

Phenomenology of hadrons in the eLSM

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in collaboration with

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Outline

Development of the eLSM: general considerations

Meson sector

Baryon sector

Nonzero density

Development of the eLSM: general considerations

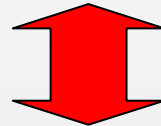
Objectives

- Development of a chirally symmetric model for mesons and baryons **including (axial-)vector d.o.f.**

‘Extended Linear Sigma Model (eLSM)’

- Study of the model for $T = \mu = 0$ (spectroscopy in vacuum)

(Masses, decay, scattering lengths,...)



Interrelation between these two aspects!

- Second goal: properties at nonzero T and μ

(condensates and masses in thermal/matter medium,...)

Fields of the model

- Quark-antiquark mesons: **scalar**, pseudoscalar, vector and axial-vector quarkonia.
- Additional mesons: The scalar and the pseudoscalar glueballs
- Baryons: nucleon doublet and its partner
(in the so-called mirror assignment)

Criteria for the construction of the model

We construct the Lagrangian of the so-called Extended Linear Sigma Model (ELSM) according to:

chiral invariance

and

dilatation symmetry .

The breaking of the dilatation symmetry is only included in the „gluonic part“...(scalar glueball and axial anomaly)

Moreover, invariance under **C** and **P** is also taken into account.

These are basic properties of QCD:

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$

Development of the eLSM ($N_f = 3$): mesons

(Pseudo)scalar sector

9 pseudoscalar fields: $L = S = 0 \rightarrow J^{PC} = 0^{-+}$

$$P = P_a \lambda^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = i\gamma^5$$

$$\pi^+ \equiv u\bar{d}$$

$$K^+ \equiv u\bar{s}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_\eta & \sin \theta_\eta \\ -\sin \theta_\eta & \cos \theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S \equiv \bar{s}s \end{pmatrix}$$

$$-36^\circ < \theta_\eta < -45^\circ$$

...and 9 scalar fields: $L = S = 1 \rightarrow J^{PC} = 0^{++}$

$$S = S_a \lambda^a = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = 1$$

$$a_0^+ = a_0(1450) \equiv u\bar{d} \quad \text{and not } a_0(980)!!!$$

$$K_S^+ = K_0^{*+}(1430) \equiv u\bar{s} \quad \text{and not } k(800)!!!$$

$$\sigma_N \equiv \sqrt{1/2}(u\bar{u} + d\bar{d}) \approx f_0(1370)$$

and not $f_0(500)!!!$

$$\sigma_S \equiv u\bar{s} \approx f_0(1500) \text{ or } f_0(1710)$$

and not $f_0(980)!!!$

$$\Phi = S + iP$$

$$\Phi \rightarrow U_L \Phi U_R^+$$

(Axial-)Vector sector

9 vector fields... $L = 0, S = 1 \rightarrow J^{PC} = 1^{--}$

$$V^\mu = V^\mu_a \lambda^a = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & \rho^+ & K_*(892)^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_N}{\sqrt{2}} & K_*(892)^0 \\ K_*(892)^- & \bar{K}_*(892)^0 & \phi_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \Gamma = \gamma^\mu$$

$$\rho^+ \equiv u\bar{d}, \dots$$

$$K_*^+(892) \equiv u\bar{s}$$

$$\omega \approx \omega_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

$$\phi \approx \phi_S \equiv \bar{s}s$$

...and 9 axial-vector fields... $L = S = 1 \rightarrow J^{PC} = 1^{++}$

$$A^\mu = A^\mu_a \lambda^a = \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_{1,N}}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_{1,N}}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1,S} \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix} \quad \Gamma = \gamma^\mu \gamma^5$$

$$a_1^+ = a_1^+(1260) \equiv u\bar{d}$$

$$K_1^+ = K_1^+(1270) \equiv u\bar{s}$$

$$f_1(1285) \approx f_{1,N} \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

$$f_1(1510) \approx f_{1,S} \equiv \bar{s}s$$

$$\begin{aligned} L^\mu &= V^\mu + A^\mu & R^\mu &\rightarrow U_R R^\mu U_R^+ \\ R^\mu &= V^\mu - A^\mu & L^\mu &\rightarrow U_L L^\mu U_L^+ \end{aligned}$$

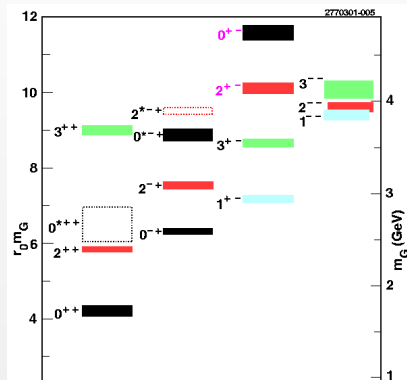
Meson sector: how many fields do we have?

36 + 2 fields



G : Scalar glueball (trace anomaly)

\tilde{G} : Pseudoscalar glueball (axial anomaly)



Morningstar (1999)

For $N_f = 3$ there are 38 mesons
36 quark-antiquark fields + 2 glueballs

Model of QCD – eLSM

$$\begin{aligned}
 \mathcal{L} = & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right) + \text{Tr} [(D^\mu \Phi)^\dagger (D_\mu \Phi)] \\
 & - m_0^2 \left(\frac{G}{G_0} \right)^2 \text{Tr} [\Phi^\dagger \Phi] - \lambda_1 (\text{Tr} [\Phi^\dagger \Phi])^2 - \lambda_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \\
 & + \left(\frac{G}{G_0} \right)^2 \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) ((L^\mu)^2 + (R^\mu)^2) \right] \\
 & - \frac{1}{4} \text{Tr} [(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \text{Tr} [H (\Phi^\dagger + \Phi)] \\
 & + c_1 [\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2} \text{Tr} [\Phi^\dagger \Phi] \text{Tr} [L_\mu L^\mu + R_\mu R^\mu] \\
 & + h_2 \text{Tr} [\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr} [\Phi R_\mu \Phi^\dagger L^\mu] + \dots
 \end{aligned}$$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$L^\mu, R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^\pm \pm a_1^\pm & K^{*+} \pm K_1^+ \\ \rho^\mp \pm a_1^\mp & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{*0} \pm K_1^0 \\ K^{*-} \pm K_1^- & \bar{K}^{*0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011)** arXiv: 1103.3238

