2+1 Flavour Phase Diagram and Thermodynamics of Strongly Interaction Matter at Nonzero Isospin

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Bundesministerium für Bildung und Forschung EMMI HIC for FAIR Helmholtz International Center HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

work done in collaboration with
Rainer Stiele and Eduardo Fraga
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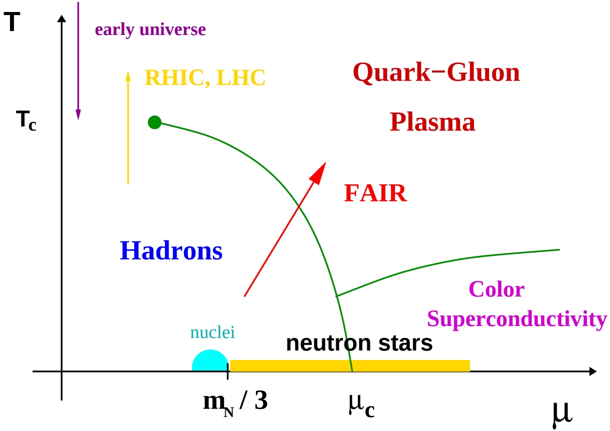
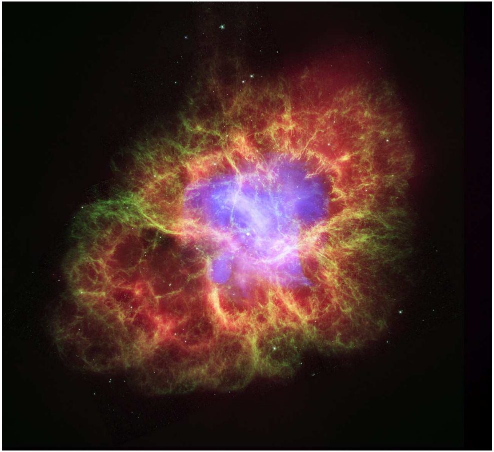
QCD Phase diagram



- chiral phase transition at high densities and temperatures
- early universe: high temperatures
- supernovae, neutron stars: high baryon densities
- QCD unter extreme conditions!

Neutron Stars und Supernovae



- neutron stars are born in core-collapse supernova explosionen
- compact, massive objects: radius $\approx$ 10 km, masses $1 - 2M_\odot$
- extreme densities, several times normal nuclear matter density
- extremely neutron-rich material, isospin asymmetric!

QCD Matter at Nonzero Isospin

- heavy-ion collisions of Au or Pb nuclei at RHIC and LHC: proton to neutron ratio of about 2:3
- core-collapse supernovae: initial proton fraction of $Y_e \approx 0.4$, reduces to ~ 0.2 at bounce
- neutron stars: proton fractions of less than 0.1!
- early universe: sizable nonzero lepton chemical potential of $-0.38 < \mu_\nu/T < 0.02$ cosmologically allowed (relates to a nonzero isospin due to weak equilibrium)
- $\rightarrow$ investigate the QCD phase transition at nonzero isospin
- use effective chiral model in 2+1 flavours

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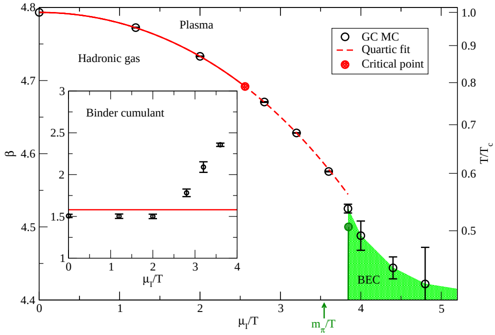
Testing effective chiral models of QCD

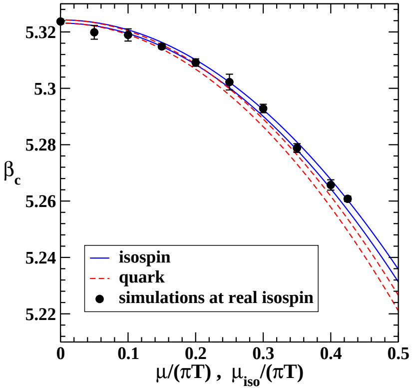
- usual approach of effective models of QCD:
  fit to lattice data and extrapolate
  (to nonzero baryon density)
- does the extrapolation work? $\rightarrow$ test it with lattice data!
- three possible tests so far:
  1. scaling of results with quark masses
     (EoS, susceptibilities, hadron masses, phase transition line)
  2. dependence on magnetic fields
     (does not work! inverse magnetic catalysis!)
  3. extending to nonzero isospin chemical potential
     ($\mu_{iso} = \mu_u - \mu_d \neq 0$ at $\mu_b = 0 \rightarrow$ requires $\mu_s = 0$)

