Bourses postdoctorales Banting Postdoctoral Fellowships

McGill

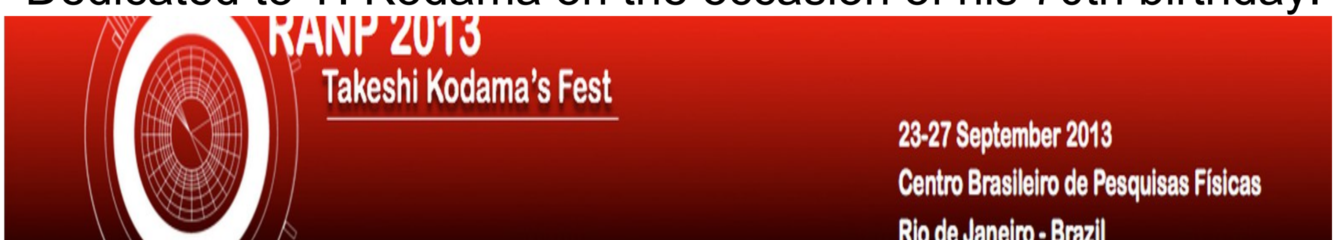
Extracting the bulk viscosity of the quark-gluon plasma

Gabriel S. Denicol

with:

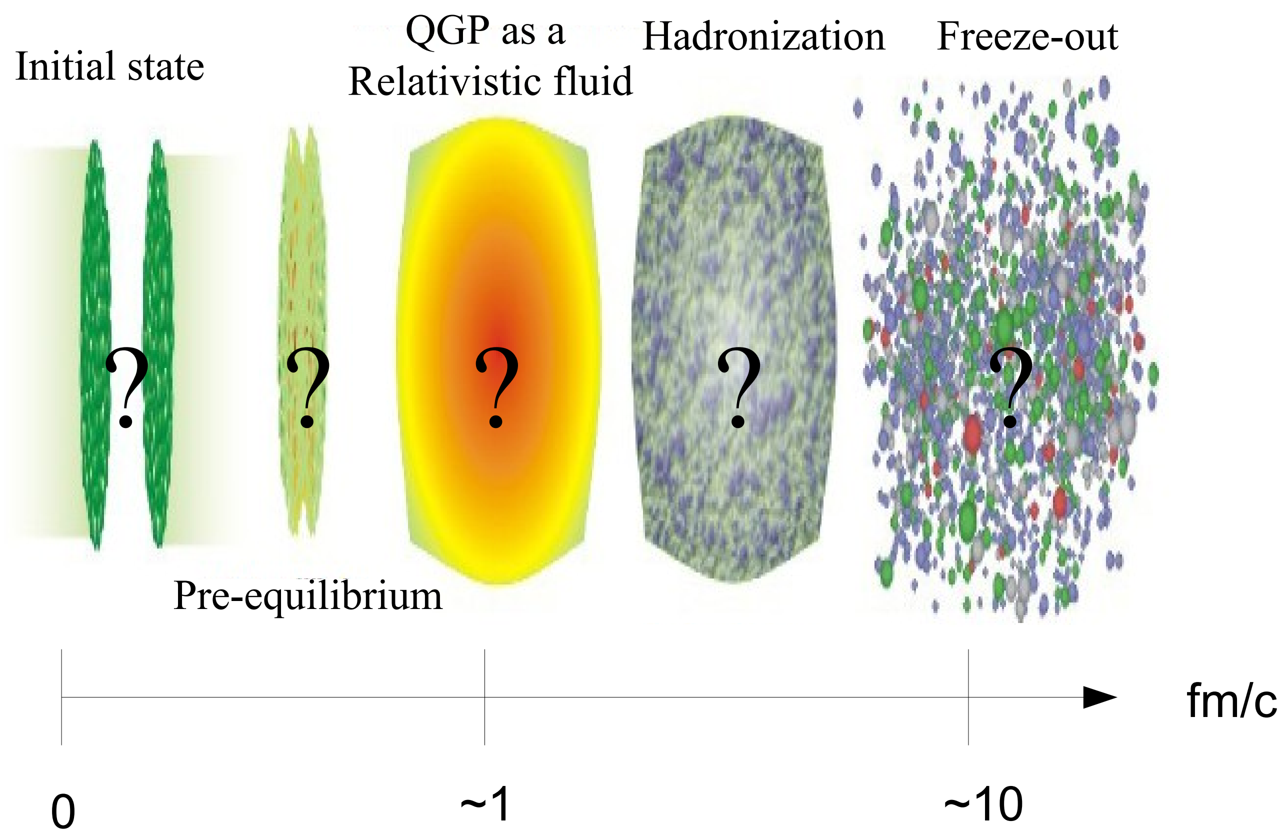
<u>J.B. Rose</u>, J.F. Paquet, I. Kozlov, M. Luzum, S. Jeon, C. Gale

Dedicated to T. Kodama on the occasion of his 70th birthday.



Heavy Ion Collisions in a Nutshell

(fluid dynamical modeling)



The Quark-Gluon Plasma (QGP) behaves as a "**nearly perfect**" fluid !

very small η/s

Basics of fluid dynamics

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

Charge conservation

$$\partial_\mu N^\mu = 0$$

$$N^\mu = nu^\mu + n^\mu,$$
$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu}(P_0 + \Pi) + \pi^{\mu\nu}$$

Particle diffusion current

Bulk viscous pressure

Shear stress tensor

Spatial projector

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

$$u_\mu u^\mu = 1$$

Always true, but not enough

What most people solve

- Most simulations neglect nonlinear terms
- Most simulations neglect bulk viscous pressure
- All simulations neglect heat flow

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta$$

Majority of conclusions of our field are based on these equations
(e.g., MUSIC 1.0, Ohio Group)

Is there any point in improving this?