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011** arXiv:1208.0585

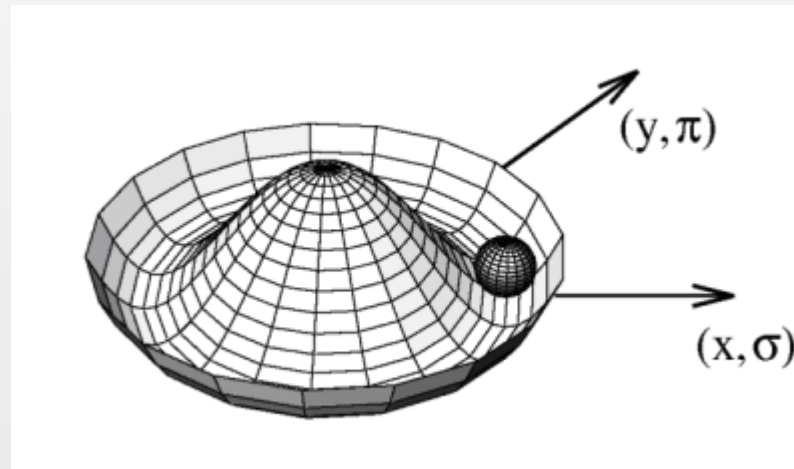
Basic feature

$$V = \frac{m_0^2}{2} (\sigma^2 + \pi^2) + \frac{\lambda_1 + \lambda_2}{4} (\sigma^2 + \pi^2)^2$$

$m_0^2 < 0 \rightarrow$ Mexican hat

$\pi =$ neutral pion

Spontaneous Symmetry Breaking (SSB): $\sigma = \sigma_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \equiv f_0(1370)$
...and not to $f_0(500)$...

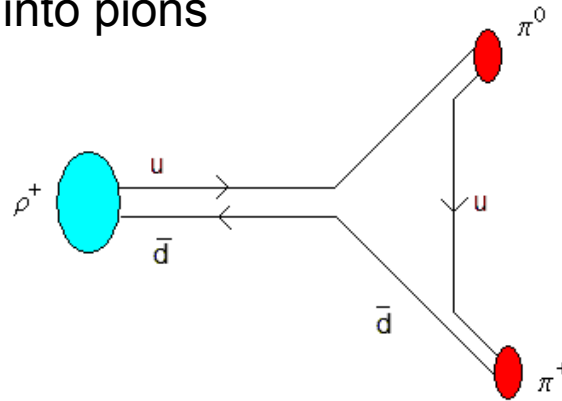


Because of dilatation invariance: only a finite number of terms is present!

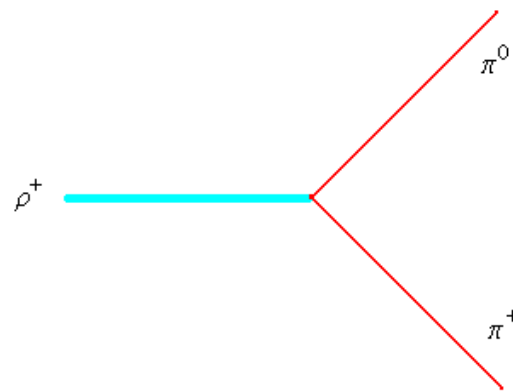
We calculate: masses, decays, and scattering lengths.

Example: ρ -meson decay into pions

Microscopic



eLSM



Results of the fit (11 parameters, 21 exp. quantities)

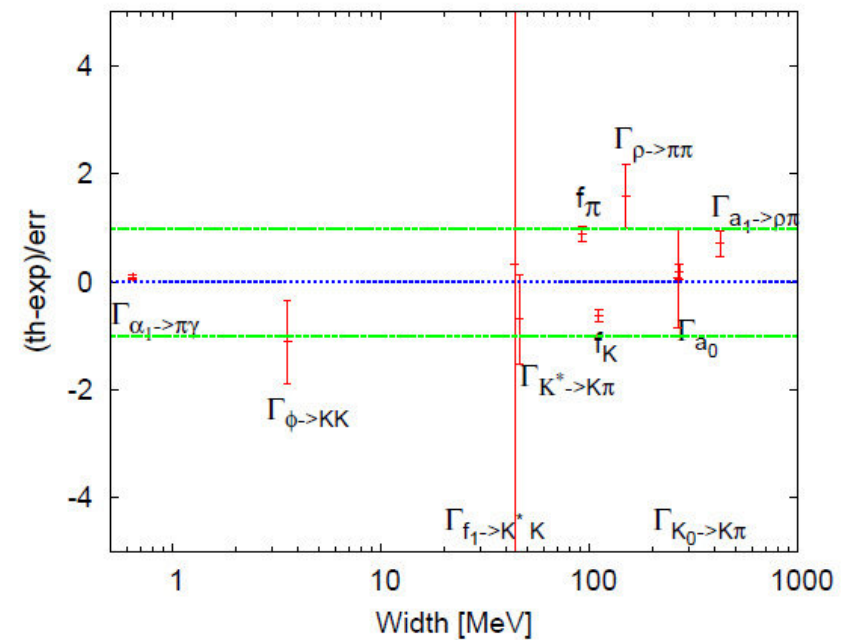
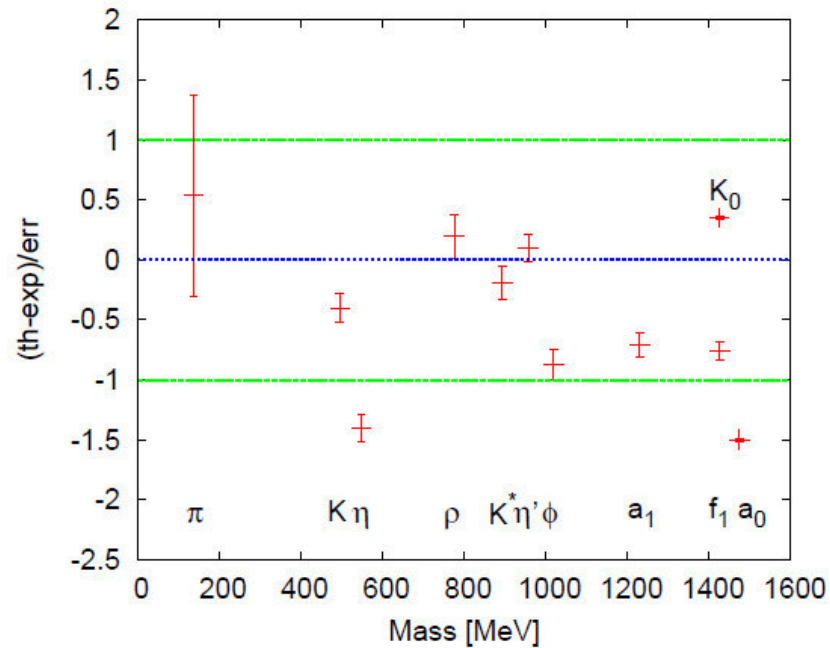
Error from PDG or 5% of exp.
Scalar-isoscalar sector not
included.