Lattice Data at Nonzero Isospin



- lattice data for $N_f = 8$ QCD (four u- and d-quark species) (de Forcrand, Stephanov, Wenger 2007)
- solid line: first order, dashed line: crossover, green area: pion condensation
- critical temperature decreases with isospin

Lattice Data at Nonzero Isospin II



- lattice data for two-flavour QCD
  (Cea, Cosmai, D'Elia, Papa, Sanfilippo 2012)
- quark masses corresponding to $m_\pi \approx 400$ MeV
- (pseudo-)critical line decreases with isospin

Former work on QCD matter at nonzero isospin

- chiral perturbation theory and fuzzy bag model (Palhares, Fraga, Villavicencio 2009)
- hadron resonance gas model (Toublan and Kogut 2005)
- O(2N) symmetric ϕ^4-model (Andersen 2007)
- two- and three-flavour Nambu-Jona-Lasinio model (Toublan and Kogut 2003; He, Jin, Zhuang 2005; Andersen and Kyllingstad 2009; Xia, He, Zhuang 2013)
- Polyakov-loop extended Nambu-Jona-Lasinio (two-flavour) (Sasaki, Sakai, Kouno, Yahiro 2010; Cavagnoli, Menezes, Providencia 2013; Ueda, Nakano, Ohnishi, Ruggieri, Sumiyoshi 2013)
- two-flavour quark-meson model (within FRG) (Kamikado, Strodthoff, von Smekal, Wambach 2013)

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Chiral Effective Lagrangian

Investigation of the chiral phase transition of QCD within a chiral effective Lagrangian (Pisarski and Wilczek 1984):

$$\begin{aligned}\mathcal{L}_{\mathrm{SU(3)}} &= \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \mathrm{M}\partial^\mu \mathrm{M}^\dagger\right] + \frac{1}{2}\mu^2\mathrm{Tr}\left[\mathrm{MM}^\dagger\right] - \lambda\mathrm{Tr}\left[\mathrm{MM}^\dagger\mathrm{MM}^\dagger\right] \\ &\quad -\lambda'\left(\mathrm{Tr}\left[\mathrm{MM}^\dagger\right]\right)^2 + c\left[\det\mathrm{M} + \det\mathrm{M}^\dagger\right]\end{aligned}$$

equivalent to the linear σ Modell (for SU(3): Levy, 1967)

- $\mathrm{Tr} M^\dagger M \longrightarrow O(18)$ (norm of a vector)
- $\mathrm{Tr} M^\dagger M M^\dagger M \longrightarrow U(3) \times U(3)$ ($M \to UMU^{-1}$)
- $\det M + \det M^\dagger \longrightarrow SU(3) \times SU(3)$
  ($\det \exp i\lambda_0 = \exp \mathrm{Tr}\lambda \neq 1$) breaks $U_A(1)$ symmetry

Breaking of Chiral Symmetry and Masses

- spontaneous breaking of chiral symmetry by two order parameters

$$< \bar{u}u + \bar{d}d > \to \sigma_u + \sigma_d \qquad \text{and} \qquad < \bar{s}s > \to \sigma_s$$

- explicit breaking of chiral symmetry:

$$\mathcal{L}_{\rm esb} = \epsilon \cdot (\sigma_u + \sigma_d) + \epsilon' \cdot \sigma_s$$

- vacuum values fixed by PCAC relations:
  $\sigma_u^0 = \sigma_d^0 = f_\pi = 92.4$ MeV
  $\sigma_s^0 = \sqrt{2} f_K - f_\pi/\sqrt{2} = 94.5$ MeV
- coupling constants μ^2, λ, λ', c fitted to meson masses: m_π, m_K, $m_\eta^2 + m_{\eta'}^2$, m_σ.

Quark contribution

Add quarks to the thermodynamic potential including the coupling to the Polyakov-loop and meson fields:

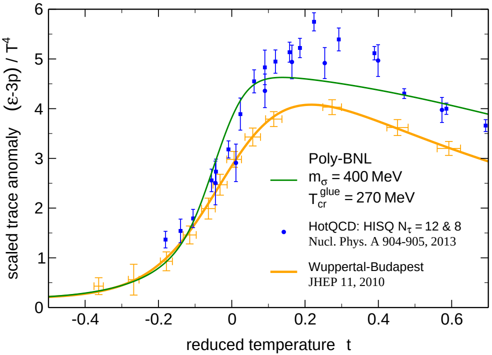
$$\Omega_q^{th} = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \times$$
$$\left\{ \ln\left[1 + 3\left(\Phi + \bar{\Phi} e^{-(E_f-\mu_f)/T}\right) \times e^{-(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}\right]\right.$$
$$\left. + \ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-(E_f+\mu_f)/T}\right) \times e^{-(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}\right]\right\}$$