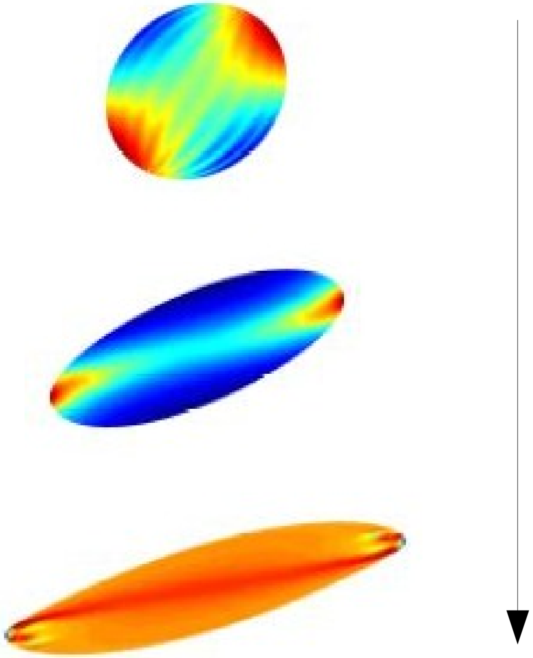
Viscous Effects

Bulk

Resistance to expansion

Shear

Resistance to deformation



We ignore this one

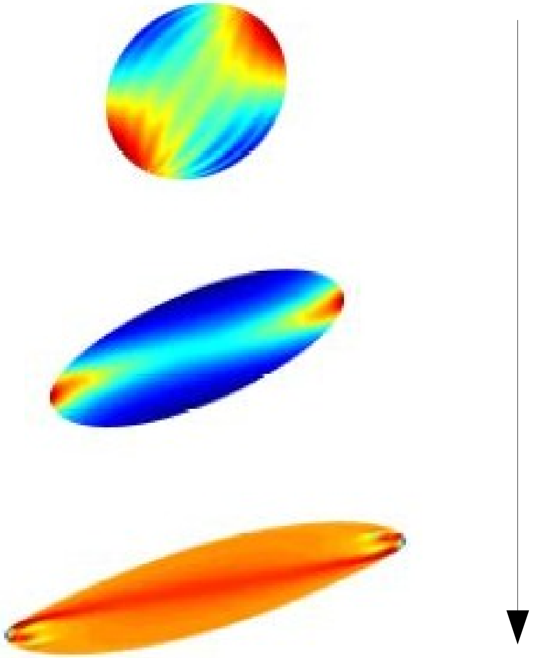
Bulk

Resistance to expansion

We like this one

Shear

Resistance to deformation

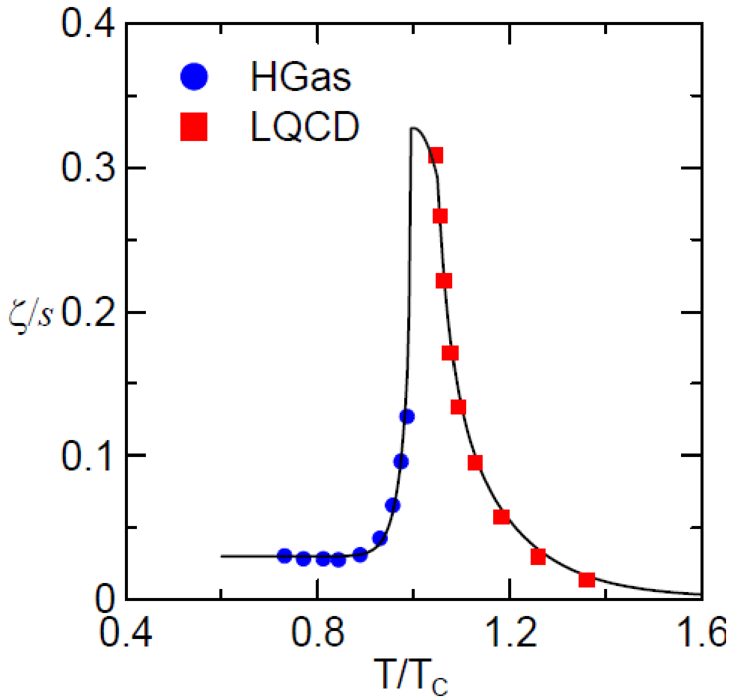


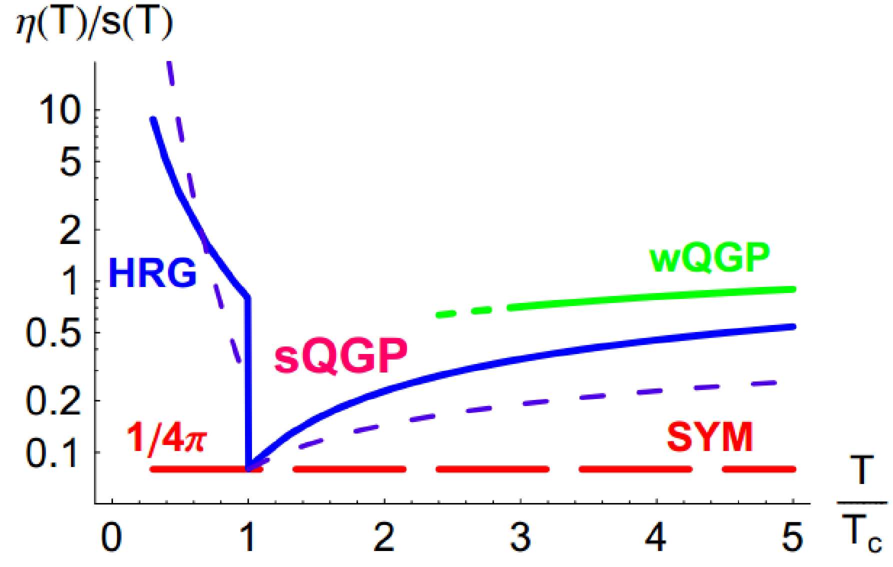
Why?

because it's small? some estimates ...