$$\chi_{red}^2 = 1.2$$

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
m_{K^*}	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^*}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho \rightarrow \pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^* \rightarrow K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi \rightarrow \bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1 \rightarrow \rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1 \rightarrow \pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420) \rightarrow K^*K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^* \rightarrow K\pi}$	285 ± 12	270 ± 80

arXiv:1208.0585

Results of the fit: pictorial representation



arXiv:1208.0585

Overall phenomenology is good.

Scalar mesons $a_0(1450)$ and $K_0(1430)$ above 1 GeV and are quark-antiquark states.

Importance of the (axial-)vector mesons

There are many consequences of the fit.
Example: $a_0(1450)$

Theory

$$\frac{\Gamma_{a_0 \rightarrow \eta' \pi}}{\Gamma_{a_0 \rightarrow \eta \pi}} = 0.19 \pm 0.02, \quad \frac{\Gamma_{a_0 \rightarrow K K}}{\Gamma_{a_0 \rightarrow \eta \pi}} = 1.12 \pm 0.07$$

Exp (PDG)

$$\frac{\Gamma_{a_0(1450) \rightarrow \eta' \pi}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}} = 0.35 \pm 0.16, \quad \frac{\Gamma_{a_0(1450) \rightarrow K K}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}} = 0.88 \pm 0.23 .$$

An important ongoing work: where is the scalar glueball?

The calculation of the full mixing problem in the $I=J=0$ sector is ongoing:

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = B \begin{pmatrix} \sigma_N \equiv \bar{nn} = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ G \equiv gg \\ \sigma_S \equiv \bar{ss} \end{pmatrix}$$

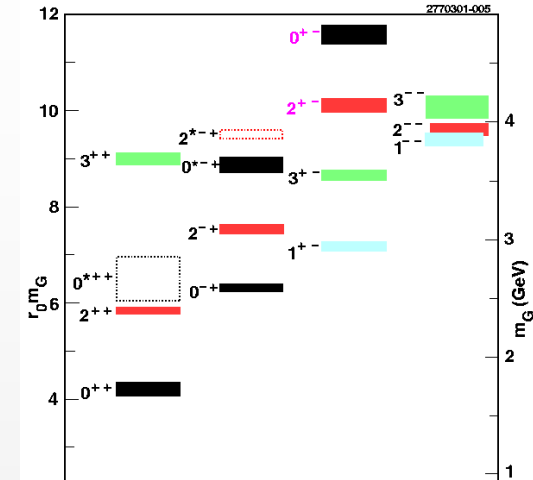
where B is a 3×3 orthogonal matrix

A new entry: the pseudoscalar glueball

$$\mathcal{L}_{\tilde{G}\text{-mesons}}^{int} = ic_{\tilde{G}\Phi} \tilde{G} (\det\Phi - \det\Phi^\dagger) \quad M_{\tilde{G}} \approx 2.6 \text{ GeV}$$

Quantity	Value
$\Gamma_{\tilde{G} \rightarrow KK\eta} / \Gamma_{\tilde{G}}^{tot}$	0.049
$\Gamma_{\tilde{G} \rightarrow KK\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.019
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta} / \Gamma_{\tilde{G}}^{tot}$	0.016
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.0017
$\Gamma_{\tilde{G} \rightarrow \eta\eta'\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.00013
$\Gamma_{\tilde{G} \rightarrow KK\pi} / \Gamma_{\tilde{G}}^{tot}$	0.46
$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.16
$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.094

Quantity	Value
$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G}}^{tot}$	0.059
$\Gamma_{\tilde{G} \rightarrow a_0\pi} / \Gamma_{\tilde{G}}^{tot}$	0.083
$\Gamma_{\tilde{G} \rightarrow \eta\sigma_N} / \Gamma_{\tilde{G}}^{tot}$	0.028
$\Gamma_{\tilde{G} \rightarrow \eta\sigma_S} / \Gamma_{\tilde{G}}^{tot}$	0.012
$\Gamma_{\tilde{G} \rightarrow \eta'\sigma_N} / \Gamma_{\tilde{G}}^{tot}$	0.019



$$\Gamma_{\tilde{G} \rightarrow \pi\pi\pi} = 0$$

PANDA/FAIR will be able to scan the energy above 2.5 GeV

Details in:

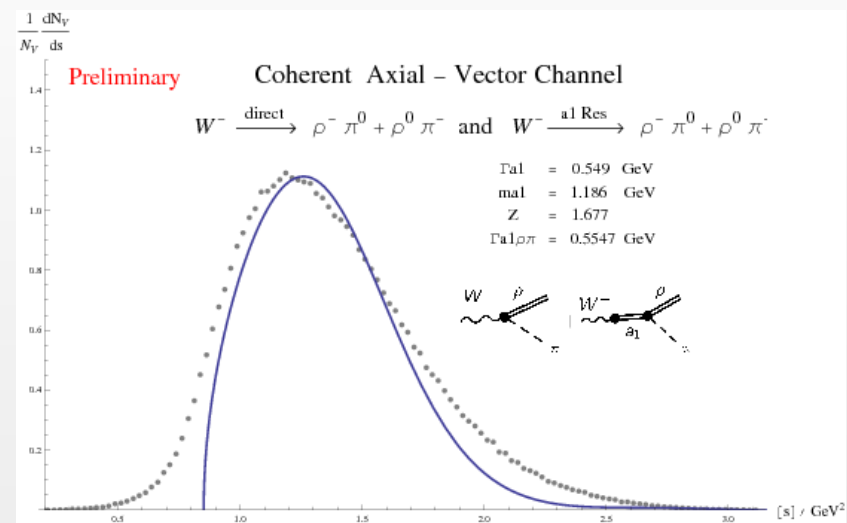
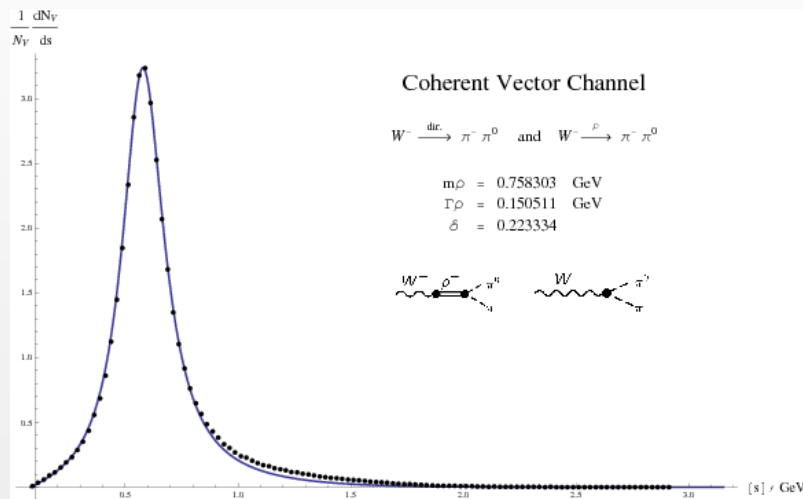
W. Eshraim, S. Janowski, F.G., D. Rischke, **Phys.Rev. D87 (2013) 054036**. [arxiv: 1208.6474](#) .

W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Prc. Suppl. 5/4, [arxiv: 1209.3976](#)

The weak tau-decay into mesons

$$\tau \rightarrow W^- \nu_\tau \rightarrow \pi\pi\nu_\tau$$

$$\tau \rightarrow W^- \nu_\tau \rightarrow \pi\pi\pi\nu_\tau$$



Thanks to Anja Habersetzer

Development of a hadronic model ($N_f = 2$): baryons

Baryon sector in the EISM

($N_f = 2$ only)

Nucleon and its chiral partner; chiral symmetry and dilatation invariance
(Axial-)vector mesons are included

Mirror assignment: C. De Tar and T. Kunihiro, **PRD 39 (1989) 2805**)

$$\begin{array}{ll} \Psi_{1,R} \rightarrow U_R \Psi_{1,R} & \Psi_{1,L} \rightarrow U_L \Psi_{1,L} \\ \Psi_{2,R} \rightarrow U_L \Psi_{2,R} & \Psi_{2,L} \rightarrow U_R \Psi_{2,L} \end{array}$$

A chirally invariant mass-term is possible!

$$m_0 \left(\bar{\Psi}_{1,L} \Psi_{2,R} - \bar{\Psi}_{1,R} \Psi_{2,L} - \bar{\Psi}_{2,L} \Psi_{1,R} + \bar{\Psi}_{2,R} \Psi_{1,L} \right)$$

Lagrangian in the baryon sector

Interaction of baryons with (pseudo)scalar and (axial-)vector mesons

$$\mathcal{L}_{mirror} = \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) + \mathcal{L}_{mass}$$

$$D_{1R}^\mu = \partial^\mu - ic_1 R^\mu, D_{1L}^\mu = \partial^\mu - ic_1 L^\mu$$

$$D_{2R}^\mu = \partial^\mu - ic_2 R^\mu, D_{2L}^\mu = \partial^\mu - ic_2 L^\mu$$

$$\mathcal{L}_{mass} = -m_0 (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L})$$

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \frac{1}{\sqrt{2 \cosh \delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \quad \delta = ar \cosh \left[\frac{M_N + M_{N^*}}{2m_0} \right]$$

$$N = N(940)$$

$$N^* = N^*(1535)$$

Mass of the nucleon

$$m_{N,N^*} = \sqrt{m_0^2 + \left(\frac{\hat{g}_1 + \hat{g}_2}{4}\right)^2 \phi^2} \pm \frac{(\hat{g}_1 - \hat{g}_2)\phi}{4}$$

$N = N(940)$
 $N^* = N^*(1535)$

If $m_0 = 0 \rightarrow$ only the quark condensate generates the masses. $m_N \sim \phi$

$$m_0 = 460 \pm 136 \text{ MeV}$$

Using $g_A^N = 1.26$ (exp), $g_A^{N^*} \approx 0.2$ (latt) and $\Gamma_{N^* \rightarrow N\pi} \approx 67 \text{ MeV}$

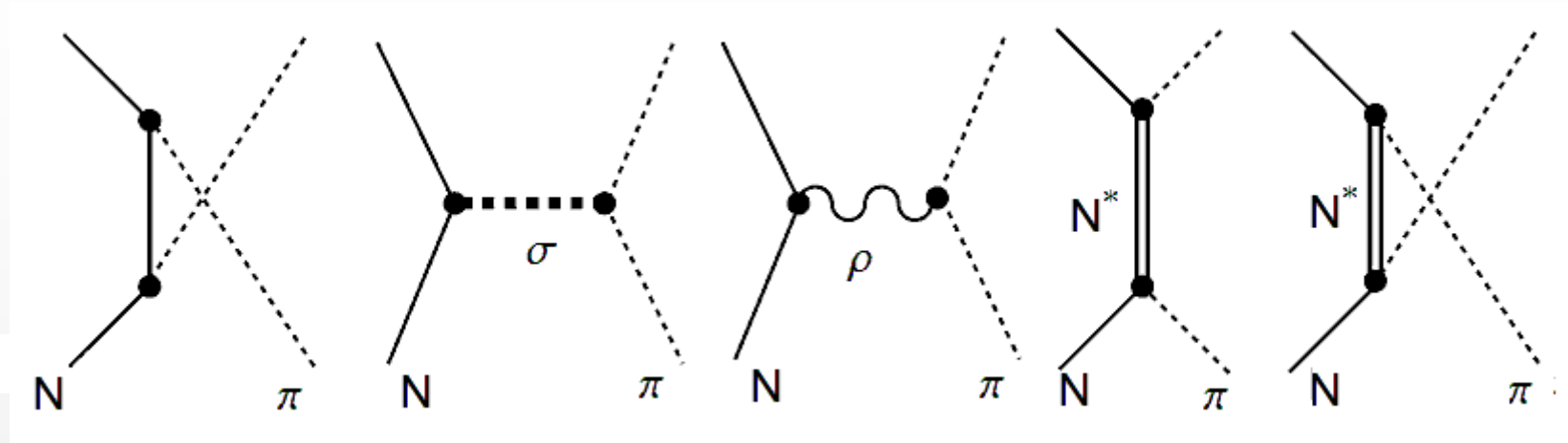
Details in S. Gallas, F. G., D. H. Rischke, **Phys.Rev. D82 (2010) 014004**, [arXiv:0907.5084](https://arxiv.org/abs/0907.5084)

m_0 parameterizes the contribution which does not stem from the quark condensate

Crucial also at nonzero temperature and density

also in the so-called quarkyonic phase: L. McLerran, R. Pisarski **Nucl.Phys.A796:83-100,2007**

