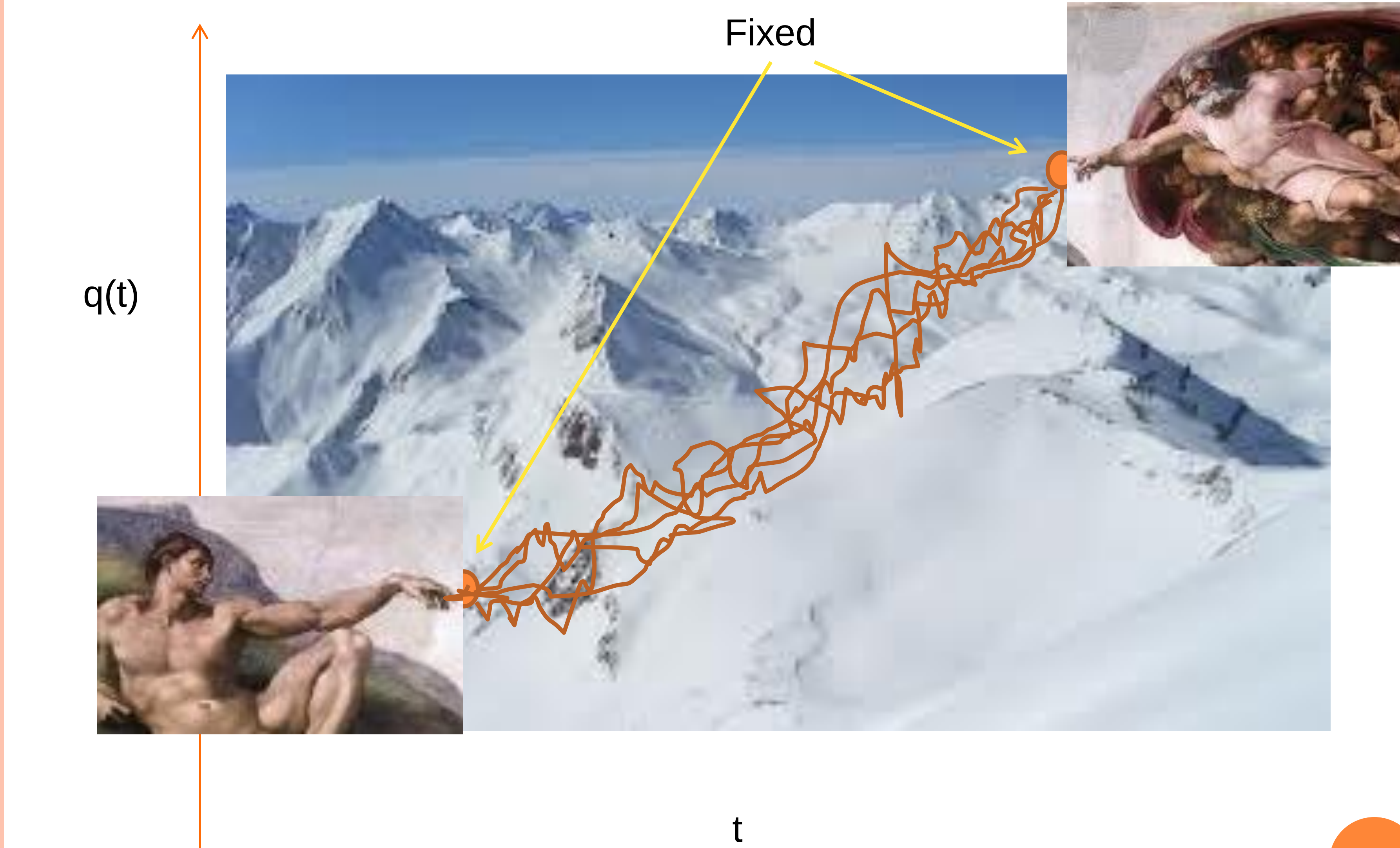
Stochastic

$$\frac{d}{dt}X(t) = V(t) + \xi(t)$$

$$\frac{d}{dt}X(t)=V(t)$$

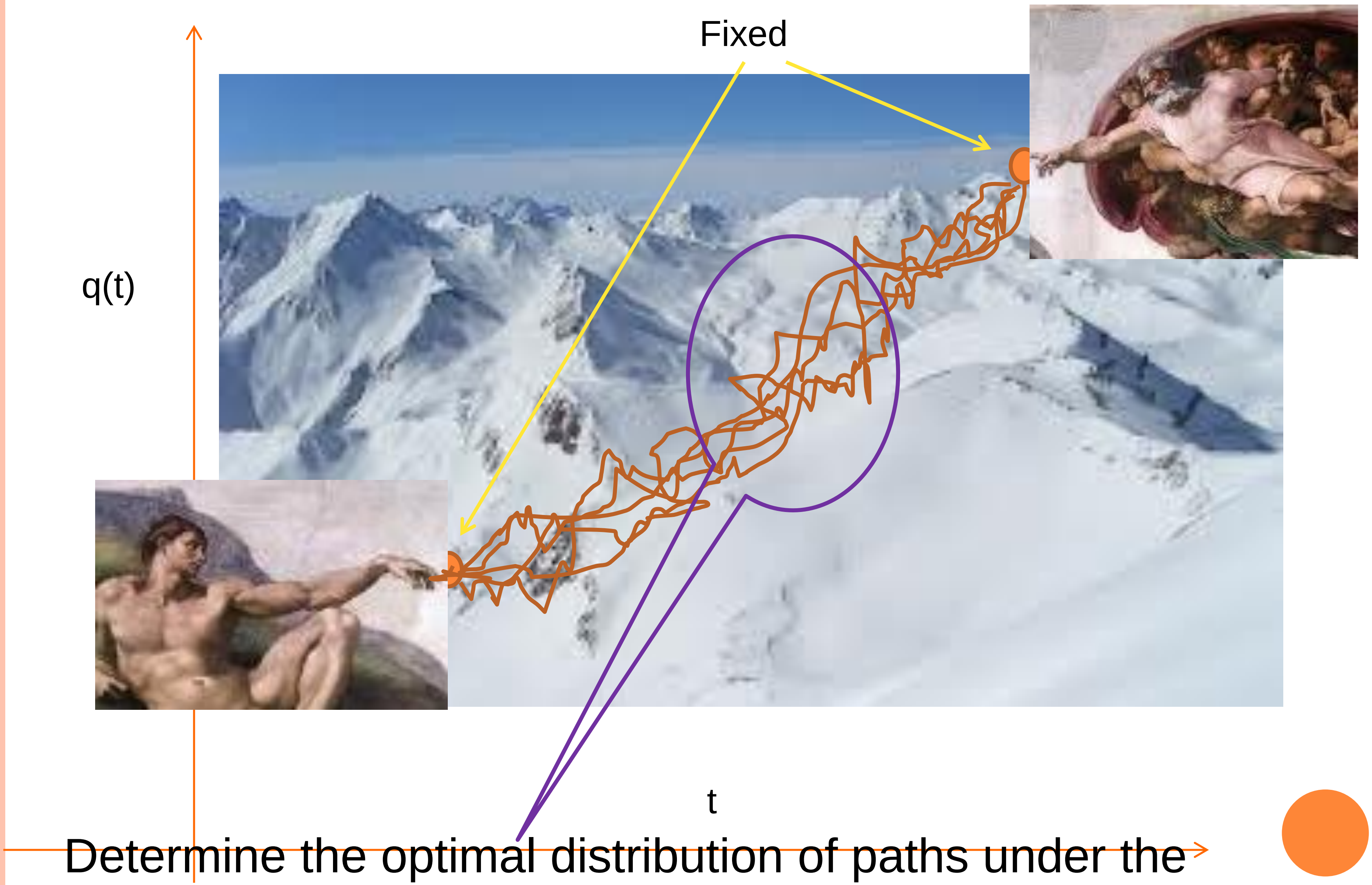
$$\frac{d}{dt}X(t)=V(t)+\xi(t)$$

Variational Approach with Noises?



Determine the optimal distribution of paths under the influence of random noise….

VARIATIONAL APPROACH WITH NOISES?



Determine the optimal distribution of paths under the influence of random noise….

For a given stochastic motion under the influence of noise, we can write Fokker-Plank equation,

For the Probability Density as

$$\rho(\vec{x},t) = \left\langle \delta(\vec{x} - \vec{x}(t)) \right\rangle$$

Average over all events.

One event given by a SDE

