Di-hadron azimuthal angular correlations at RHIC and LHC (or Pomerons, Odderons and more... from CGC)

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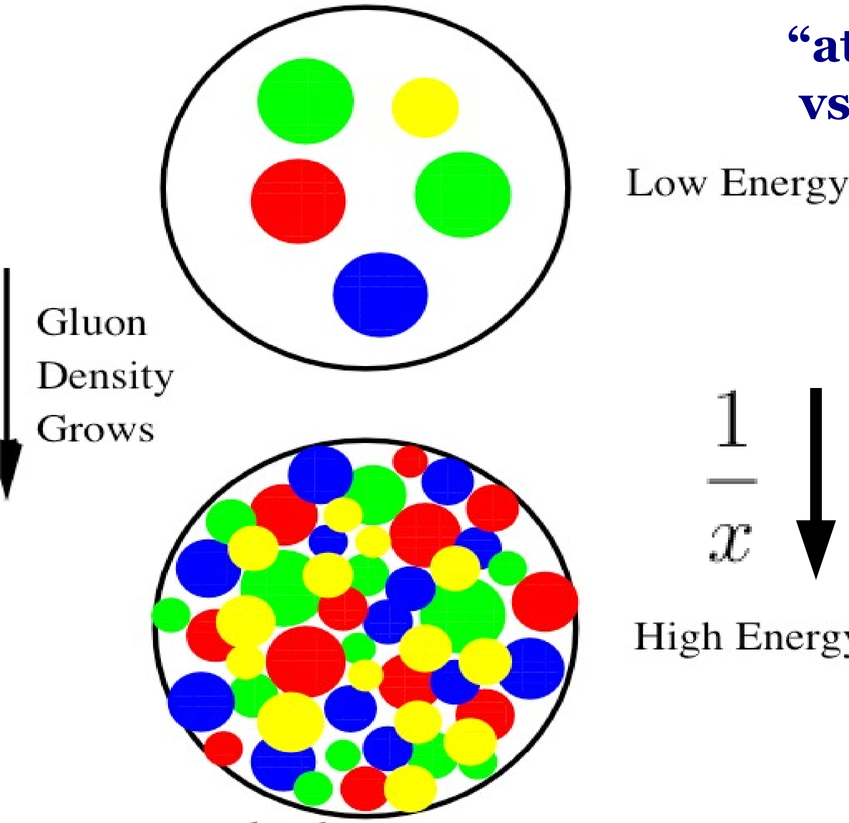
(celebrating Prof. Kodama's 70th birthday)

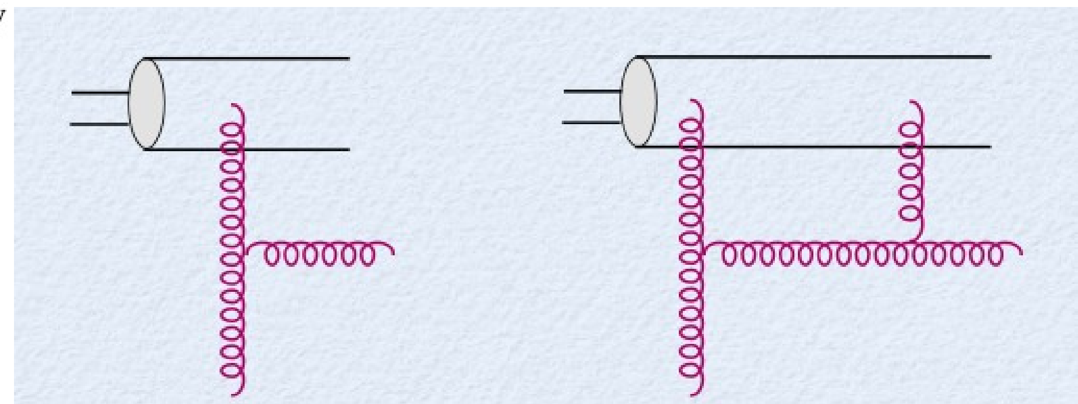
Gluon saturation/CGC

Gribov-Levin-Ryskin

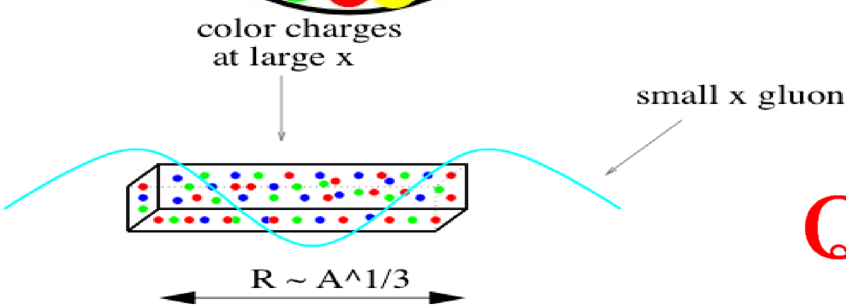
$$\mathbf{S \to \infty, \quad Q^2 \;\; fixed, \quad x_{Bj} \equiv \frac{Q^2}{S} \to 0}$$

“attractive” bremsstrahlung
vs. “repulsive” recombination



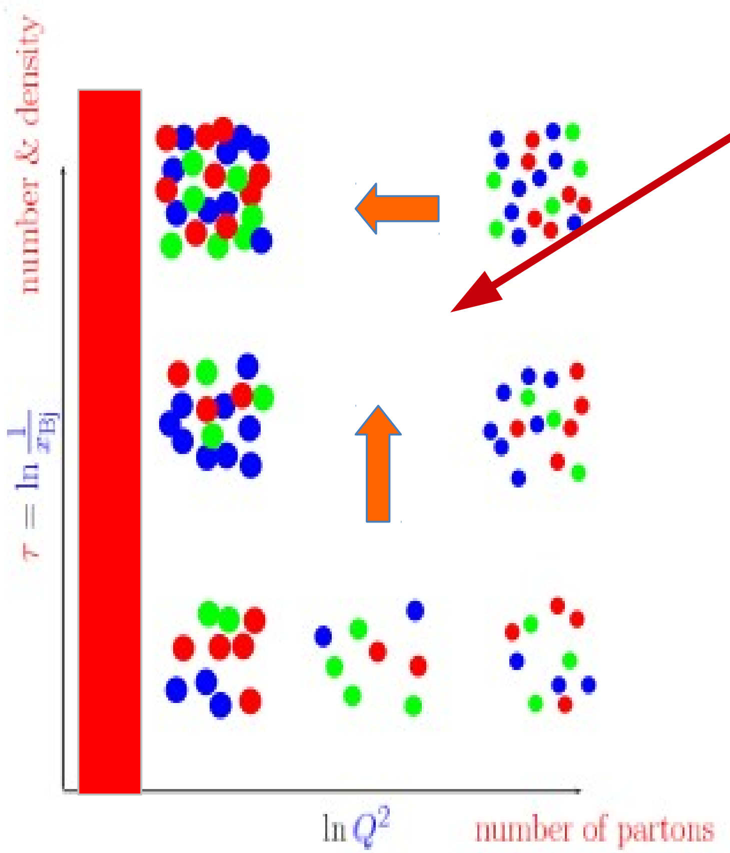


$$\frac{\alpha_s \, \mathbf{xG(x, b_t, Q^2)}}{\mathbf{S_\perp \, Q^2}} \sim \mathbf{1}$$



$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

Many-body dynamics of universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

Can this provide a first-principles understanding of the initial Conditions-thermalization in Heavy ion collisions?