Test: pion-nucleon scattering lengths



$$a_0^- = (6.04 \pm 0.63) \cdot 10^{-4} \text{ MeV}^{-1} \quad a_0^{-(\text{exp})} = (6.4 \pm 0.1) \cdot 10^{-4} \text{ MeV}^{-1}$$

$$a_0^+ \approx (\text{from } -20 \text{ to } +20 \cdot 10^{-4}) \text{ MeV}^{-1} \quad a_0^{+(\text{exp})} = (-8.8 \pm 7.2) \cdot 10^{-4} \text{ MeV}^{-1}$$

Large theoretical uncertainty due to the scalar-isoscalar sector

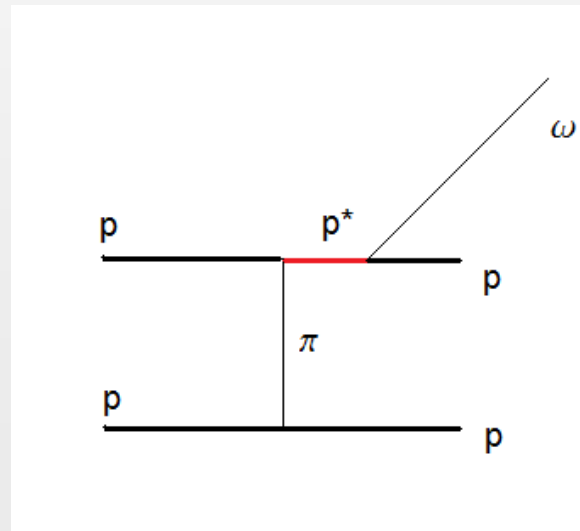
Importance of both vector mesons and mirror assignment in order to get these results

What we are studying right now...

$$p + p \rightarrow p + p + X$$

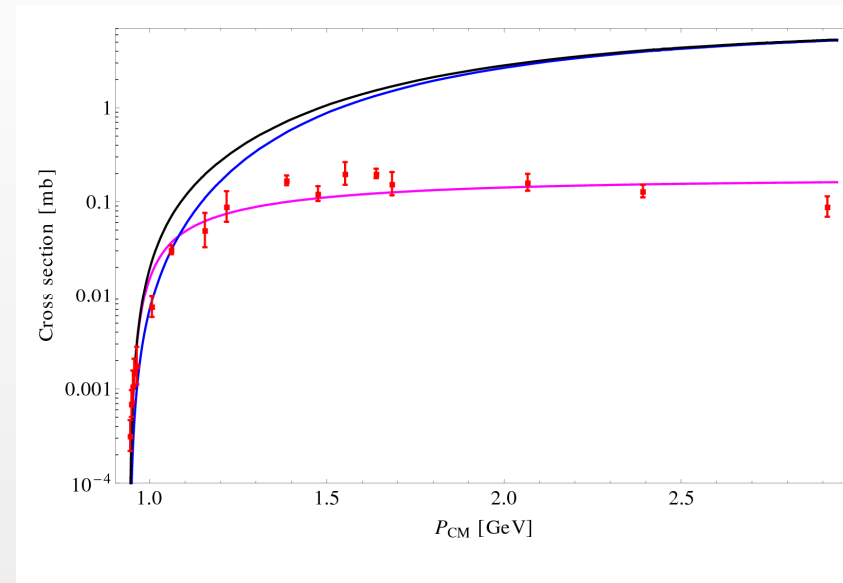
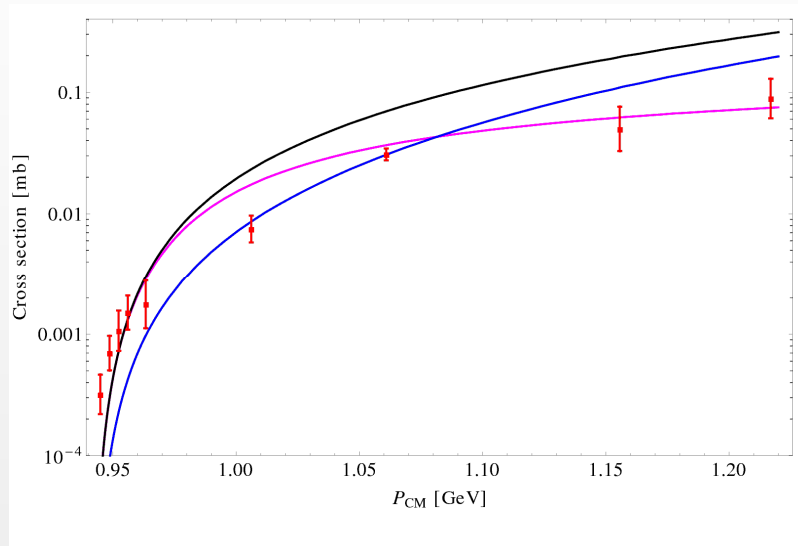
$$X = \omega, \eta, \text{ lepton pair, ...}$$

Many diagrams to calculate; the advantage is : chiral symmetry (and also g.i.) built in.



$$p + p \rightarrow p + p + \omega$$

Preliminary!



Thanks to Dr. Khaled Teilab.

Results at nonzero density

Basic considerations for nonzero density

The σ -field of our model corresponds to the resonance $f_0(1370)$
...and not to the lightest scalar resonance $f_0(500)$.

The question is: what is $f_0(500)$ and, more in general, what are
the scalar states below 1 GeV?