$$\partial_t \rho = -\nabla \left(\vec{u} - \nu \nabla \right) \rho$$

once the velocity field $\vec{u}$ is known.

Variational Principle for Stochastic trajectories…

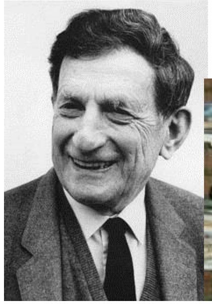
Define the Action for Stochastic Variables.

$$I = \left\langle \int_a^b dt L(X, DX) \right\rangle$$

We are talking necessarily about the distribution of trajectories and not a particular trajectory…

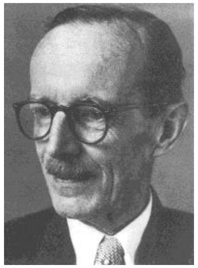
Yasue, J. Funct. Anal, 41, 327 (‘81), Guerra&Morato, Phys. Rev. D27, 1774 (‘83), Nelson, “Quantum Fluctuations” (‘85).

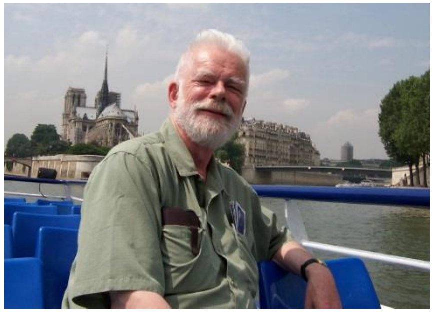
Genealogy of non conventional formulation of Quantum Mechanics