Bulk viscosity

Shear viscosity





Karsh&Kharzeev&Tuchin
Noronha&Noronha&Greiner

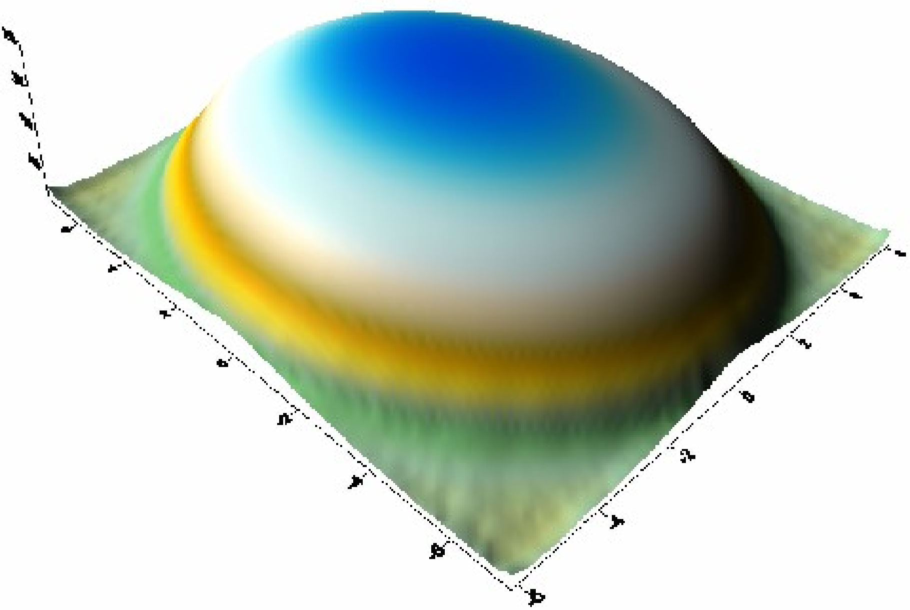
Hirano&Gyulassy

But in the region of interest, we don't really know ...

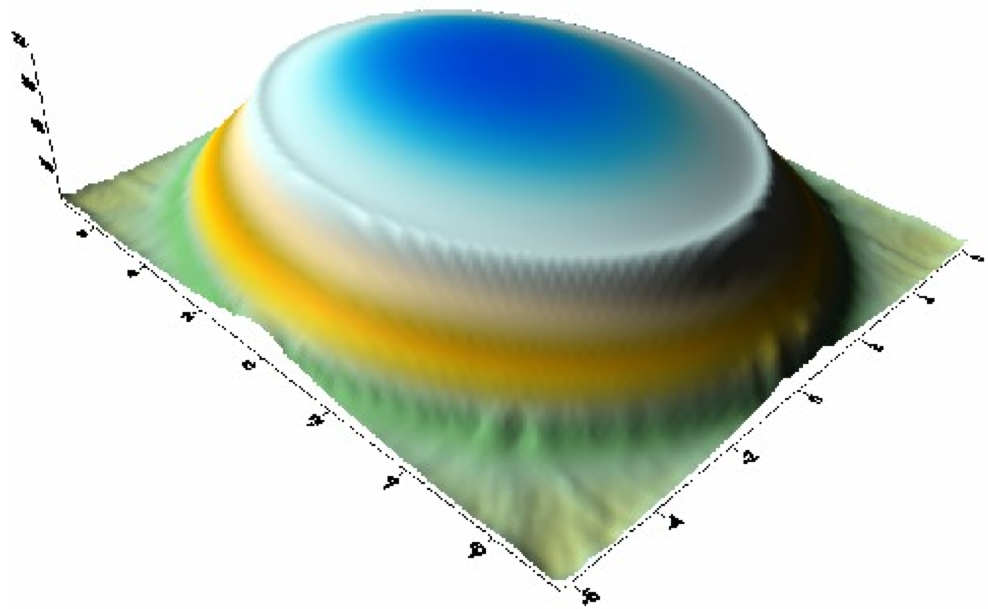
My old calculation (with Kodama)

Temperature profile (Glauber IC, τ_0=0.6fm)