A good phenomenology (masses and decays) is achieved when
interpreting the light scalar states as tetraquarks: $f_0(500) \approx [\bar{u}, \bar{d}][u, d]$
(bound states of a diquark and an anti-diquark)

Details in: F.G, Phys.Rev. D **75** (2007) 054007

Back to nucleons: where does m_0 comes from?

$$m_0 \left(\bar{\Psi}_{1,L} \Psi_{2,R} - \bar{\Psi}_{1,R} \Psi_{2,L} - \bar{\Psi}_{2,L} \Psi_{1,R} + \bar{\Psi}_{2,R} \Psi_{1,L} \right)$$

By requiring dilatation invariance one should modify the mass-term as:

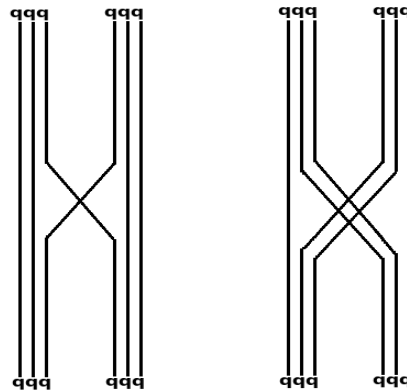
$$a\chi \left(\bar{\Psi}_{1,L} \Psi_{2,R} - \bar{\Psi}_{1,R} \Psi_{2,L} - \bar{\Psi}_{2,L} \Psi_{1,R} + \bar{\Psi}_{2,R} \Psi_{1,L} \right)$$

Tetraquark
New field: $f_0(500)$

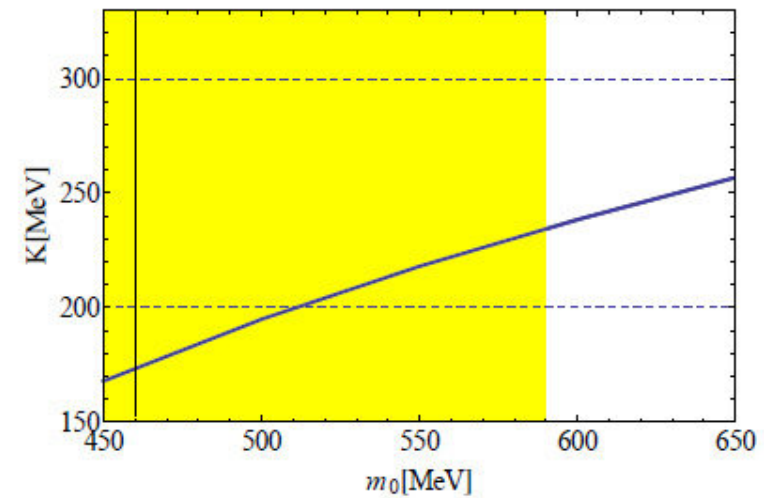
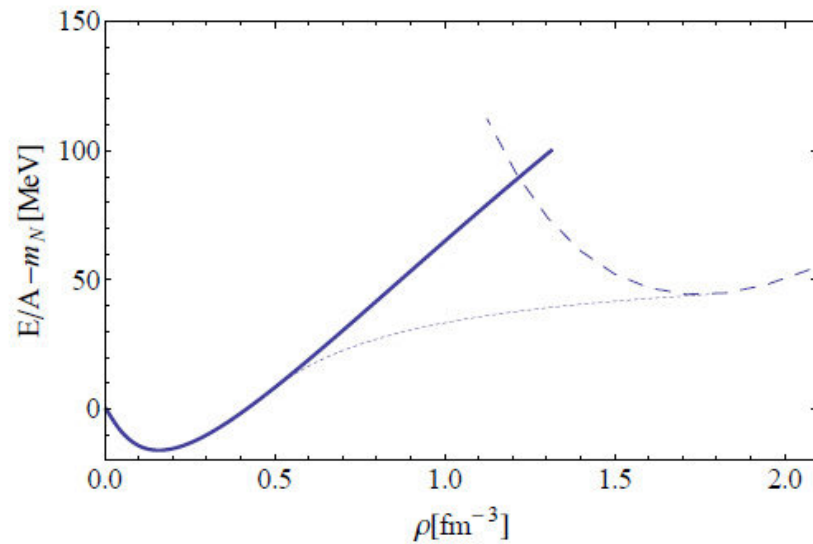
By shifting : $\chi \rightarrow \chi_0 + \chi$ one has : $m_0 = a\chi_0$

m_0 originates form the tetraquark condensate

Note, also, a tetraquark exchange naturally arises in nucleon-nucleon interactions



Nuclear matter saturation and compressibility



Details in: S. Gallas, F. G., G. Pagliara, **Nucl.Phys. A872 (2011) 13-24** [arXiv:1105.5003](https://arxiv.org/abs/1105.5003)

Nuclear matter: why does it bind?

The resonance $f_0(500)$, here interpreted as a tetraquark, plays an important role for the stability of nuclear matter.

Related 'amusing' question: does nuclear matter binds at large N_c ?

As soon as the lightest scalar $f_0(500)$ is not a quarkonium, nuclear matter ceases to exist already for $N_c=4$.

Details in: L. Bonanno and F.G., **Nucl.Phys.A859:49-62,2011** [arXiv:1102.3367](https://arxiv.org/abs/1102.3367) [hep-ph]

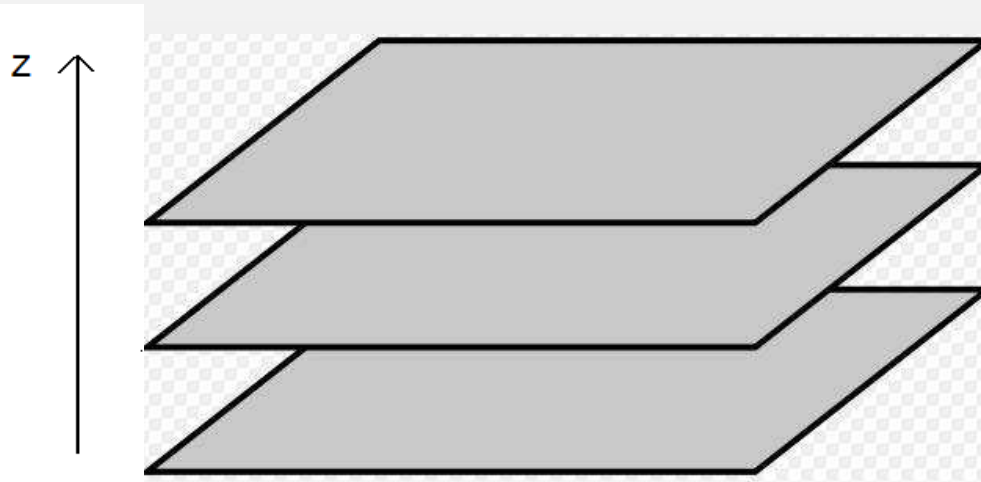
Inhomogeneous condensation at nonzero density