MV effective Action + RGE

$$\mathbf{S}[\mathbf{A},\rho] = -\frac{\mathbf{1}}{\mathbf{4}}\int \mathbf{d^4x\,F^2_{\mu\nu}} + \frac{\mathbf{i}}{\mathbf{N_c}}\int \mathbf{d^2x_t dx^-}\,\delta(\mathbf{x^-})\mathbf{Tr}[\rho(\mathbf{x_t})\mathbf{U}(\mathbf{A^-})]$$

Large x: color source ρ small x: gluon field $\mathbf{A}^\mu$

$$\mathbf{U}(\mathbf{A^-}) = \hat{\mathbf{P}}\,\mathbf{Exp}\left[\mathbf{ig}\int \mathbf{dx^+\,A^-_a\,T_a}\right]$$

$$\mathbf{Z}[\mathbf{j}] = \int [\mathbf{D}\rho]\,\mathbf{W}_{\mathbf{\Lambda^+}}[\rho]\left[\frac{\int^{\mathbf{\Lambda^+}}[\mathbf{DA}]\delta(\mathbf{A^+})\mathbf{e}^{\mathbf{iS}[\mathbf{A},\rho]-\int \mathbf{j}\cdot\mathbf{A}}}{\int^{\mathbf{\Lambda^+}}[\mathbf{DA}]\delta(\mathbf{A^+})\mathbf{e}^{\mathbf{iS}[\mathbf{A},\rho]}}\right]$$

weight functional:

$\mathbf{W}_{\mathbf{\Lambda^+}}[\rho]$ probability distribution of color source ρ

at longitudinal scale $\mathbf{\Lambda^+}$

invariance under change of $\mathbf{\Lambda^+} \longrightarrow$ RGE for $\mathbf{W}_{\mathbf{\Lambda^+}}[\rho]$

The classical field

saddle point of effective action-> Yang-Mills equations

$$\mathbf{D}_\mu \, \mathbf{F}_{\mathbf{a}}^{\mu\nu} = \delta^{\nu\,+} \delta(\mathbf{x}^-)\, \rho_{\mathbf{a}}(\mathbf{x_t})$$

solutions are non-Abelian
Weizsäcker-Williams fields

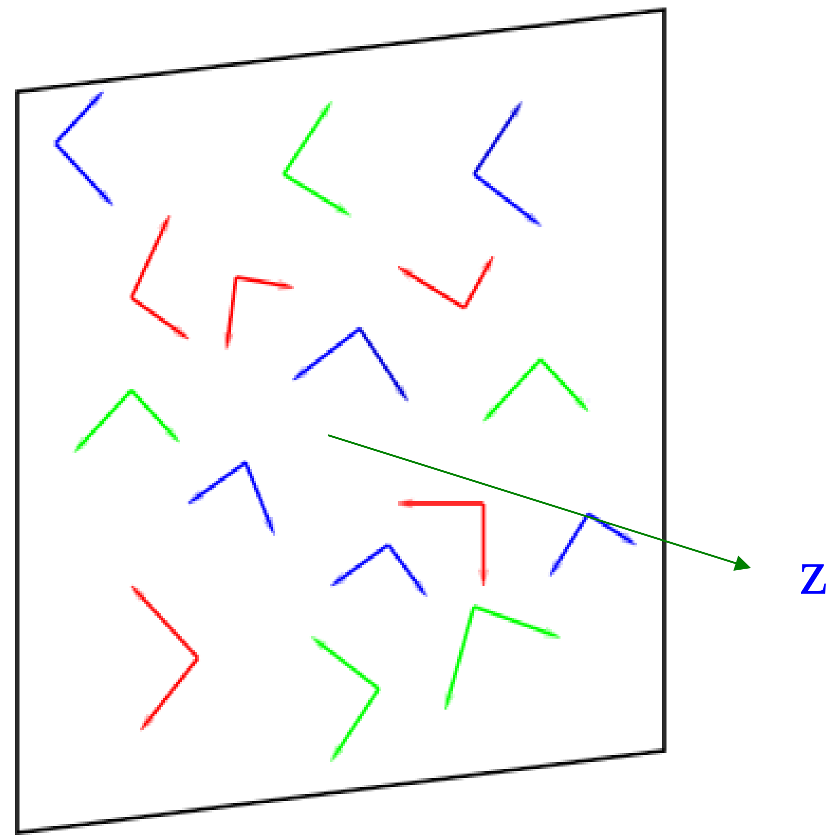
$$\mathbf{A}^+ = \mathbf{0}$$

$$\mathbf{A}^- = \mathbf{0}$$

$$\mathbf{A}_{\mathbf{a}}^{\mathbf{i}} = \theta(\mathbf{x}^-)\, \alpha_{\mathbf{a}}^{\mathbf{i}}(\mathbf{x_t})$$

$$\partial^{\mathbf{i}}\, \alpha_{\mathbf{a}}^{\mathbf{i}} = \mathbf{g}\, \rho_{\mathbf{a}}$$

pure (2d) gauge



$$\mathbf{E}_\perp \,, \, \mathbf{B}_\perp$$

Quantum corrections: JIMWLK evolution equation

$$\frac{d}{d\,ln 1/x}\langle O\rangle = \frac{1}{2}\left\langle \int d^2x\, d^2y\, \frac{\delta}{\delta\alpha_x^b}\,\eta_{xy}^{bd}\,\frac{\delta}{\delta\alpha_y^d}\,O\right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi}\int \frac{d^2z}{(2\pi)^2}\,\frac{(x-z)\cdot(y-z)}{(x-z)^2(y-z)^2}\left[\underbrace{1+U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}}\right]^{bd}$$

U is a Wilson line in adjoint representation

QCD at low x: CGC

(a high gluon density environment)

two main effects:

"multiple scatterings"
evolution with ln (1/x)

CGC observables: $<\mathrm{Tr}\,\mathrm{V}\cdots\cdots\mathrm{V}^{\dagger}>$ with $\mathbf{V(x_t) = \hat{P}e^{ig\int dx^-\,A_a^+\,t_a}}$

$$\mathbf{A_a^\mu(x_t, x^-) \sim \delta^{\mu\,+}\,\delta(x^-)\,\alpha_a(x_t)} \qquad \mathbf{\alpha^a(k_t) = g\,\rho^a(k_t)/k_t^2}$$

gluon distribution: $\mathbf{xG(x, Q^2) \sim \int^{Q^2} \frac{d^2k_t}{k_t^2}\,\phi(x, k_t)}$ with $\mathbf{\phi(x, k_t^2) \sim <\rho_a^\star(k_t)\,\rho_a(k_t)>}$

<u>pQCD with collinear factorization:</u>

single scattering
evolution with ln Q²

Observables

DIS:

structure functions (diffraction)
particle production

dilute-dense (pA, forward pp) collisions:

multiplicities
p_t *spectra*
di-hadron angular correlations

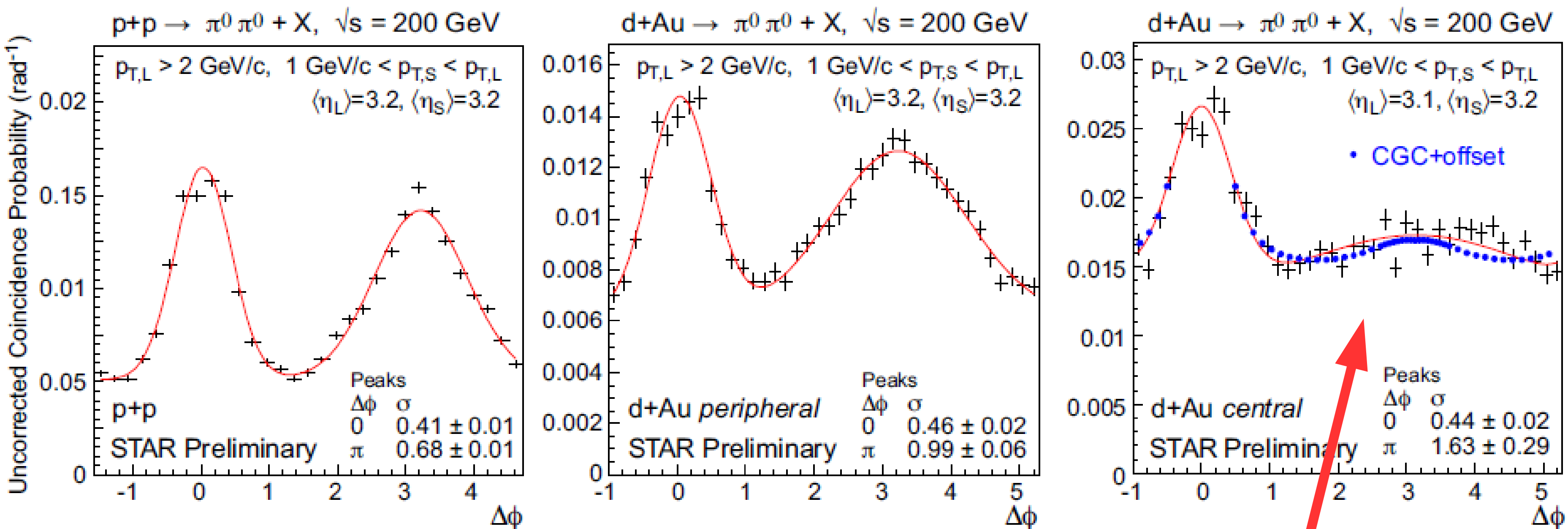
dense-dense (AA, pp) collisions:

multiplicities, spectra
long range rapidity correlations

Spin asymmetries

disappearance of back to back hadrons

Recent STAR measurement (arXiv:1008.3989v1):



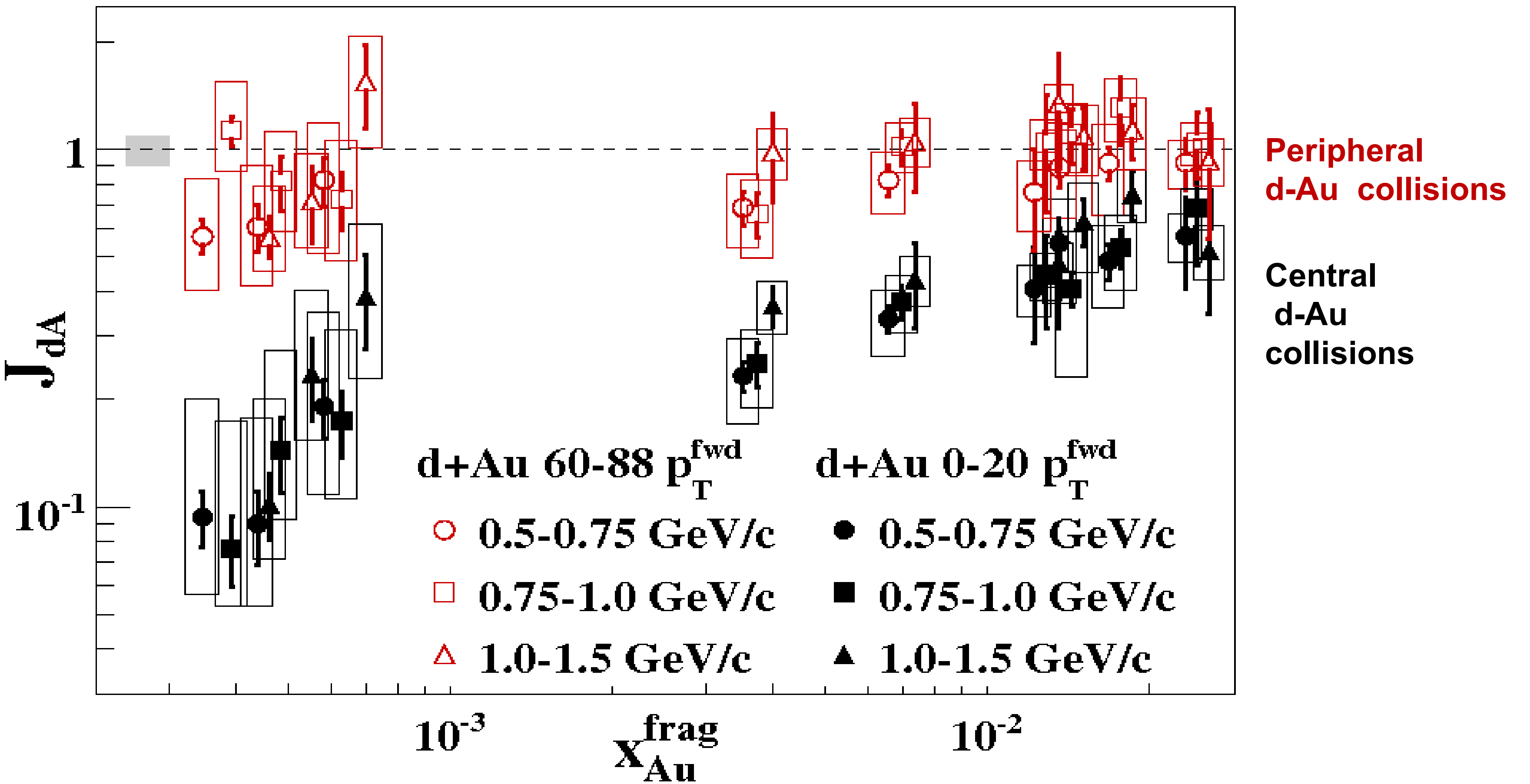
CGC fit from Albacete + Marquet, PRL (2010)
Tuchin, NPA846 (2010)
A. Stasto, B-W. Xiao, F. Yuan, PLB716 (2012)
T. Lappi, H. Mantysaari, NPA908 (2013)