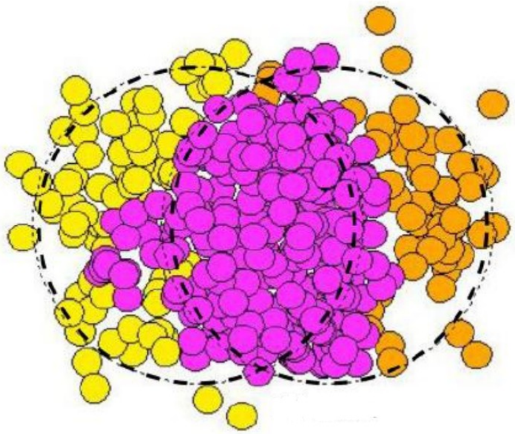
Ideal - 2.1 fm



Viscous - 2.1 fm



Also: Extreme time and spatial **scales**



~10 fm

~10 fm

➔**very small** system

➔**very large** gradients

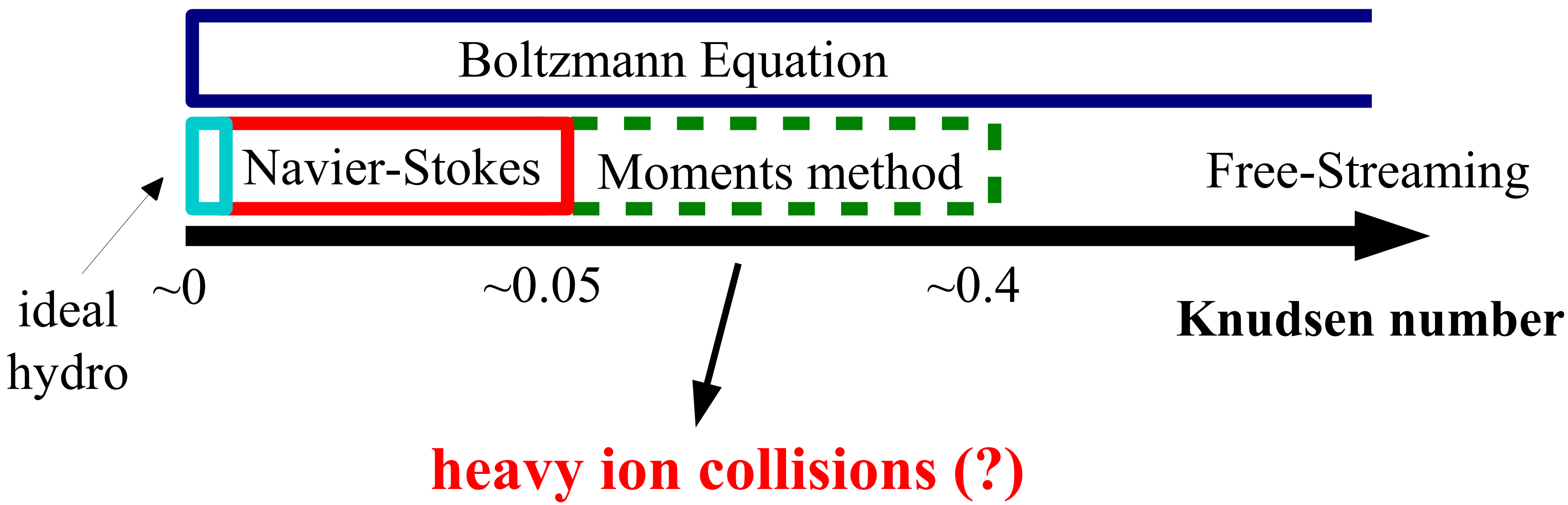
➔**very large** expansion rate

- From the fluid-dynamical point of view, challenging to describe

➔**It is possible that higher-order terms also matter**

For a dilute gas

In terms of Knudsen number $\mathrm{Kn} = \dfrac{\ell_{\mathrm{micro}}}{L_{\mathrm{macro}}}$



In a heavy ion collision ... $\mathrm{Kn} \sim \tau_\pi \nabla_\mu u^\mu \sim \dfrac{\eta}{s}\dfrac{1}{T\tau} \sim 0.2 - 1$

Higher order terms can be important …

In this talk

➔are there effects from nonlinear terms ?

➔are there effects form bulk viscosity? How they differ from shear

➔Difficulties in ultracentral collisions ...

Transport equations

MUSIC 2.0

Inclusion of bulk viscous pressure, shear-stress tensor, and all couplings

theory part discussed by Rischke

$$
\begin{aligned}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi} \Pi \theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu} \pi_{\mu\nu} \,, \\
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu} \omega^{\nu\rangle\alpha} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \varphi_7 \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha} - \tau_{\pi\pi} \pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} \\
&\quad + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} + \varphi_6 \Pi \pi^{\mu\nu} .
\end{aligned}
$$

Transport coefficients computed within the 14-moment approximation

Transport equations

MUSIC 2.0

Inclusion of bulk viscous pressure, shear-stress tensor, and all couplings

theory part discussed by Rischke

$$\boxed{\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2} + \boxed{\lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu}},$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + \boxed{2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}}$$
$$\boxed{+\lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu}}.$$

Second-Order Nonlinear source terms

Bulk viscous pressure

Coupling between bulk viscous pressure and shear-stress tensor

Coefficients employed **MUSIC 2.0**

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \varphi_1 \Pi^2 + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} + \varphi_3 \pi^{\mu\nu}\pi_{\mu\nu} \,,$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \varphi_6 \Pi\pi^{\mu\nu} .$$

Transport coefficients computed within the 14-moment approximation

$$\beta_\pi = \frac{\varepsilon_0 + P_0}{5}, \; \delta_{\pi\pi} = \frac{4}{3}\tau_\pi \,, \quad \tau_{\pi\pi} = \frac{10}{7}\tau_\pi \,, \quad \varphi_7 = \frac{9}{70 P_0}\tau_\pi \,.$$

$$\beta_\Pi = \frac{\zeta}{\tau_\Pi} = 14.55 \times \left(\frac{1}{3} - c_s^2\right)^2 (\varepsilon_0 + P_0) + \mathcal{O}\left(z^5\right),$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = 1 - c_s^2 + \mathcal{O}\left(z^2 \ln z\right) \,,$$

$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right) + \mathcal{O}\left(z^4\right) \,,$$

$$z \equiv m/T,$$

Bulk viscosity from kinetic theory

Weinberg (matter coupled to radiation)

$$\boxed{\zeta \;=\; 15\,\eta\left(\frac{1}{3}-c_s^2\right)^2}$$

Dusling et al (pure glue)

$$\boxed{\zeta \;\approx\; 50\,\eta\left(\frac{1}{3}-c_s^2\right)^2}$$

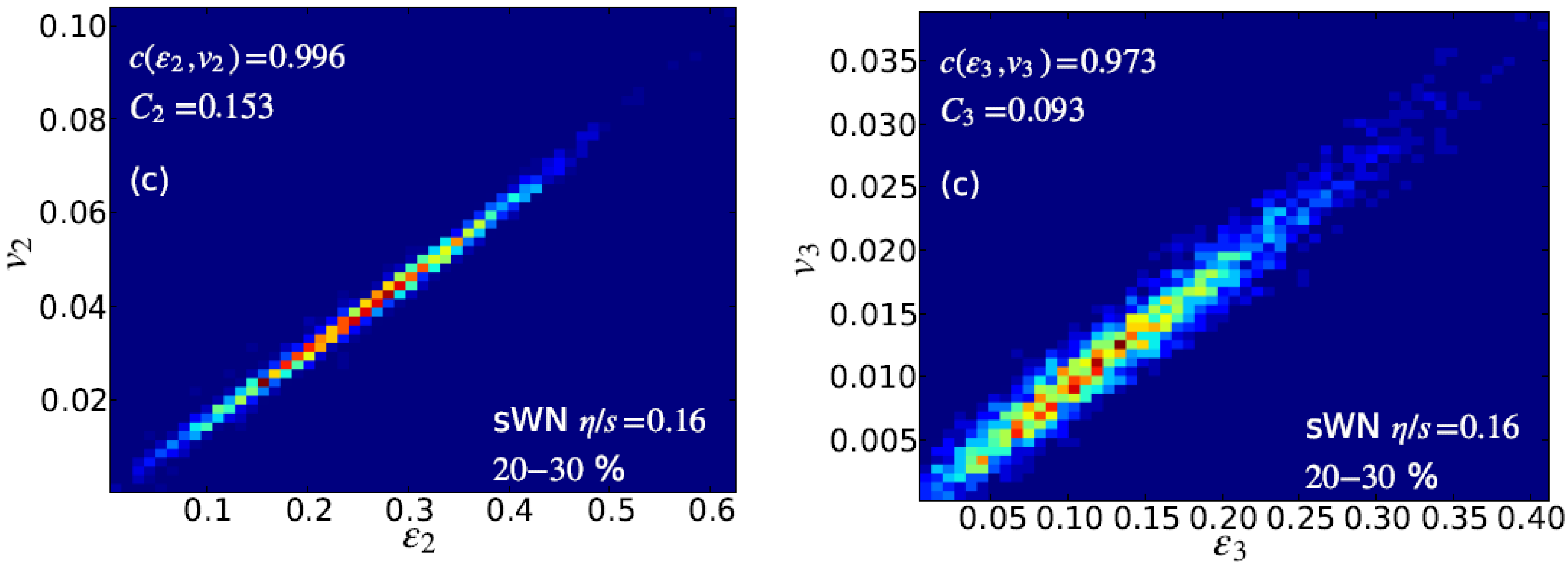
QGP?