Up to now : $\phi = const$

...but one can have a Chiral Density Wave:

$$\phi(z) = \phi \cos(2 fz)$$

$$\langle \pi^0 \rangle = \phi \sin(2 fz) / Z$$

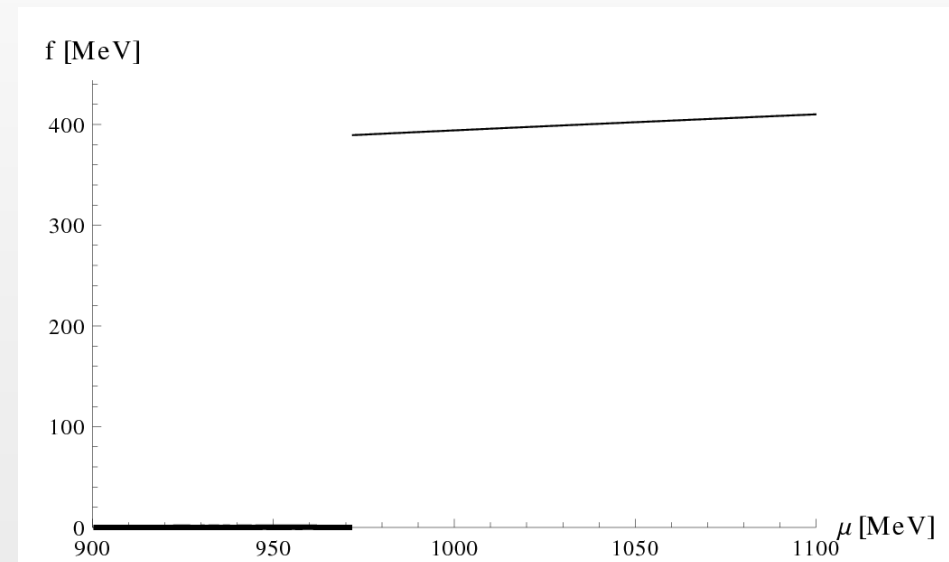
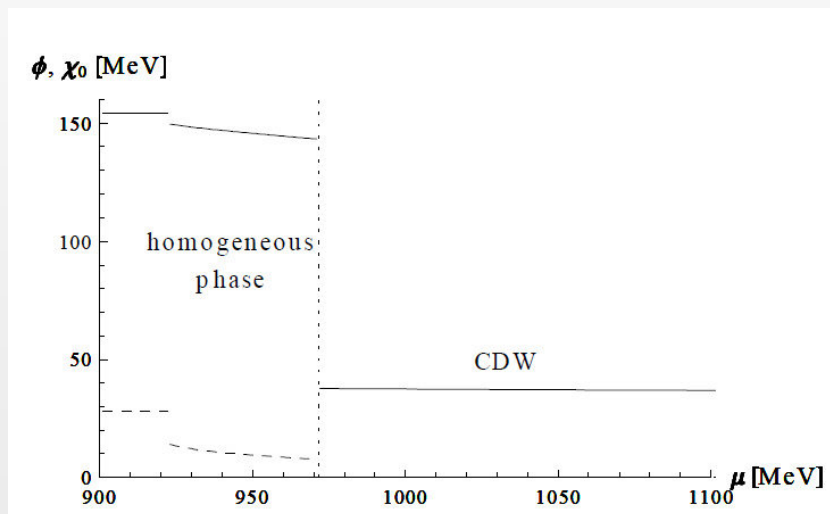


Inhomogeneous condensation/2

$$\phi(z) = \langle \sigma \rangle = \phi \cos(2 fz)$$

$$\langle \pi^0 \rangle = \phi \sin(2 fz) / Z$$

$$m_0 = 460 \text{ MeV}$$



$$\rho_{CDW} / \rho_0 = 2.4$$

A. Heinz, F.G., D. H. Rischke, in preparation.

Summary

Summary

Hadronic Theory (eLSM) based on chiral symmetry and **dilatation invariance**

Important role of (axial-)vector mesons in all phenomenology

Scalar quarkonia and glueball above 1 GeV (effects in the medium)

Nucleon mass contribution which does not stem from the chiral condensate (but from the tetraquark and glueball condensates)

Ongoing works: $N_f = 4$, additional tetraquark states, weak decays, proton-proton scattering, unitarization (loops)...

Planned: nonzero temperature

Thank You

Tetraquark: outlook and short excursus at nonzero T

A possibility is to interpret the light scalar states below 1 GeV
[$f_0(600)$, $k(800)$, $f_0(980)$ and $a_0(980)$]
as diquark-antidiquark objects: these are the Jaffe's tetraquarks.

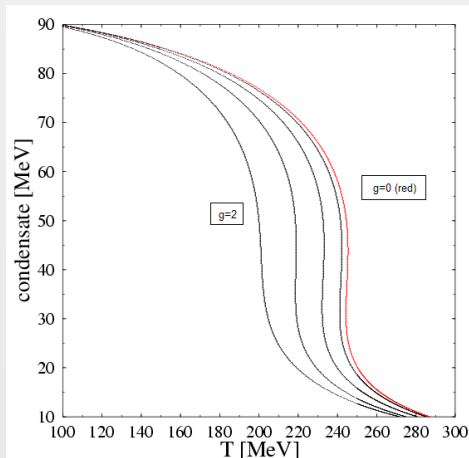
The $N_f=3$ case is an outlook. Mixing of these tetraquark-quarkonia takes place.

Black et al, **Phys. Rev. D 64** (2001), F.G., **Phys.Rev.D 75**,(2007)

For $N_f=2$ only one tetraquark survives. In this case we studied a simplified system at nonzero T.

The resonance $f_0(1370) \approx \sigma \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ is the chiral partner of the pion $\vec{\pi}$.