multiple scatterings de-correlate the hadrons

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

disappearance of back to back jets

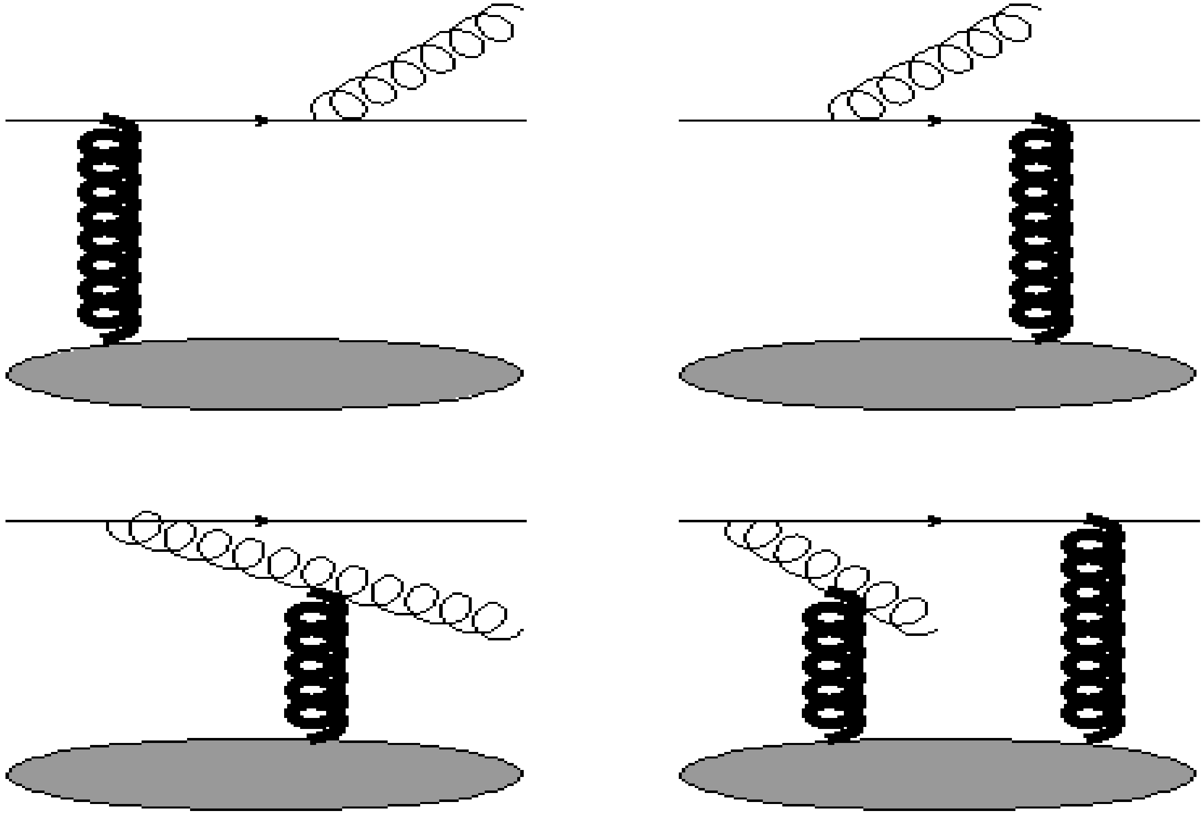
$$J_{dA} \sim \frac{N_{dAu}^{\text{back-to-back}} \,/\, N_{\text{N-N-collision}}}{N_{pp}^{\text{back-to-back}}}$$



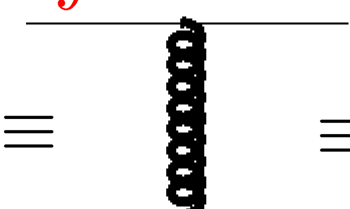
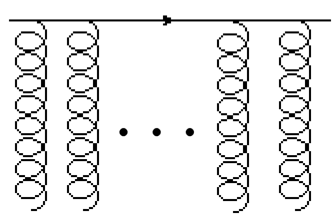
A. Adare, et al., PRL 107 (2011) 172301

Di-jet production: pA

$$\mathbf{q(p)\,T \to q(q)\,g(k)}$$

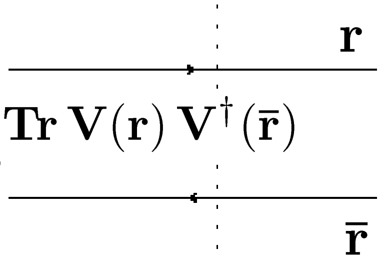


$$\mathbf{d}\sigma \sim \int \mathbf{K} \otimes \left[< \mathbf{Tr V\, V^\dagger} > + < \mathbf{Tr V\, V^\dagger\, V\, V^\dagger} > + \cdots \right]$$

$\mathbf{V} \equiv$  $\equiv$  $\sim \mathbf{1} + \mathbf{O}(\mathbf{g}\,\rho) + \mathbf{O}(\mathbf{g^2}\,\rho^{\mathbf{2}})$

Evolution (energy dependence) of the 2-point function (dipole): DIS, single inclusive production

$$\frac{d}{dy}\,\mathbf{Tr\,V(r)\,V^\dagger(\bar{r})}$$



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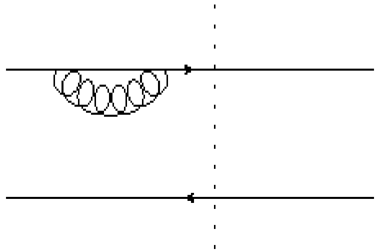

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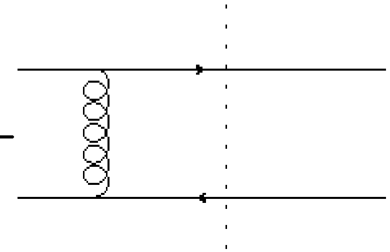
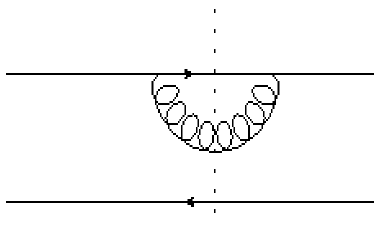


$\mathbf{Tr\,V(r)\,V^\dagger(\bar{r})}$

$+$



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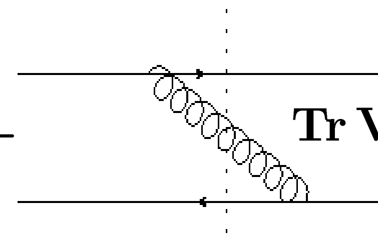


$\mathbf{Tr\,V(r)\,V^\dagger(z)\,Tr\,V(z)\,V^\dagger(r)}$

$$\frac{\mathbf{1}}{(\mathbf{r}-\mathbf{z})^2} \qquad \frac{(\mathbf{r}-\mathbf{z})\cdot(\bar{\mathbf{r}}-\mathbf{z})}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}$$

$$\frac{d}{dy} < \mathbf{TrV(r)\,V^\dagger(\bar{r}} >= -\frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{z}\,\frac{(\mathbf{r}-\bar{\mathbf{r}})^2}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}\times$$

$$\left[< \mathbf{TrV(r)\,V^\dagger(\bar{r})} > -\frac{1}{N_c} < \mathbf{TrV(r)\,V^\dagger(z)\,TrV(z)\,V^\dagger(\bar{r})} >\right]$$

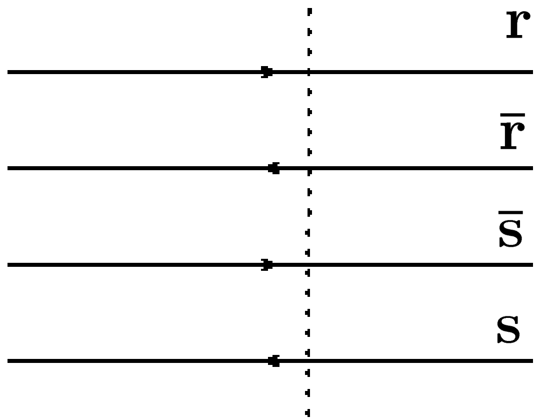
BK equation (known to NLO)

Evolution of quadrupole from JIMWLK

$$\mathbf{Q}(\mathbf{r},\bar{\mathbf{r}},\bar{\mathbf{s}},\mathbf{s}) \equiv \frac{1}{\mathbf{N_c}} < \mathbf{Tr}\,\mathbf{V}(\mathbf{r})\,\mathbf{V}^{\dagger}(\bar{\mathbf{r}})\,\mathbf{V}(\bar{\mathbf{s}})\,\mathbf{V}^{\dagger}(\mathbf{s}) >$$

radiation kernels
as in dipole

$$\frac{\mathbf{d}}{\mathbf{dy}}$$



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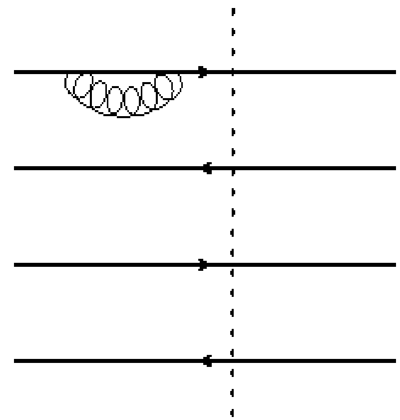