14-moment approximation

$$\boxed{\zeta \;\approx\; 73\,\eta\left(\frac{1}{3}-c_s^2\right)^2}$$

We don't do EbE (sorry Kodama ...)

sWN $\eta/s = 0.16$



Niemi&GSD&Huovinen&Holopainen

$$\boxed{v_n = C_n \epsilon_n} \Longleftrightarrow \boxed{C_n = \langle v_n \rangle_{\mathrm{ev}} / \langle \epsilon_n \rangle_{\mathrm{ev}}}$$

Works for n=2 and n=3

In ultracentral collisions: **Works for all of them**

Gardim&Grassi&Luzum&Ollitrault

$$\boxed{v_n = C_n \epsilon_n} \quad \Longleftrightarrow \quad \boxed{C_n = \langle v_n \rangle_{\rm ev} / \langle \epsilon_n \rangle_{\rm ev}}$$

no dependence on initial state; can be computed using any IC

<u>What we do:</u>

We compute this coefficient for an arbitrary IC, but with the **correct multiplicity** and **average pT**

In this talk ...

- ✔ We solve the fluid-dynamical equations using a relativistic version of the KT algorithm

  Schenke&Jeon&Gale

  Phys.Rev. C82 (2010) 014903

- ✔ Freeze-out via Cooper-Frye, T=140 MeV

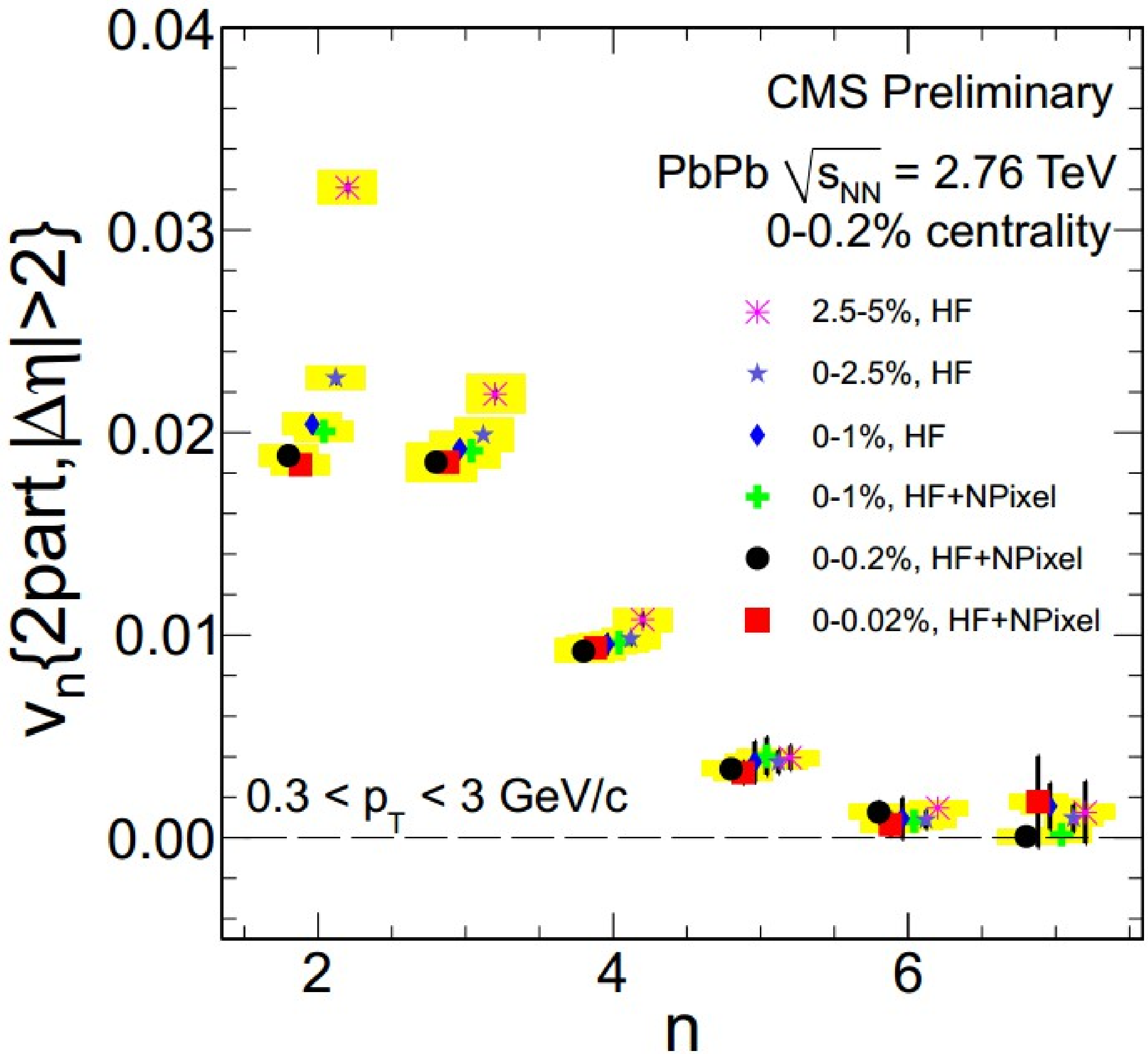
- ✔ **no** δf for bulk! For shear, it is included.