Bohm-Vigier
Hidden variables

de Broglie



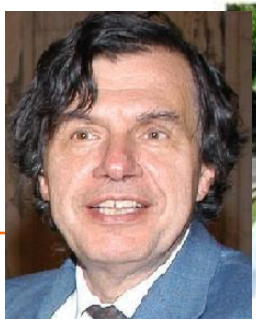


Edward Nelson
- Stochastic Method

A. Eddington

E. Madelung

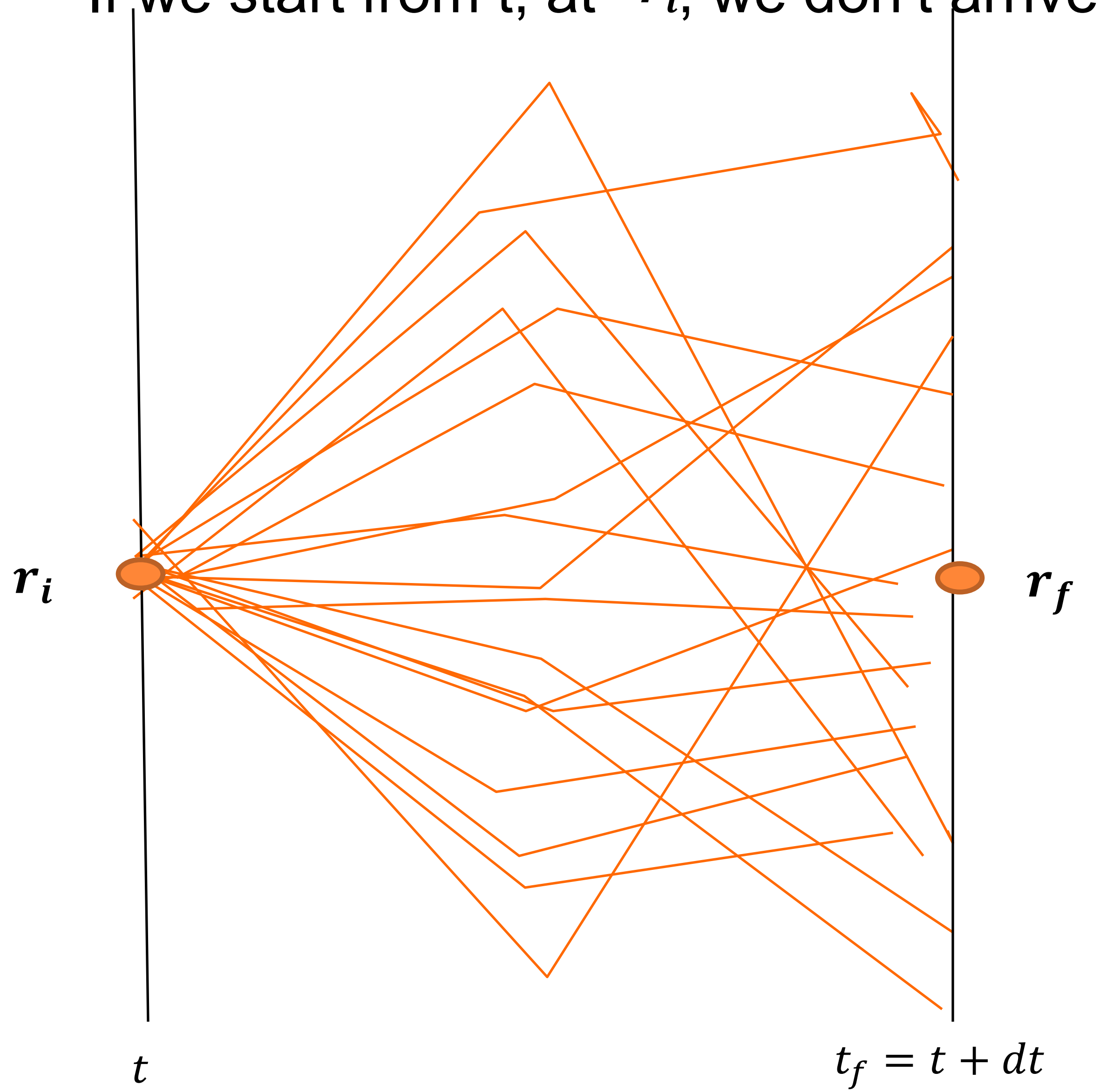
Kunio Yasue
Stochastic Variational Method



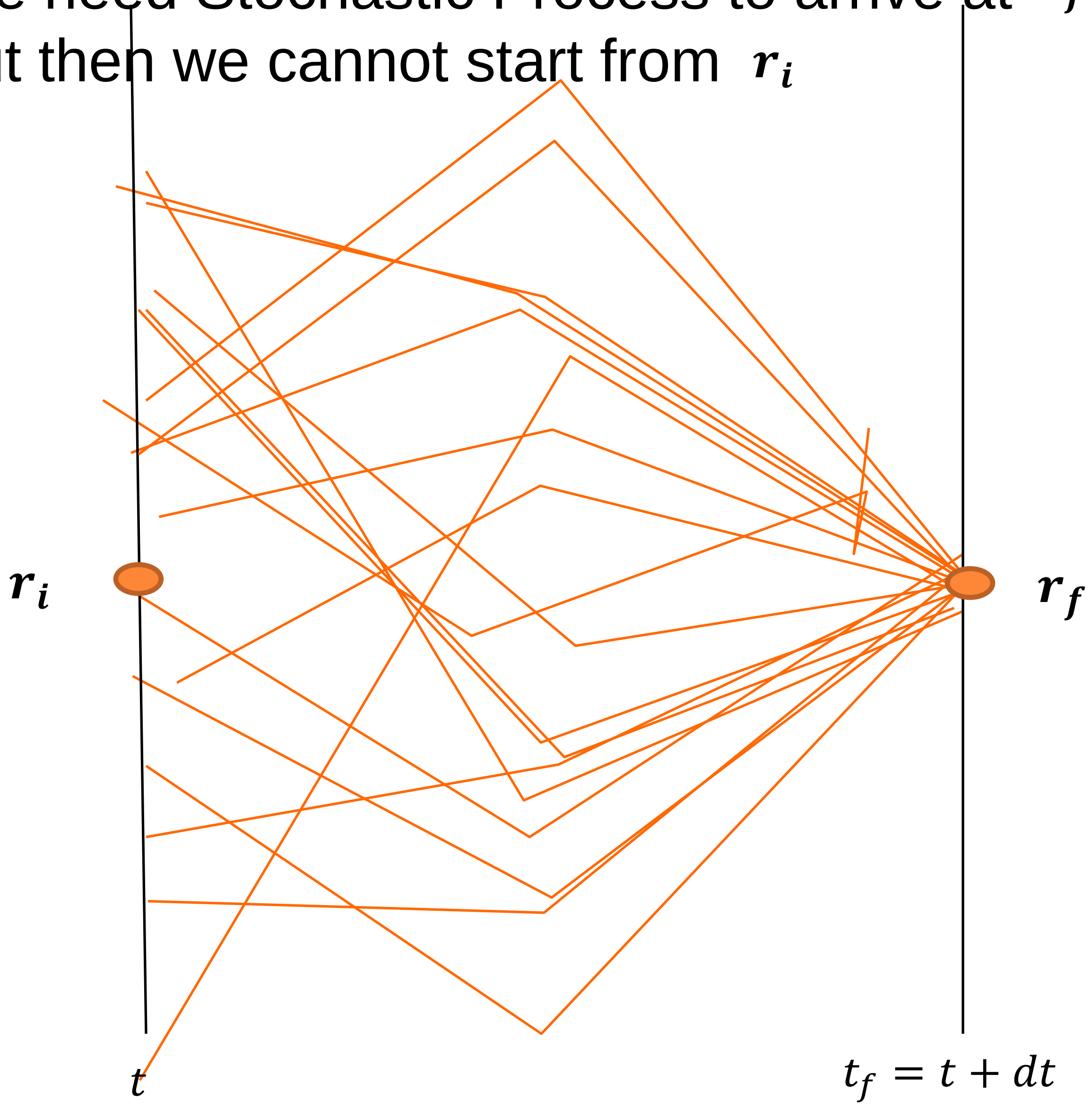
Parisi-Wu
Stochastic Quantization
5^{th} Dim (time)