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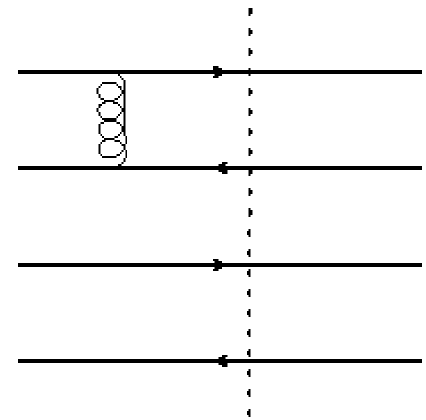
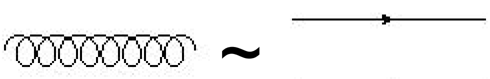
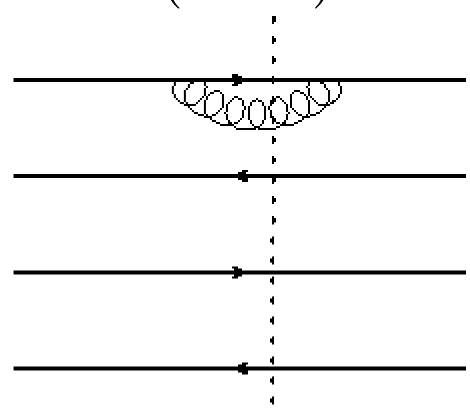
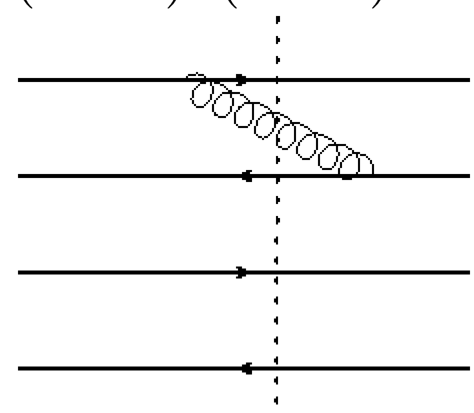


$$\frac{\mathbf{1}}{(\mathbf{r}-\mathbf{z})^{\mathbf{2}}} \qquad \frac{(\mathbf{r}-\mathbf{z})\cdot(\bar{\mathbf{r}}-\mathbf{z})}{(\mathbf{r}-\mathbf{z})^{\mathbf{2}}(\bar{\mathbf{r}}-\mathbf{z})^{\mathbf{2}}}$$

large $\mathrm{N_c}$







$+ \cdots$

Evolution of quadrupole from JIMWLK

$$
\begin{aligned}
&\frac{d}{dy}\,\langle Q(r,\bar r,\bar s,s)\rangle\\
=\;&\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z\Bigg\{\Bigg\langle\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right]Q(z,\bar r,\bar s,s)\,S(r,z)\\
+\;&\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right]Q(r,z,\bar s,s)\,S(z,\bar r)\\
+\;&\left[\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(s-z)^2(\bar r-z)^2}\right]Q(r,\bar r,z,s)\,S(\bar s,z)\\
+\;&\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right]Q(r,\bar r,\bar s,z)\,S(z,s)\\
-\;&\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}\right]Q(r,\bar r,\bar s,s)\\
-\;&\left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right]S(r,s)\,S(\bar r,\bar s)\\
-\;&\left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right]S(r,\bar r)\,S(\bar s,s)\Bigg\rangle\Bigg\}
\end{aligned}
$$

$$\frac{d}{d\,y}Q=\int P_1\,[Q\,S]-P_2\,[Q]+P_3[S\,S]$$ with $$P_1-P_2+P_3=0$$

J. Jalilian-Marian, Y. Kovchegov: PRD70 (2004) 114017
Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037

quadrupole evolution: models

$$< Q(r,\bar{r},\bar{s},s) >\equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$

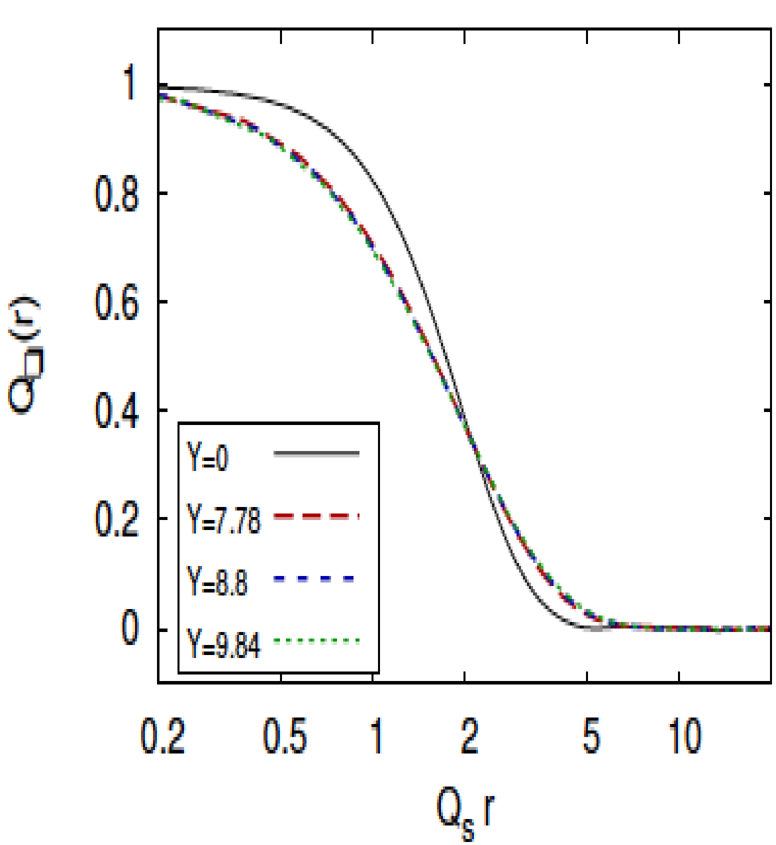
Gaussian model

$$Q_{sq}(z)=[S(z)]^2\left[\frac{N_c+1}{2}\left(\frac{S(z)}{S(\sqrt{2}z)}\right)^{\frac{2}{N_c+1}}-\frac{N_c-1}{2}\left(\frac{S(\sqrt{2}z)}{S(z)}\right)^{\frac{2}{N_c-1}}\right]$$