The resonance $f_0(600) \approx \chi \equiv \frac{1}{2}[u, d][\bar{u}, \bar{d}]$ is an extra - scalar state



Increasing of mixing:

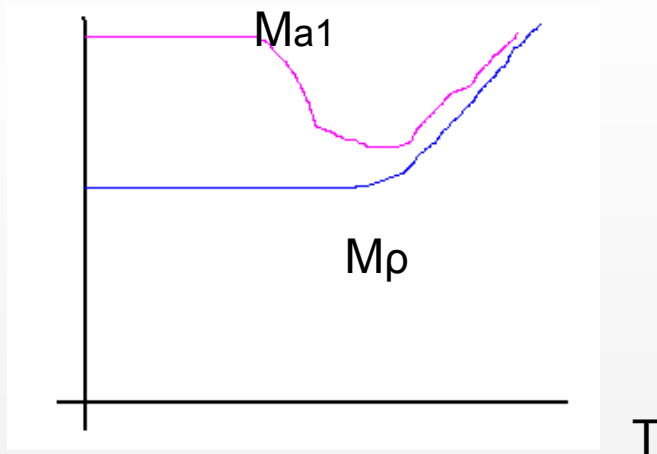
- 1) T_c decreases
- 2) First order softened
- 3) Cross-over obtained for g large enough

Achim Heinz, Stefan Strube, F.G., Dirk H. Rischke

Phys.Rev.D 79:037502,2009; arXiv:0805.1134 [hep-ph]

Digression: 3 scenarios for the ρ -meson at nonzero T

$$M_{\rho}^2 = \underbrace{\phi^2}_{\text{quark condensate}} (\dots) + \underbrace{G_0^2}_{\text{gluon condensate}} (\dots)$$

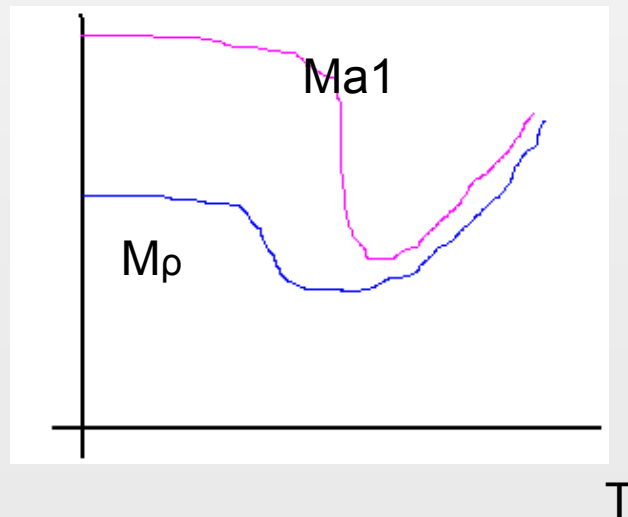


Case A: G0-term dominates

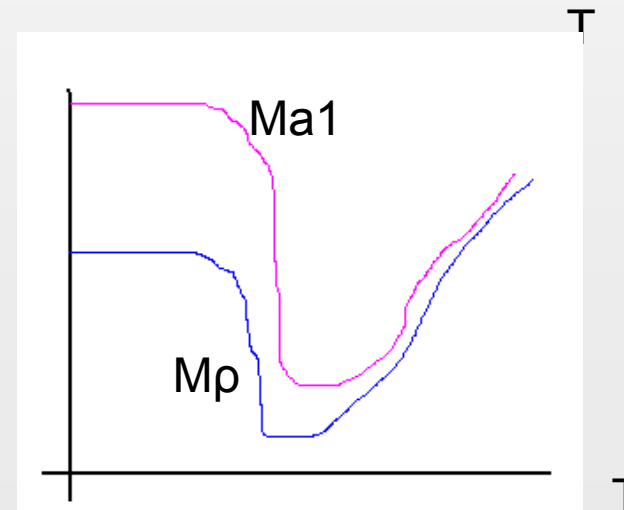


In our case: both terms comparables.

We expect case B to hold;
small drop of the masses in the medium



Case B: both terms are similar



Case C: the condensate dominates

QCD and its symmetries

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$$

SU(3)_{color}: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit.
Broken by quantum fluctuations (trace anomaly)
and by small quark masses

SU(3)_R × SU(3)_L: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is spontaneously broken to U(3)_{V=R+L}

U(1)_{A=R-L}: holds at a classical level, but is also broken by quantum fluctuations (chiral anomaly)

C and P: charge conjugation and parity: exact.

Dilaton / Scalar glueball

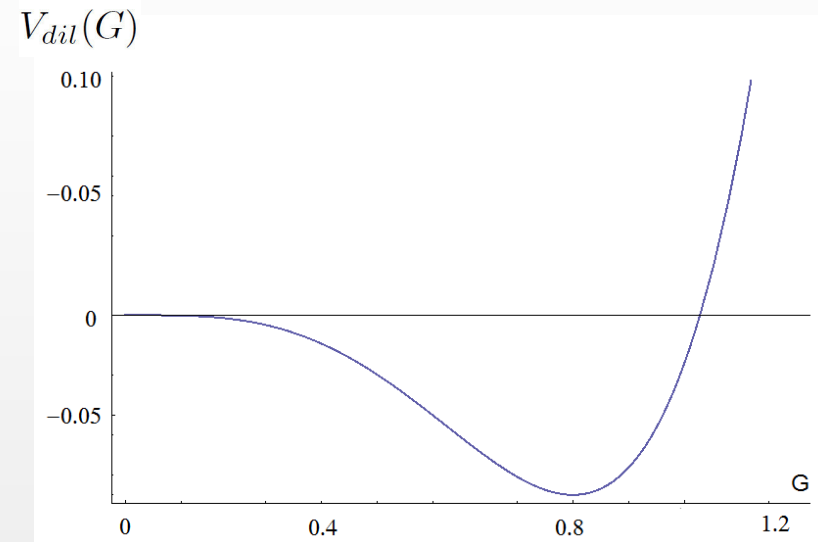
At the hadronic level, we describe these properties as:

$$G^4 \sim G_{\mu\nu}^a G^{a,\mu\nu}$$

$$\mathcal{L}_{dil} = \frac{1}{2} (\partial_\mu G)^2 - V_{dil}(G)$$

$$V_{dil}(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left[G^4 \ln \left(\frac{G}{\Lambda_G} \right) - \frac{G^4}{4} \right]$$

Λ_G dimensionful param that breaks dilatation inv!



$$\partial_\mu J^\mu = T_\mu^\mu = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4$$

In QCD it is:

$$\partial_\mu J^\mu = T_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0$$