Stochastic Process
vs.
Boundary Condition

If we start from t, at $\boldsymbol{r_i}$, we don't arrive at $\boldsymbol{r_f}$



We need Stochastic Process to arrive at $\boldsymbol{r_f}$
but then we cannot start from $\boldsymbol{r_i}$



Forward SDE

$$d\vec{r} = \vec{u}(\vec{r}(\vec{R},t),t)dt + \sqrt{2\nu}\cdot d\vec{W}(t) \qquad (dt > 0)$$

with white noise

$$\left\langle d\vec{W}(t) \right\rangle = 0 \qquad \left\langle dW^i(t)dW^j(t) \right\rangle = \delta^{ij} dt$$

Backward SDE

$$d\vec{r} = \vec{\tilde{u}}(\vec{r}(\vec{R},t),t)dt + \sqrt{2\nu}\cdot d\vec{\tilde{W}}(t) \qquad (dt < 0)$$

$$\left\langle d\vec{\tilde{W}}(t) \right\rangle = 0 \qquad \left\langle d\tilde{W}^i(t)d\tilde{W}^j(t) \right\rangle = \delta^{ij} \, |\, dt \,|$$

The Two SDE's should descrive the *same stochastic ensemble*

Forward Fokker-Planck

$$\partial_t \rho = -\nabla\left(\vec{u} - \nu\nabla\right)\rho$$

Backward Fokker-Planck

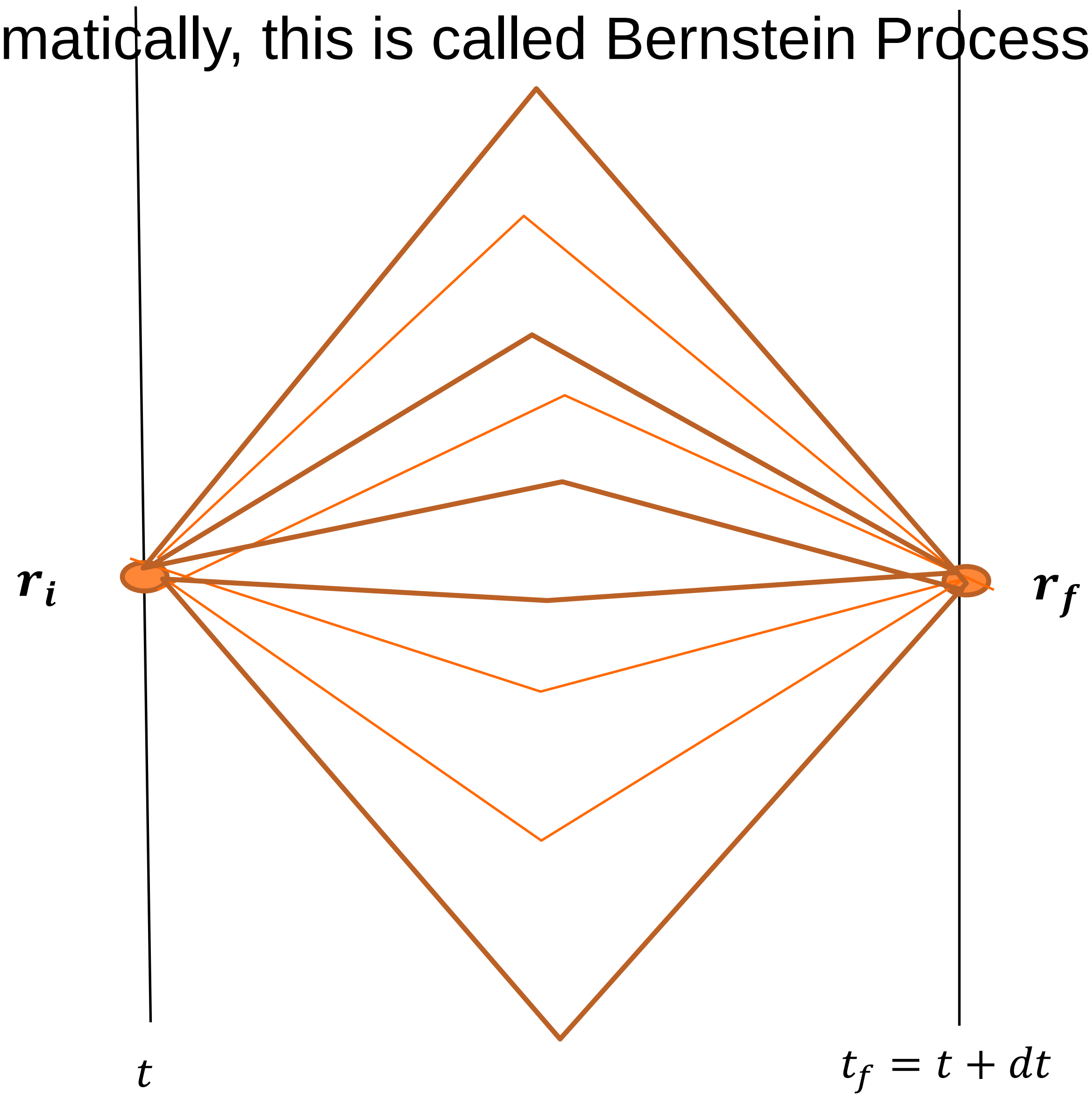
$$\partial_t \rho = -\nabla\left(\vec{\tilde{u}} + \nu\nabla\right)\rho$$