- ✔ lQCD + HRG EoS by Huovinen&Petrescky

  Nucl.Phys. A837 (2010) 26-53

- ✔ τ_0=1 fm, equilibrium

In ultracentral collisions: **data from CMS**



$v_2 \sim v_3$

hard to get with hydro

Effect of nonlinear terms

usual

$$\boxed{\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta}$$

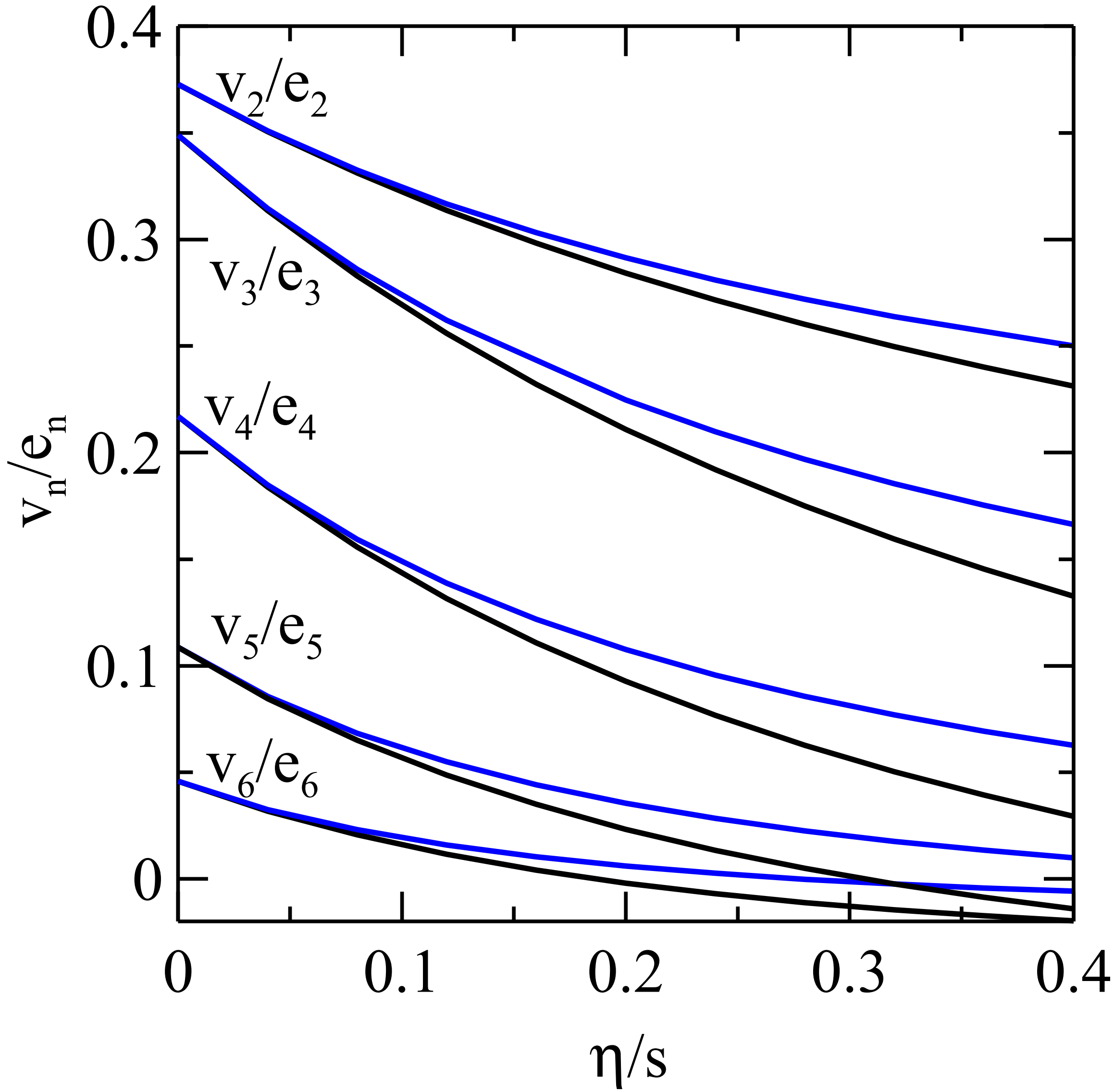
with nonlinear terms

$$\boxed{\begin{aligned}\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} \;=\;& 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta \\ & + \frac{18}{35}\tau_\pi \frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} - \frac{10}{7}\tau_\pi \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}.\end{aligned}}$$

can we see a difference?

Effect of nonlinear terms

MUSIC 2.0



0-1% - LHC

wo/ nonlinear terms

w/ nonlinear terms

nonlinear terms reduce viscous effects

Effect of bulk viscous pressure (1)

Bulk Only

$$\boxed{\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta - \left(1 - c_s^2\right) \tau_\Pi \Pi \theta}$$

Shear Only

$$\boxed{\begin{aligned} \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi \pi^{\mu\nu}\theta \\ &\quad + \frac{18}{35}\tau_\pi \frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} - \frac{10}{7}\tau_\pi \pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}. \end{aligned}}$$