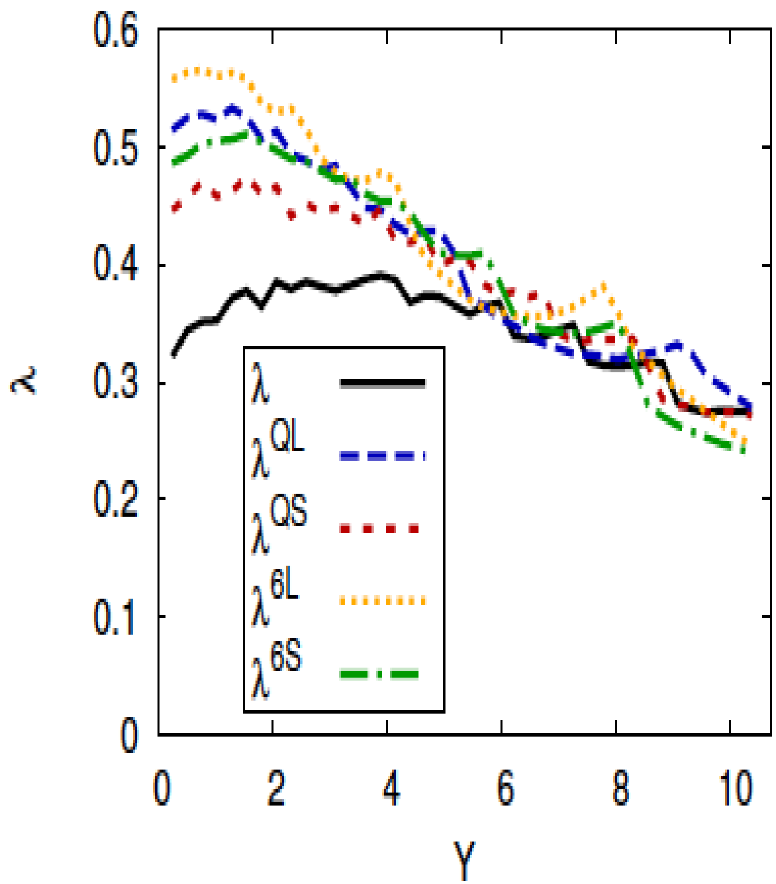
Gaussian + large N_c

$$Q_{sq}(z)=\left[1+2\ln\left(\frac{S(z)}{S(\sqrt{2}z)}\right)\right]$$

Quadrupole evolution



Geometric scaling also present in quadrupoles

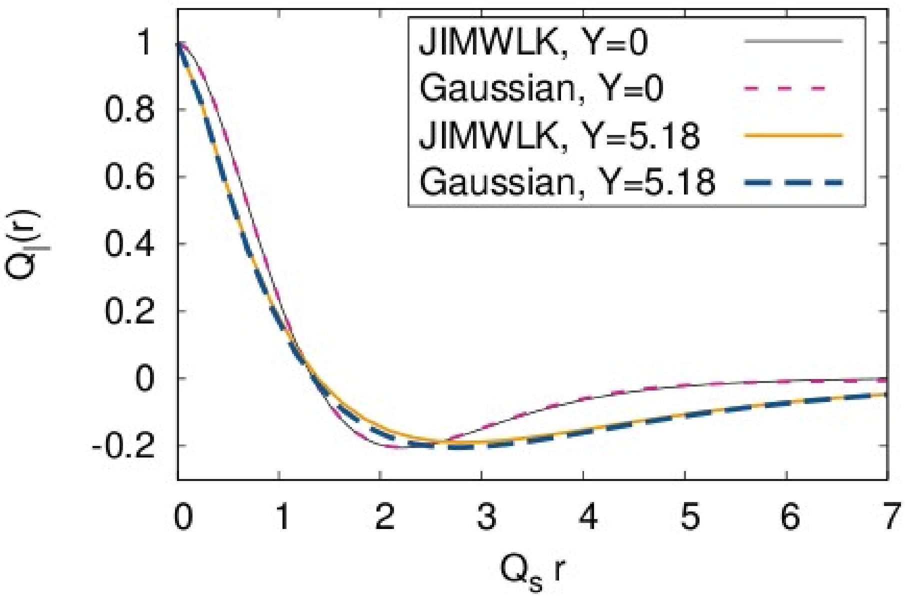


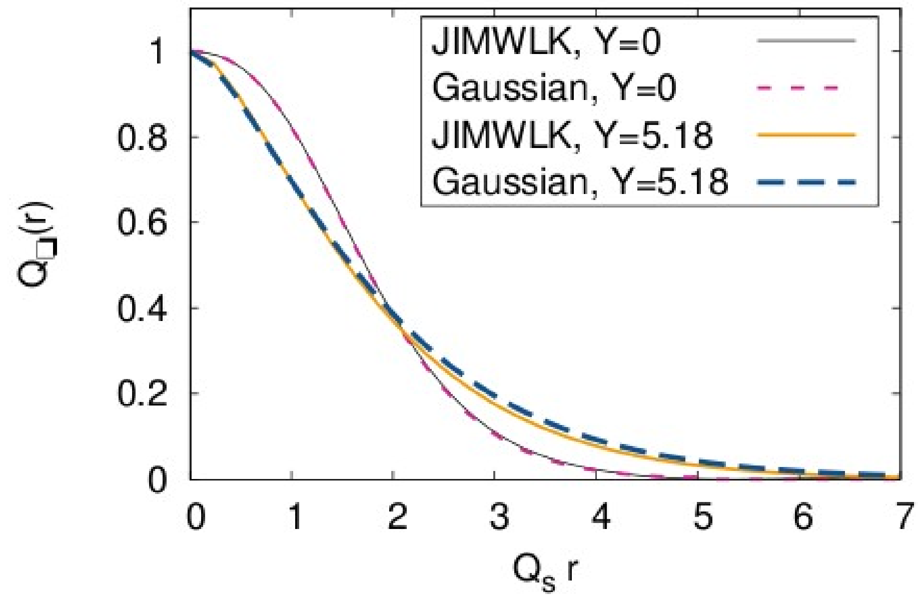
Growth of the saturation scale

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219

Quadrupole evolution

comparing with Gaussian model





quadrupole evolution: linear regime

define $\mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) \equiv \mathbf{1} - \mathbf{S}(\mathbf{r}, \bar{\mathbf{r}}) \qquad \mathbf{T}_{\mathbf{Q}}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv \mathbf{1} - \mathbf{Q}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s})$

expand in powers of gauge fields (or color charges)

ignore contribution of non-linear terms: T T and T_Q T

$\mathbf{O}(\alpha^{\mathbf{2}}) \qquad \mathbf{T}_{\mathbf{Q}}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \rightarrow \mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) + \mathbf{T}(\mathbf{r}, \mathbf{s}) + \cdots$

with $\mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) \sim \alpha^{\mathbf{2}}(\mathbf{r}, \bar{\mathbf{r}})$

quadrupole evolution reduces to a sum of BFKL evolution eqs

Dominguez, Mueller, Munier, Xiao: PLB705 (2011) 106
J. Jalilian-Marian: Phys.Rev. D85 (2012) 014037
D. Triantafyllopoulos

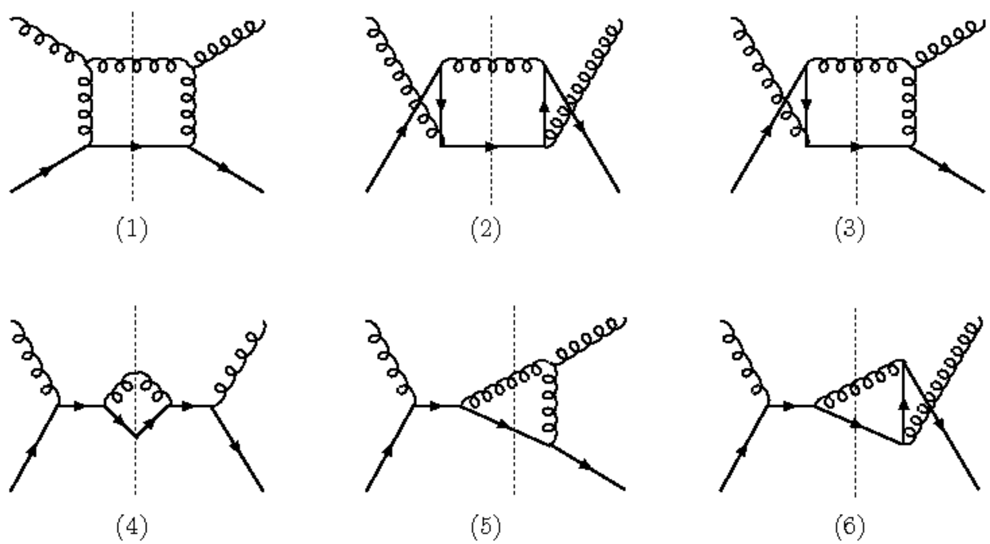
di-hadron correlations: high $\mathbf{p_t}$ limit