The two should be the same …

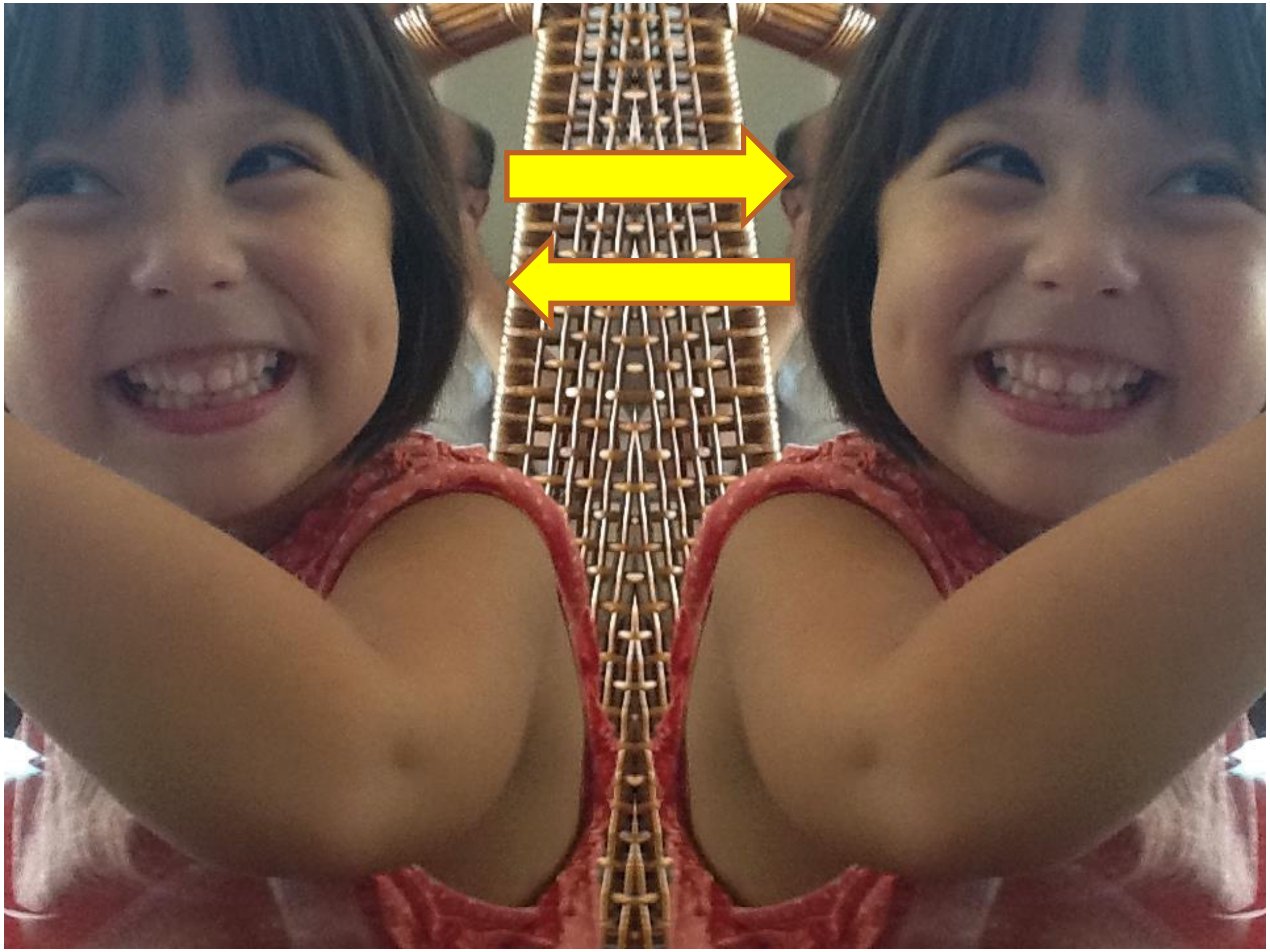
Consistency condition…

$$\vec{\tilde{u}} = \vec{u} + 2\nu\nabla \ln \rho$$

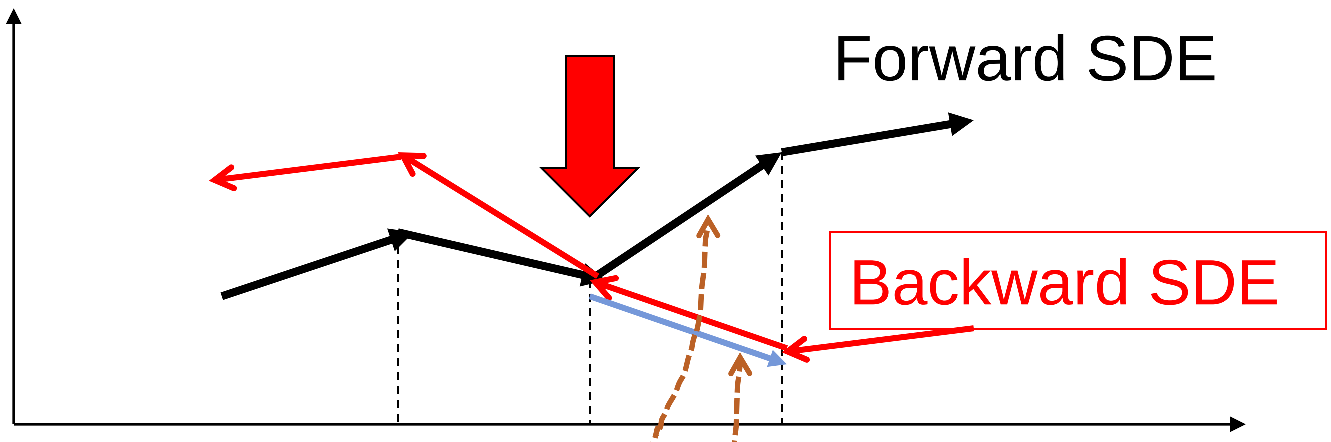
Mathematically, this is called Bernstein Process,



Or “reciprocal stochastic process”



How to define velocity for Stochastic Variables ?