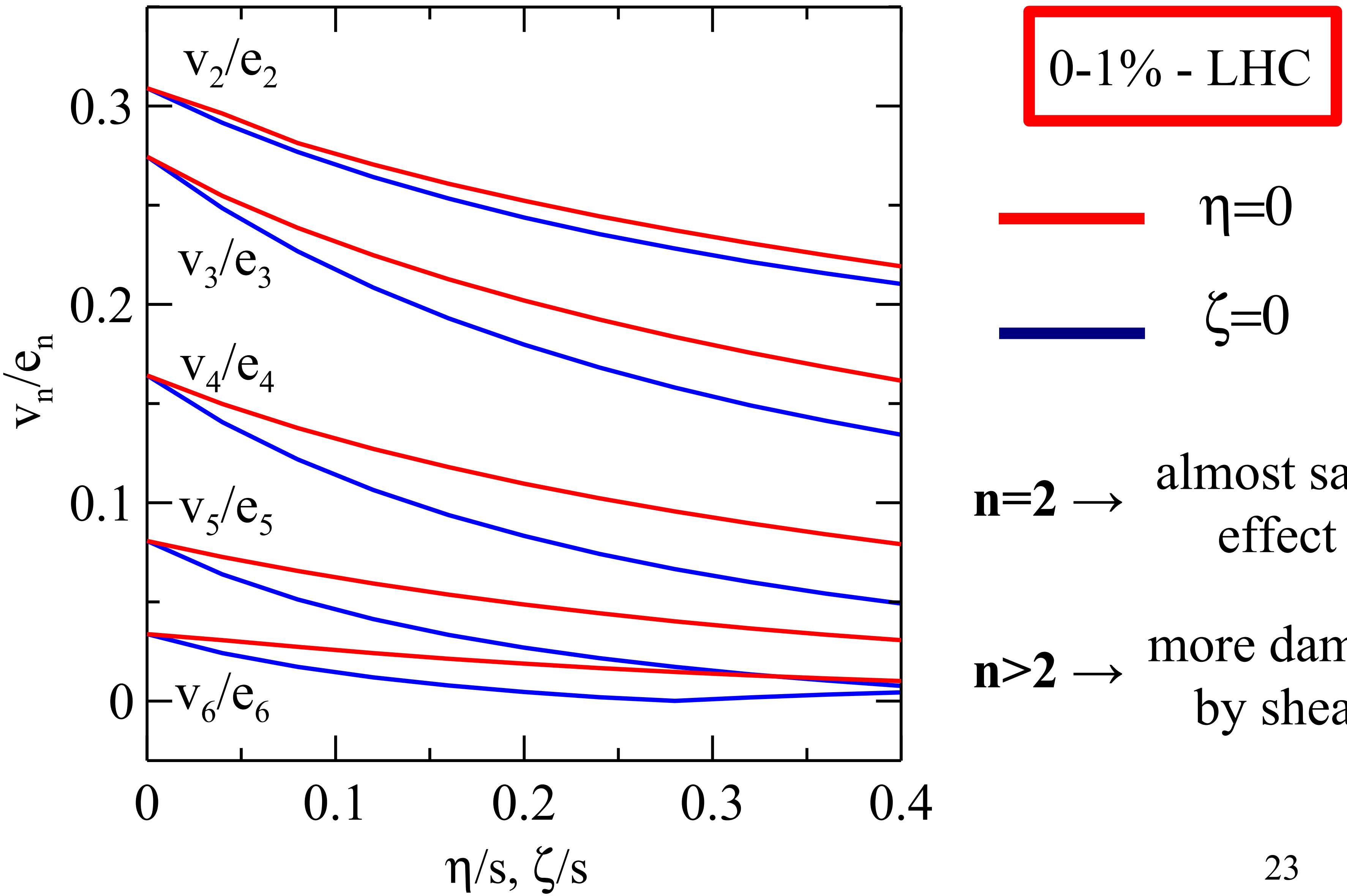
assume effective viscosities:

$$\frac{\eta}{s} = \mathrm{const}$$

$$\frac{\zeta}{s} = \mathrm{const}$$

Effect of bulk viscous pressure

MUSIC 2.0

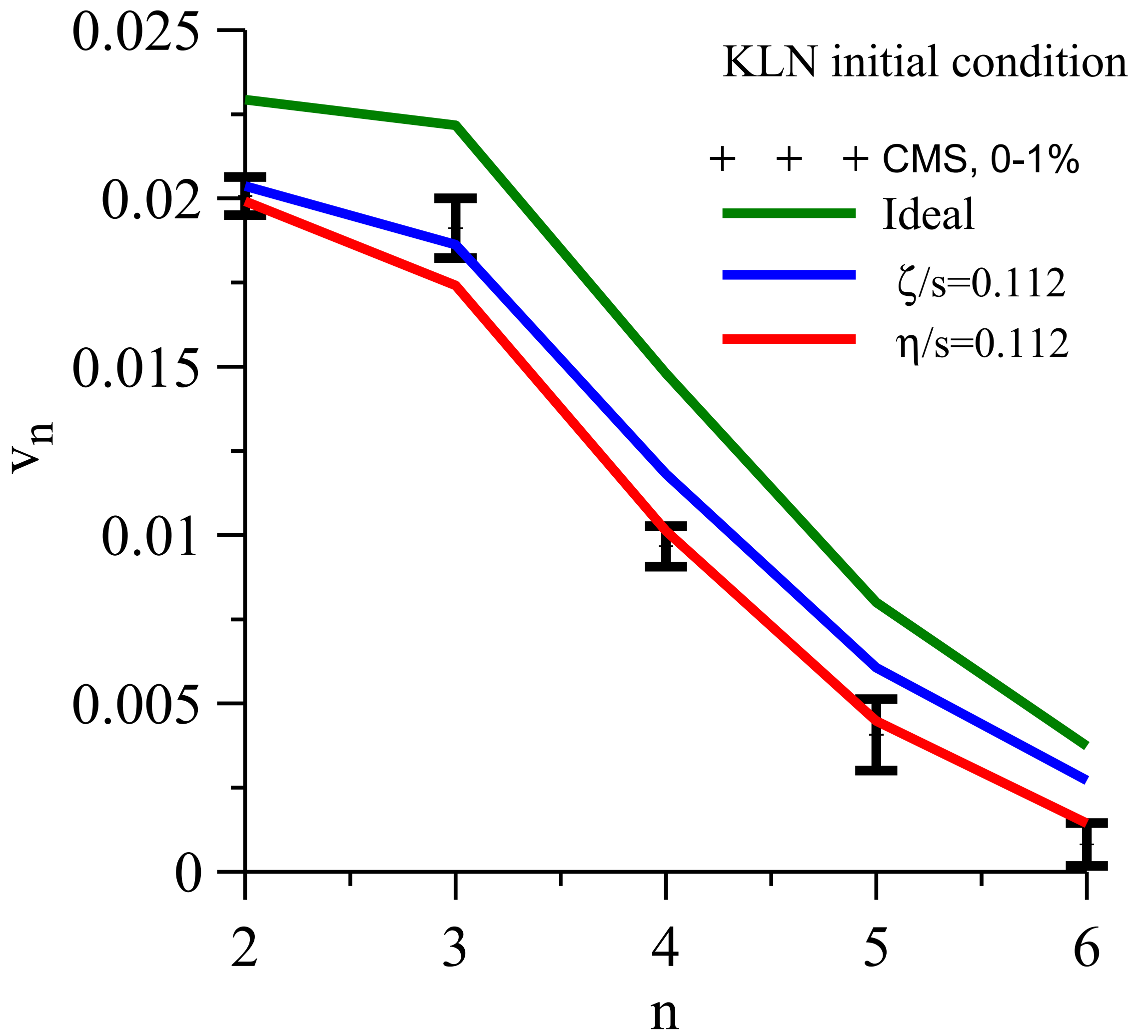


n=2 → almost same effect

n>2 → more damped by shear

Effect of bulk viscous pressure

MUSIC 2.0



0-1% - LHC

bulk → v_3 less damped
misses n=4,5,6

shear → v_3 too damped
describes n=4,5,6

Effect of bulk viscous pressure (2)

Shear Only

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi\pi^{\mu\nu}\theta + \frac{18}{35}\tau_\pi\frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} - \frac{10}{7}\tau_\pi\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha}.$$

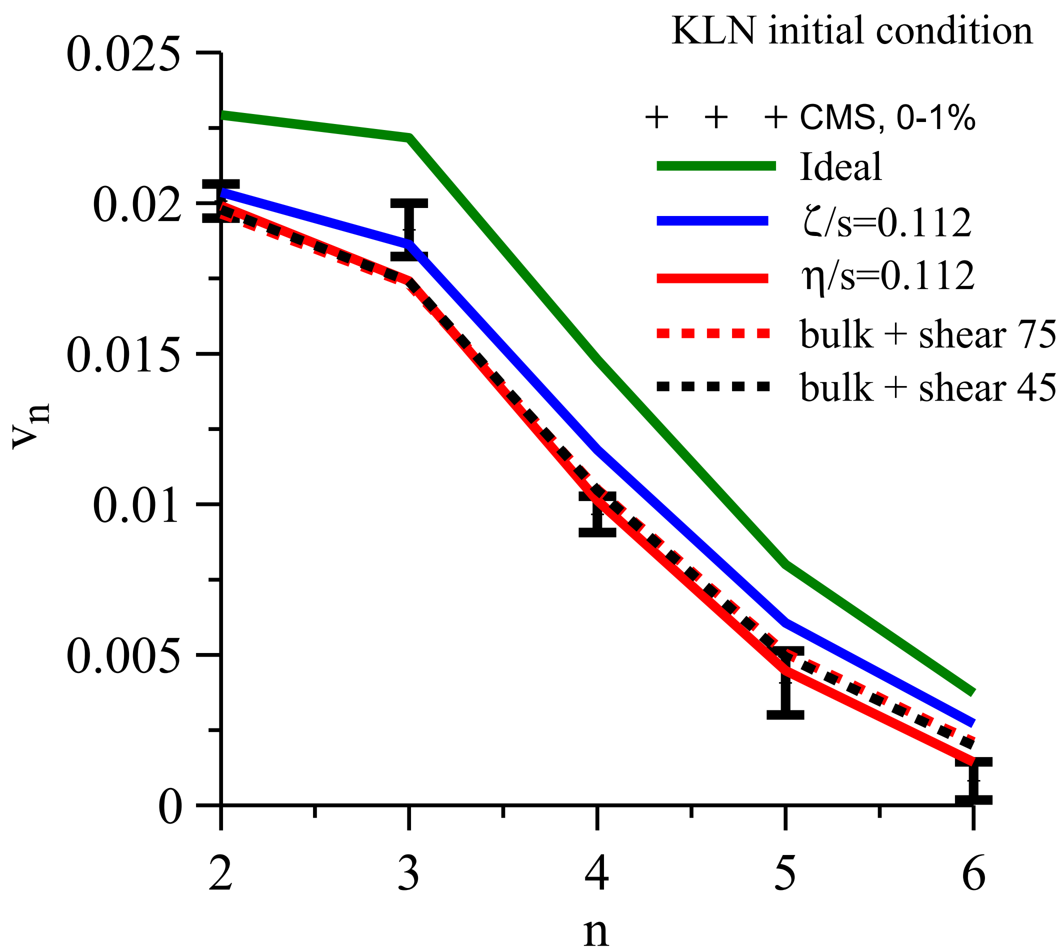
Complete equations

$$\tau_\Pi\dot{\Pi} + \Pi = -\zeta\theta - \left(1 - c_s^2\right)\tau_\Pi\Pi\theta + \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi\pi^{\mu\nu}\sigma_{\mu\nu},$$

$$\tau_\pi\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi\pi_\alpha^{\langle\mu}\omega^{\nu\rangle\alpha} - \frac{4}{3}\tau_\pi\pi^{\mu\nu}\theta - \frac{10}{7}\tau_\pi\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \frac{18}{35}\tau_\pi\frac{\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha}}{\varepsilon_0 + P_0} + \frac{6}{5}\tau_\pi\Pi\sigma^{\mu\nu}.$$

Effect of bulk viscous pressure

MUSIC 2.0



0-1% - LHC

η/s=0.072

η/s=0.08

$$\zeta \approx 45\,\eta\left(\frac{1}{3}-c_s^2\right)^2 ,$$

$$\zeta \approx 75\,\eta\left(\frac{1}{3}-c_s^2\right)^2 ,$$

Not good enough ...

Summary/conclusions

We studied the effect of bulk viscosity and nonlinear terms on the azimuthal momentum anisotropies

✔ We see a clear effect of bulk viscosity on flow

✔ When viscosity becomes large, nonlinear terms also become relevant

✔ Ultracentral collisions are a challenge to hydro models

Ongoing:

✗ Actual extraction of bulk viscosity from data

✗ Inclusion of heat flow

✗ Inclusion of δf (discussed in previous talk)

✗ Effects on photons and dileptons

My special thanks to Kodama

- ✔ My undergrad supervisor
- ✔ My Masters supervisor
- ✔ My neighbor (thanks for all the rides)

ありがとう