Quark dispersion relation: $E_f = \sqrt{k^2 + m_f^2}$ with

$$m_u = \frac{g}{2}\sigma_u \qquad m_d = \frac{g}{2}\sigma_d \qquad m_s = \frac{g}{\sqrt{2}}\sigma_s$$

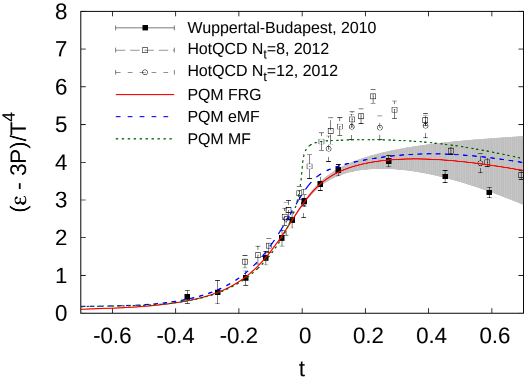
Fix Yukawa coupling g by setting $m_l = 300$ MeV

Comparison to lattice data



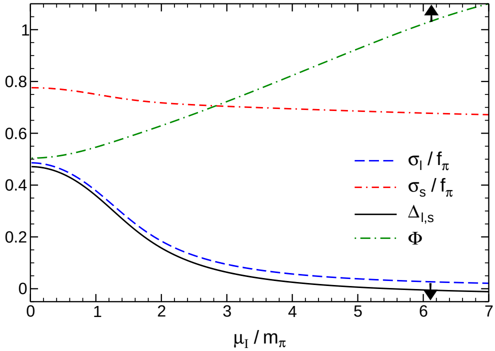
- check lattice data at zero chemical potentials
- use improved description of Polyakov loop potential (Haas, Stiele, Braun, Pawlowski, JSB 2013)
- include vacuum term
- reasonable description of scaled trace anomaly

Comparison to lattice data II



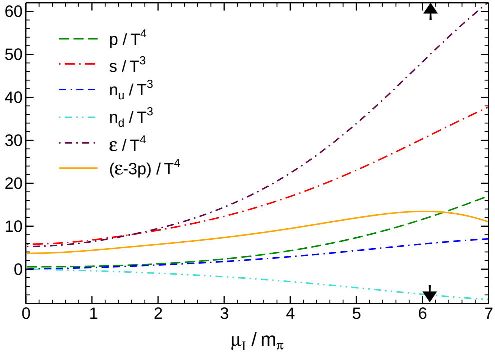
- comparison to functional renormalisation group (FRG) (Herbst, Mitter, Pawlowski, Schaefer, Stiele 2013)
- mean-field results are close to FRG results
- some difference without (MF) and with vacuum term (eMF)

Order parameters at nonzero μ_{iso}



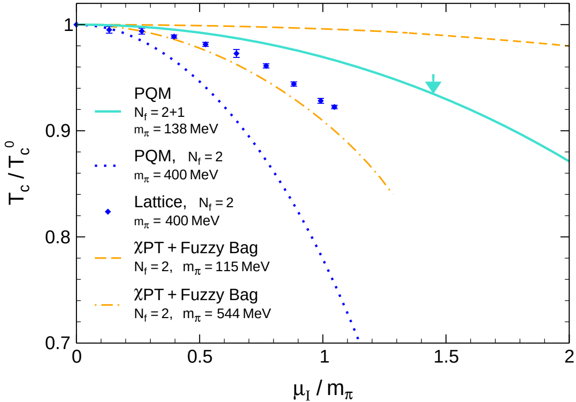
- order parameters at $T_c^0 = 164$ MeV versus μ_I / m_π
- drop of light quark condensate $\sigma_l \rightarrow$ chiral restoration
- arrow: onset of pion condensation

Thermodynamics at nonzero μ_{iso}



- thermodynamic quantities at $T_c^0 = 164$ MeV
- small change in pressure, sizable increase of energy density and entropy
- arrow: onset of pion condensation

Test with lattice data at nonzero μ_{iso}



- lattice data for $N_f = 2$ at $m_\pi \approx 400$ MeV (Cea, Cosmai, D'Elia, Papa, Sanfilippo 2012)
- PQM model ($N_f = 2$, $m_\pi = 400$ MeV): drops too fast
- χPT+Fuzzy bag: closer to lattice data

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Summary

- new ways to test effective models of QCD by lattice data (isospin, quark masses, magnetic field, . . .)
- pseudo critical temperature for chiral phase transition: drops with isospin chemical potential
- PQM model: reasonable description of lattice data at zero chemical potentials
- PQM model at nonzero isospin chemical potential: shows right trend, but phase transition line drops too strongly
- quark mass effects are important for a correct comparison with lattice data!
- missing terms? missing physics? related to problem of inverse magnetic catalysis?

To Takeshi

Happy Birthday Takeshi!