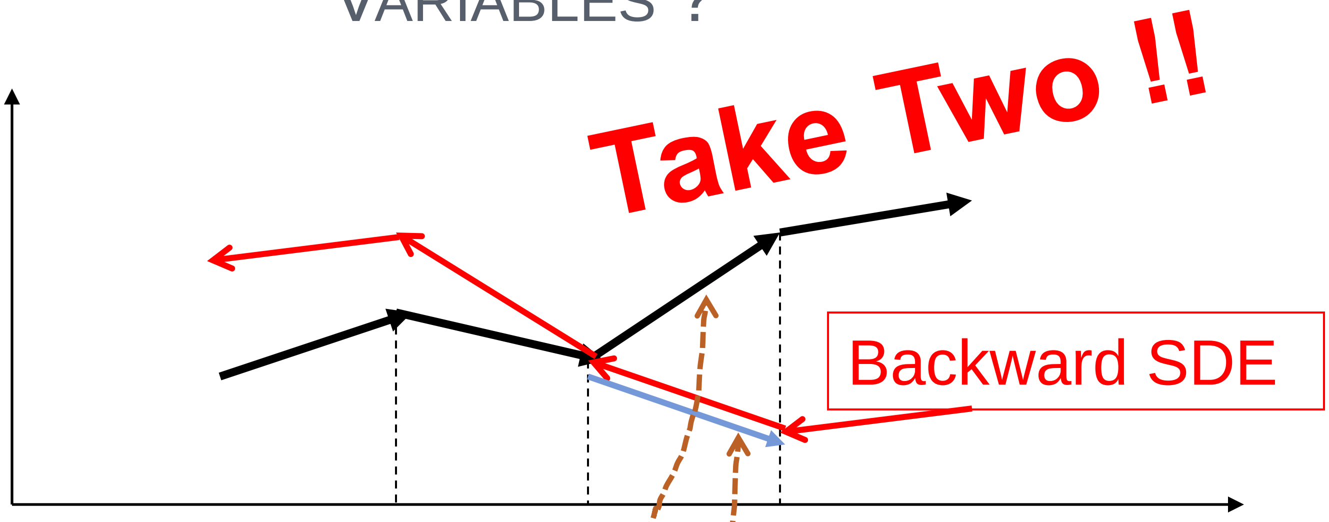
$$\vec{v} = \lim_{dt \to 0+} \frac{\vec{r}(\vec{R}, t+dt) - \vec{r}(\vec{R}, t)}{dt}$$

Forward SDE

$$\vec{\tilde{v}} = \lim_{dt \to 0+} \frac{\vec{r}(\vec{R}, t) - \vec{r}(\vec{R}, t-dt)}{dt}$$

Backward SDE

HOW TO DEFINE VELOCITY FOR STOCHASTIC VARIABLES ?



$$\vec{v} = \lim_{dt \to 0+} \frac{\vec{r}(\vec{R}, t+dt) - \vec{r}(\vec{R}, t)}{dt}$$

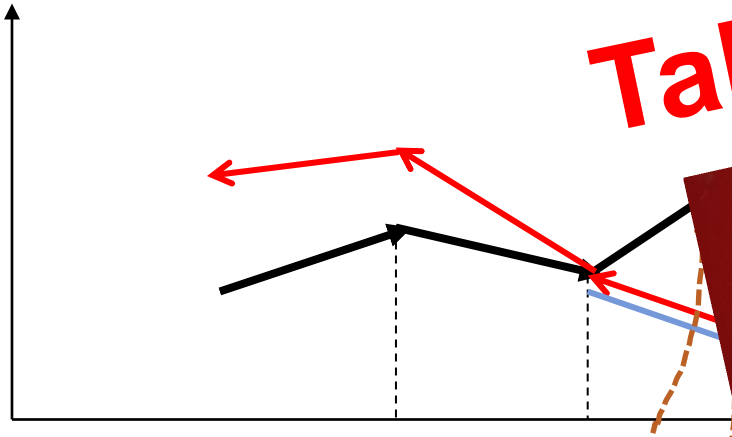
Forward SDE

$$\tilde{\vec{v}} = \lim_{dt \to 0+} \frac{\vec{r}(\vec{R}, t) - \vec{r}(\vec{R}, t-dt)}{dt}$$

Backward SDE

How to define velocity for Stochastic Variables ?

Take Two !!



$$\vec{v} = \lim_{dt \to 0+} \frac{\vec{r}(\vec{R}, t+dt) - \vec{r}(\vec{R}, t)}{dt}$$

Forward SDE

$$\vec{\tilde{v}} = \lim_{dt \to 0+} \frac{\vec{r}(\vec{R}, t) - \vec{r}(\vec{R}, t-dt)}{dt}$$

Backward SDE

From the variation with respect to u's

$$\delta I=\delta\left\langle\int_a^b dtL(X,DX,\tilde{D}X)\right\rangle=0$$

$$\left(\partial_t+\vec{u}_m\cdot\nabla\right)\vec{u}_m-2\nu^2\nabla\left(\rho^{-1/2}\nabla^2\sqrt{\rho}\right)=-\frac{1}{m}\nabla V$$

And the Fokker-Planck $\partial_t\rho+\nabla\cdot\left(\rho\vec{u}_m\right)=0,$

$$\partial_t\rho+2\nu\nabla\cdot\left(\rho\nabla\vartheta\right)=0,\qquad \nabla\vartheta=\vec{u}_m/(2\nu)$$

$$\nabla\left[\partial_t\vartheta+\nu\left(\nabla\vartheta\right)^2-\nu\left(\rho^{-1/2}\nabla^2\sqrt{\rho}\right)+\frac{1}{m}\nabla V\right]=0$$