$\mathbf{O}(\alpha^{\mathbf{2}})$

Dominguez, Marquet, Xiao, Yuan (2011)
Dominguez, Xiao, Yuan (2011)

factorization of target distribution functions and hard scattering matrix element

$$\mathbf{d}\sigma \sim \mathbf{\Phi} \otimes \frac{\mathbf{d}\sigma}{\mathbf{dt}}^{\mathbf{2\to 2}}$$



$$\frac{d\sigma}{dt}^{q\,g\to q\,g} \sim \frac{1}{s^2}\left[\frac{4}{9}\frac{s^2+u^2}{-s\,u} + \frac{s^2+u^2}{t^2}\right]$$

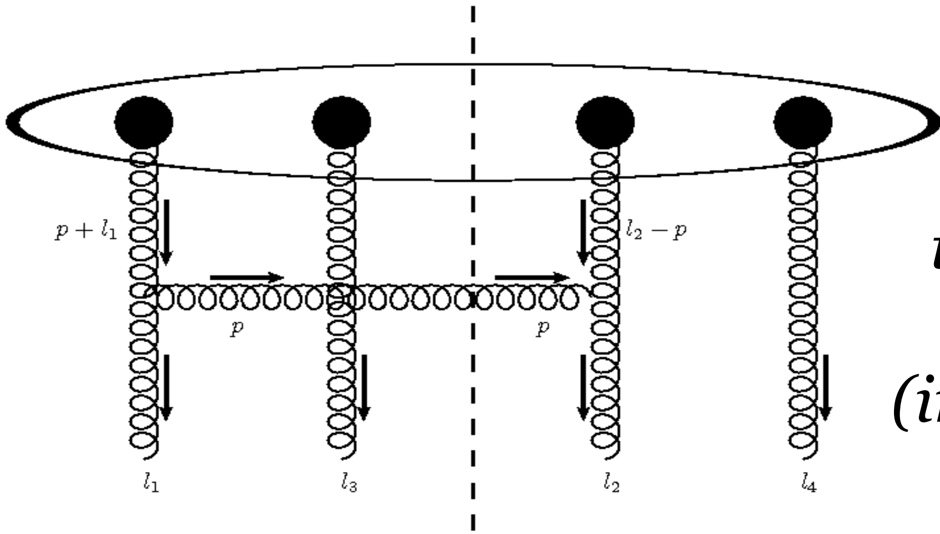
partons are back to back

quadrupole evolution: linear regime

BJKP equation

$\mathbf{O}(\alpha^4)$: 4-gluon exchange

J. Jalilian-Marian, PRD85 (2012) 014037



the color structure is identical on both sides of this eq. (independent of color averaging)

$$\frac{d}{dy}\hat{T}_4(l_1,l_2,l_3,l_4) = \frac{N_c\,\alpha_s}{\pi^2}\int d^2p_t\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_1^i)}{(p_t+l_1)^2}\right]\cdot\left[\frac{p^i}{p_t^2}-\frac{(p^i-l_2^i)}{(p_t+l_2)^2}\right]$$
$$\hat{T}_4(p_t+l_1,l_2-p_t,l_3,l_4)+\cdots$$
$$-\frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2p_t\left[\frac{l_1^2}{p_t^2(l_1-p_t)^2}+\{l_1\to l_2,l_3,l_4\}\right]\hat{T}_4(l_1,l_2,l_3,l_4)$$

this will <u>de-correlate</u> the produced partons at high $p_t > Q_s$

color structure

$$\hat{\mathbf{T}}_{\mathbf{4}}(\mathbf{l_1}, \mathbf{l_2}, \mathbf{l_3}, \mathbf{l_4}) \equiv \frac{\mathbf{1}}{\mathbf{N_c}} \mathbf{Tr}\, \rho(\mathbf{l_1})\, \rho(\mathbf{l_2})\, \rho(\mathbf{l_3})\, \rho(\mathbf{l_4}) = \mathbf{Tr}\,(\mathbf{t^a}\, \mathbf{t^b}\, \mathbf{t^c}\, \mathbf{t^d})\, \rho^{\mathbf{a}}(\mathbf{l_1})\, \rho^{\mathbf{b}}(\mathbf{l_2})\, \rho^{\mathbf{c}}(\mathbf{l_3})\, \rho^{\mathbf{d}}(\mathbf{l_4})$$

$$\begin{aligned} Tr\,(t^a\, t^b\, t^c\, t^d) &= \frac{1}{4N_c}\left[\delta^{ab}\delta^{cd} - \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right] \\ &+ \frac{1}{8}\left[d^{abr}d^{cdr} - d^{acr}d^{bdr} + d^{adr}d^{bcr}\right] \\ &+ \frac{i}{8}\left[d^{abr}f^{cdr} - d^{acr}f^{bdr} + d^{adr}f^{bcr}\right] \end{aligned}$$

overall state is a singlet, how about pairwise?

for $\mathbf{N_c = 3}$

$$\left[\delta^{\mathbf{ab}}\delta^{\mathbf{cd}} + \delta^{\mathbf{ac}}\delta^{\mathbf{bd}} + \delta^{\mathbf{ad}}\delta^{\mathbf{bc}}\right] = \mathbf{3}\left[\mathbf{d^{abr}d^{cdr}} + \mathbf{d^{acr}d^{bdr}} + \mathbf{d^{adr}d^{bcr}}\right]$$

the linear regime

$O(\alpha^3)$: 3-gluon (odderon) exchange

Dipole odderon: *Kovchegov, Szymanowski, Wallon*

$V^\dagger V^\dagger V^\dagger$ *Hatta, Iancu, Itakura, McLerran* **BJKP equation**

BJKP equation describes evolution of n-Reggeized gluons in a singlet state

JIMWLK (linear) and BJKP eqs. agree for n=2,3,4

non-linear interactions:

1) Non-linear JIMWLK evolution

2) Triple pomeron vertex: *Chirilli, Szymanowski, Wallon (2011)*

“$n \to n+1$ vertices ? ” **Work in Progress**

Non-linear regime: triple pomeron vertex

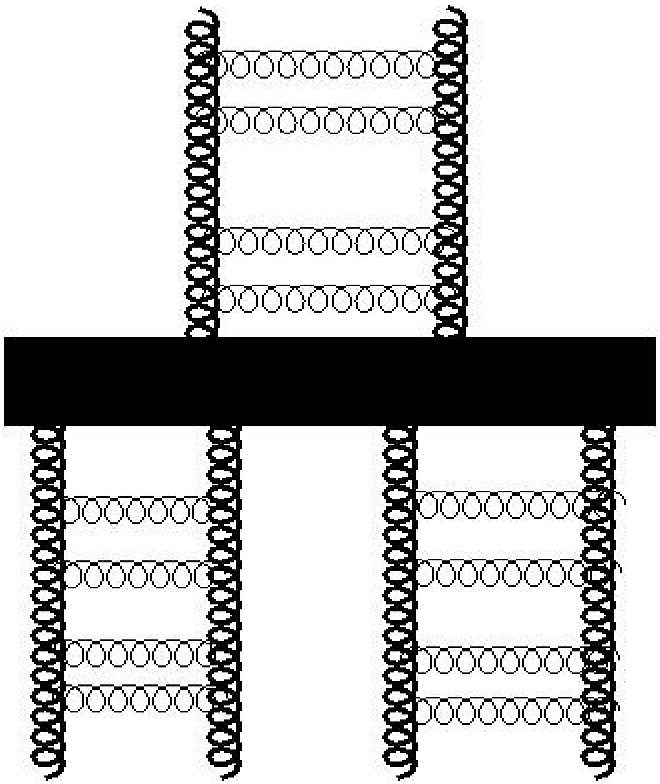
Chirilli, Szymanowski, Wallon PRD83 (2011) 014020