These two equations can be combined in a complex form,

$$i\partial_t\varphi=\left[-\nu\nabla^2+\frac{1}{2\nu m}V\right]\varphi,$$ with $$\varphi\equiv\sqrt{\rho}e^{i\vartheta},$$

In resume,

Classical Action
$$I_{cla} = \int_a^b dt \left(\frac{m}{2}\left(\frac{d\vec{r}(t)}{dt}\right)^2 - V(\vec{r}(t)) \right)$$

Estocasitic action

$$I_{sto} = \int_a^b dt \left\langle \frac{m}{2} \frac{(D\vec{r})^2 + (\tilde{D}\vec{r})^2}{2} - V(\vec{r}) \right\rangle$$

Fokker-Planck equation and the
Euler-Lagrange equation for estocastic variables

$$\partial_t \rho + \nabla \cdot \left(\rho \vec{u}_m \right) = 0,$$
$$\left(\partial_t + \vec{u}_m \cdot \nabla\right)\vec{u}_m - 2\nu^2 \nabla\left(\rho^{-1/2}\nabla^2\sqrt{\rho}\right) = -\frac{1}{m}\nabla V$$

Schrödinger's Equation !

$$i\hbar\partial_t \varphi = \left[-\frac{\hbar^2}{2m}\nabla^2 + V \right]\varphi,$$

$$\varphi \equiv \sqrt{\rho}e^{i\vartheta},$$
$$\vec{u} = 2\nu\nabla\vartheta,$$
$$\nu = \hbar / 2m.$$

Applications of Stochastic Variation Method

- Derivation of Navier-Stokes Equation
  (with the correção term of diffusive pressure)
- Derivation of Gross-Pitaevskii Equation
- Derivation of Kostin's Equation
- Quantization of Klein-Gordon Field

1) T. Koide and T. K, .J. PhysA: 45(25):255204
2) T.Koide and T.K, arXiv: 1208.0258v1
3) T.Koide and T.K., arXiv: 1306.6922v1

Some interesting facts

Relation with Parisi-Wu Quantization?

The consistency condition between the two velocity fields, “forward” and “backward”

$$u - \tilde{u} \sim \nabla \ln(n) \quad \longrightarrow \quad n \sim n_0 \exp(u - \tilde{u})\, dr$$

is nothing but the Einstein relation for the osmotic pressure.

This suggests a kind of equilíbrium between the two diffusion processes, “forward” e “backward”.

It it possible to think of the “micro” time scale in SVM is equivalent to the 5ª temporal dimension in the formalism of Parisi-Wu?

Is it possible to derive Feynmann Path Integral by Maximum entropy principle?

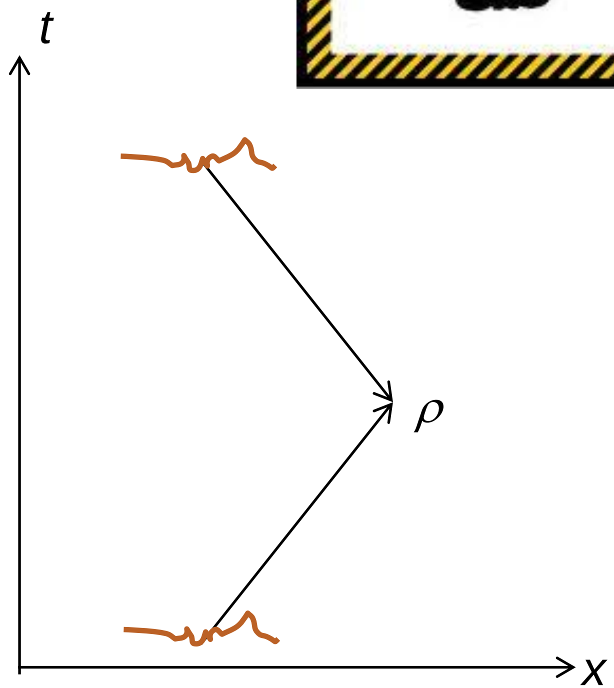
Bernstein Process is shown to be a set of real fields,

defined by

$$\theta(x,t)=\sqrt{\rho(x,t)}e^{\alpha(x,t)}$$

$$\bar{\theta}(x,t)=\sqrt{\rho(x,t)}e^{-\alpha(x,t)}$$

$$\rho(x,t)=\bar{\theta}(x,t)\theta(x,t),$$



Feynmann Path Integral ?

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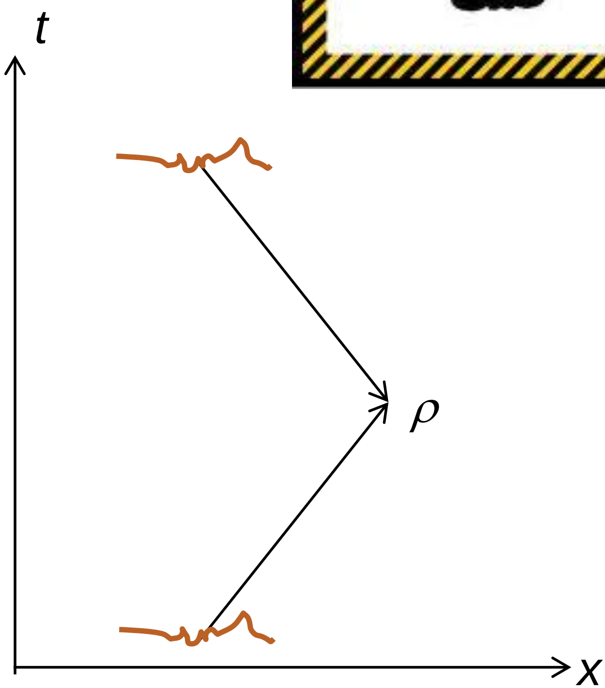
$$\theta(x,t)=\sqrt{\rho(x,t)}e^{\alpha(x,t)}$$
$$\bar{\theta}(x,t)=\sqrt{\rho(x,t)}e^{-\alpha(x,t)}$$

$$Max(\int dx\rho\ln\rho) \text{ or } Max(\int dx\rho\ln\theta)$$

with

$$H(x,p),\ H(x,\bar{p}):Const$$



Feynmann Path Integral ?