start with the dipole evolution equation

$$\frac{d}{dy}\mathbf{T}(\mathbf{r}-\bar{\mathbf{r}}) = \frac{\bar{\alpha}_s}{2\pi}\int d^2z\,\frac{(\mathbf{r}-\bar{\mathbf{r}})^2}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}\otimes$$

$$\left[\mathbf{T}(\mathbf{r}-\mathbf{z})+\mathbf{T}(\mathbf{z}-\bar{\mathbf{r}})-\mathbf{T}(\mathbf{r}-\bar{\mathbf{r}})-\mathbf{T}(\mathbf{r}-\mathbf{z})\mathbf{T}(\bar{\mathbf{r}}-\mathbf{z})\right]$$

non-linear term



recall $\quad \mathbf{T}(\mathbf{r}-\bar{\mathbf{r}}) \equiv 1-\frac{1}{N_c}\mathrm{Tr}\,\mathbf{V}(\mathbf{r_t})\,\mathbf{V}^\dagger(\bar{\mathbf{r}}_\mathbf{t})$

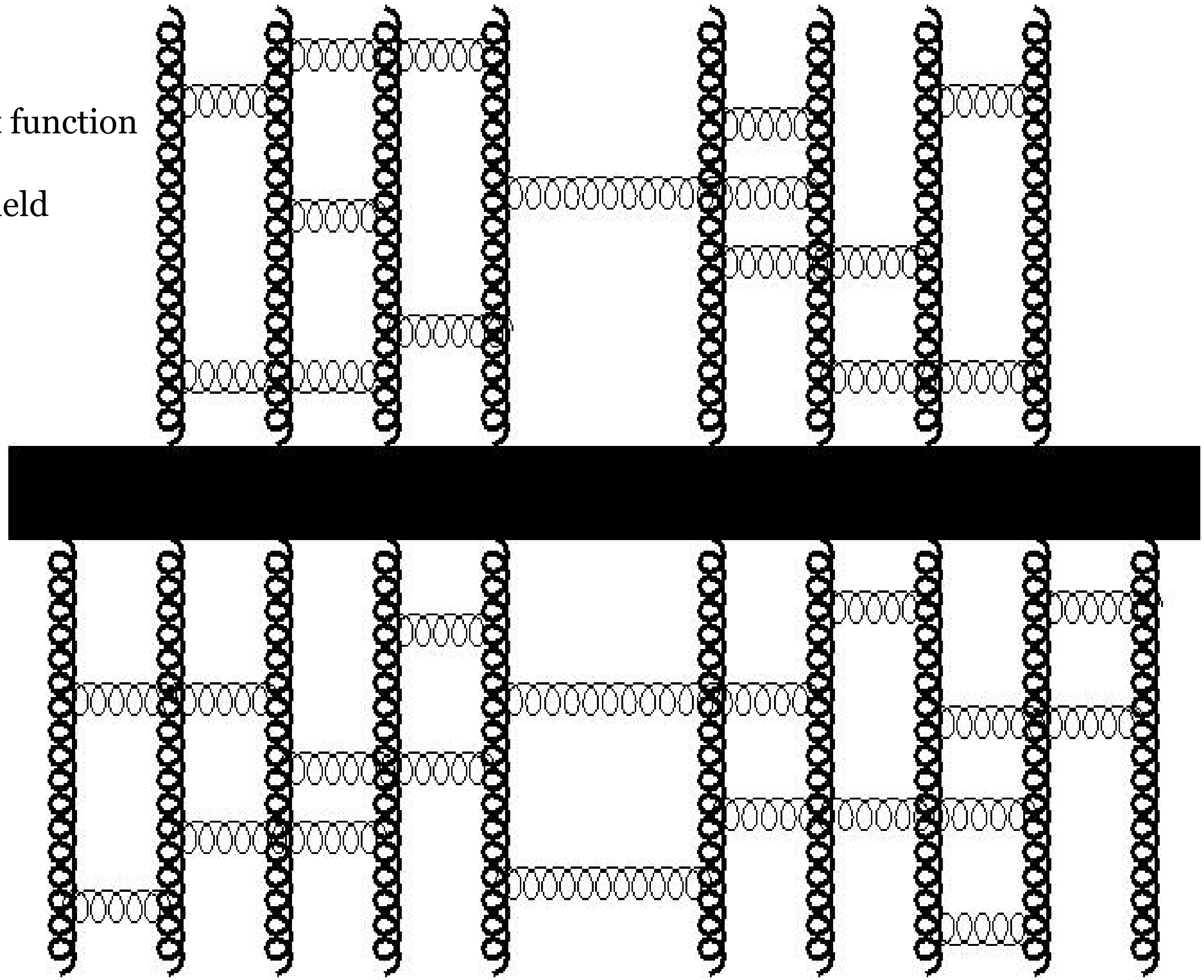
with $\quad \mathbf{V}(\mathbf{r_t}) = \hat{\mathbf{P}}\exp\left\{-ig\int_{-\infty}^{\infty}dz^-\,\alpha(z^-,\mathbf{r_t})\right\}$

Triple pomeron-vertex from O (α^4) terms in the expansion of the non-linear term

Non-linear regime: n ----> n+1 pomeron vertex

A. Ayala, L. Hernadez, J. Jalilian-Marian, M.E. Tejeda-Yeomans, in progress

Follow the same strategy:
- Evolution equation for 2n-pt function
- Keep the non-linear terms
- Expand in powers of gluon field
- Extract the vertex



First case:
2 ---> 3 vertex
(first high density correction)

(weak coupling) # QCD at high energy

Two distinct approaches:

1) CGC

McLerran-Venugopalan effective action
JIMWLK-BK evolution

2) Reggeized-gluon exchange

BJKP equation
triple,... pomeron vertex

Conjecture: CGC contains BJKP + multi-pomeron vertices

Goal: <u>hard diffraction in pp</u>

The role of initial conditions

McLerran-Venugopalan (93) $$< \mathbf{O}(\rho) > \equiv \int \mathbf{D}[\rho]\, \mathbf{O}(\rho)\, \mathbf{W}[\rho]$$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}}} \qquad \mu^{\mathbf{2}} \equiv \frac{\mathbf{g^2\, A}}{\mathbf{S}_\perp}$$

$$\mathbf{T(r_t)} \equiv \frac{\mathbf{1}}{\mathbf{N_c}} < \mathbf{Tr}\,[\mathbf{1} - \mathbf{V(r_t)^\dagger\, V(0)}] > \sim \mathbf{1 - e^{-[r_t^2\, Q_s^2]^\gamma \log(e + \frac{1}{r_t\, \Lambda_{QCD}})}}$$

with $\gamma = \mathbf{1.119}$

how about higher order terms in ρ ?

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int \mathbf{d^2 x_t} \left[\frac{\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{a}}(\mathbf{x_t})}{\mathbf{2}\,\mu^{\mathbf{2}}} - \frac{\mathbf{d^{abc}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})}{\kappa_{\mathbf{3}}} + \frac{\mathbf{F^{abcd}}\,\rho^{\mathbf{a}}(\mathbf{x_t})\rho^{\mathbf{b}}(\mathbf{x_t})\rho^{\mathbf{c}}(\mathbf{x_t})\rho^{\mathbf{d}}(\mathbf{x_t})}{\kappa_{\mathbf{4}}}\right]}$$

these higher order terms make the single inclusive spectra steeper and give <u>leading N_c</u> correlations (ridge)

Dumitru-Jalilian-Marian-Petreska, PRD84 (2011) 014018
Dumitru-Petreska, NPA9 (2012) 59