Bernstein Process is shown to be a set of real fields,

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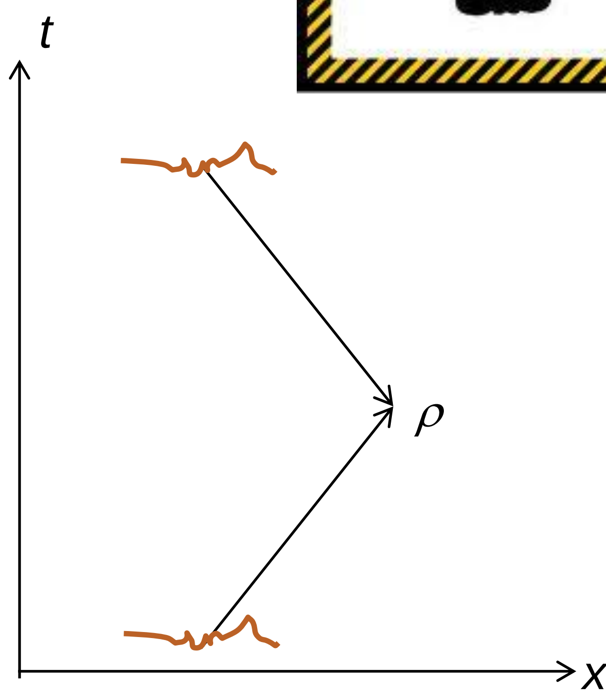

?

$$\theta(x,t) = \int dx' K(x,t;x',t-\Delta t)\theta_0(x')$$

$$K(x,t;x',t') = \exp(-\Delta t H)$$

Conclusion (if any)

- Variational formulation for stochastic process requires the consistency condition for forward and backward processes.
- Noise term generate the surface tension (quantum pressure) term.
- Schrödinger's Equation can be obtained from a Classical Lagrangian via SVM.
- Can this method be considered as alternative quantization scheme? (and useful?)
- Can the origin of Feynman's path integral be interpreted from the maximum entropy principle?

Mean-while ,…

Conferência Solvay Bruxela 1911

Conferência Solvay Bruxela 1911

The impostors of Solvay

A century ago a remarkable group of scientists met in a room at the Hôtel Métropole in Brussels. They were there at the behest of Ernest Solvay, a titan of the chemical industry who – like Alfred Nobel before him – believed that putting physicists up in posh hotels was a great way of spending his money. Between 30 October and 3 November 1911, the carefully selected participants of the first ever Solvay Conference discussed the new-fangled quantum theory of radiation. At some point, they had their picture taken (right). Then they went home, whereupon the second-youngest attendee, one A Einstein, complained that he had learned nothing new.

Despite sharing some features of ordinary meetings, the first Solvay Conference would prove a watershed in the history of physics – a moment when some of the world's finest scientific minds staged a revolution, ushering out the classical world and welcoming a quantum one in its place. Completing this transformation would not be easy, but the 24 scientists in the now-iconic photograph had every reason to feel confident in their ability to carry it off. After all, three of them had already received Nobel prizes, and six others would soon join them. Even the list of non-laureates reads like a *Who's Who* of modern physics: Langevin, Poincaré, Brillouin, De Broglie.

Yet, if our scientific forebears were anything like us, a few of them will have looked around at the glittering Métropole group and thought "I *really* don't belong here". This sense of unwarranted inadequacy is called "impostor syndrome", and sufferers may feel like frauds despite receiving prizes and invitations to prestigious conferences. It is difficult to find data on the prevalence of impostor syndrome, but one survey of medical students found that nearly a third had experienced it. Statistically speaking, an estimate of one or two Solvay "impostors" is probably conservative.

A historian could, no doubt, add some substance to this conjecture by trawling through the surviving correspondence of the 24 attendees. But even without doing such legwork, it is possible to make some informed guesses about who the "impostors" could have been.

Benjamin Couprie, 1911

If our scientific forebears were anything like us, a few of them will have looked around and thought "I *really* don't belong here"

gered in Rutherford's mind 16 years later.

The lesser lights of Solvay also offer fruitful ground for speculation. Louis de Broglie and Léon Brillouin were notable physicists, but neither appears in the photograph. The "De Broglie" and "Brillouin" pictured are actually their less-famous relatives, Maurice and Marcel. Maurice de Broglie did bring his little brother Louis to the meeting, but only as an observer, and Maurice himself was there as a secretary, not an invited scientist. His fellow secretaries Robert Goldschmidt and Frederick Lindemann were, respectively, Solvay's friend and Walther Nernst's PhD student. Goldschmidt was probably the conference's least distinguished attendee, while Lindemann was the youngest by seven years. Did they look around the room in awe?

Half a lifetime later, Lindemann would be ennobled as Winston Churchill's scientific adviser. In 1911, though, he would have had a good reason for feeling like an impostor: he was only there because another, more famous, scientist had declined Solvay's invitation. The same was true of Edouard Herzen, Georges Hostelet and Martin Knudsen. All were good scientists (Knudsen, in particular, invented a cell that is still used in molecular beam epi-

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Can I enter ??

THANKS TO

- Prof. Cesar Lattes, Alfredo Marques, Y. Fujimoto, S. Hasegawa, to whom I owe my life in Brasil.
- Prof. M. Namiki and Prof. M. Yamada for my scientific formation
- My scientific collaborators, my graduate and undergraduate students, who gave me encouragements and ideas,
- My family members and my many friends, who always helped me… such as organizers of this workshop and participants…

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And I am sorry for the bad weather……