Hydrodynamics via Kinetic Theory

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RANP 2013
Takeshi Kodama's Fest

23-27 September 2013
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro - Brazil

Team

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- Dirk Colbry, MSU iCER
- Terrance Strother, LANL (former MSU Ph.D.)
- Tobias Bollenbach, MSU M.S. (Studienstiftung)
- Rodney Picket, MSU CSE undergraduate
- James Howell, MSU CSE undergraduate
- Alec Staber, MSU AST undergraduate

- … according to fluid dynamics experts

Hydrodynamics

- Conservation Laws
  - Linear momentum (Newton's 2nd Law)
  - Energy (including Mass)
- Navier-Stokes Equation

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}\bullet\vec{\nabla})\vec{v}\right)=-\vec{\nabla}p+\vec{\nabla}\hat{T}+\vec{F}$$

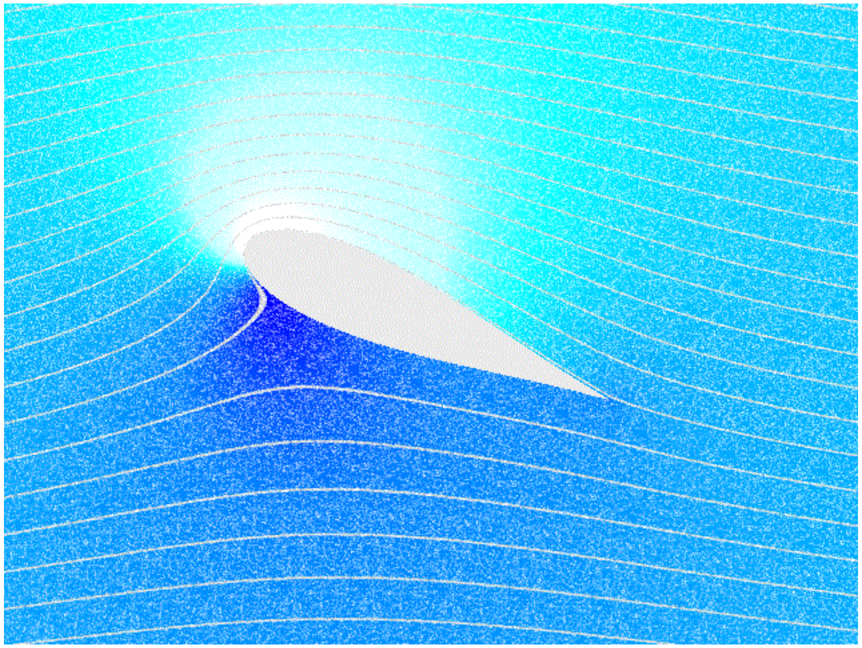
$\vec{v} =$ flow velocity

$p =$ pressure

$\rho =$ fluid density

$\hat{T} =$ stress tensor

$\vec{F} =$ external force



Credit: Thierry Dugnolle (Wikipedia)

Reynolds Number

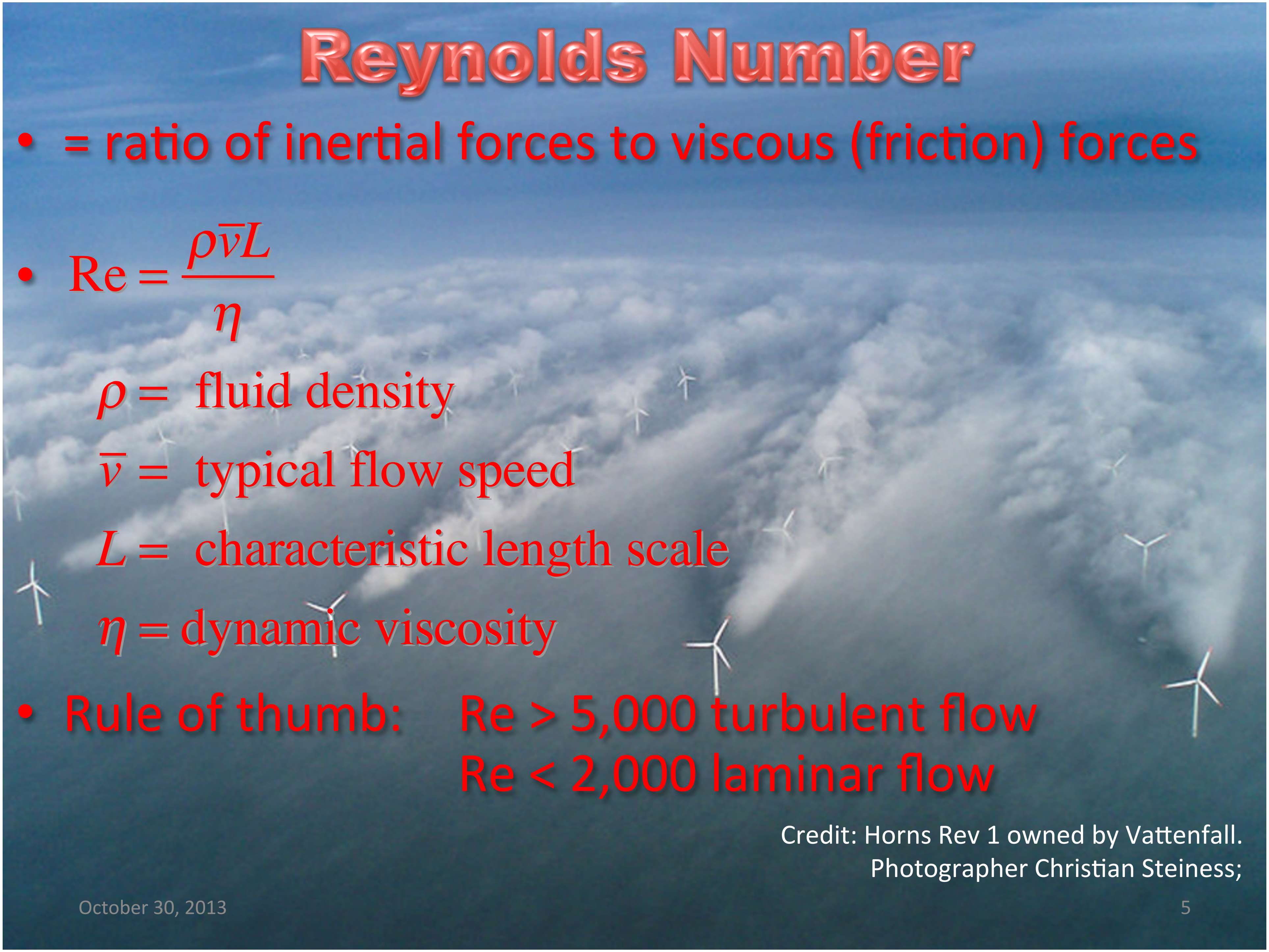
- = ratio of inertial forces to viscous (friction) forces
- $\mathrm{Re} = \dfrac{\rho \bar{v} L}{\eta}$

  $\rho =$ fluid density

  $\bar{v} =$ typical flow speed

  $L =$ characteristic length scale

  $\eta =$ dynamic viscosity
- Rule of thumb: Re > 5,000 turbulent flow
  Re < 2,000 laminar flow



Credit: Horns Rev 1 owned by Vattenfall.
Photographer Christian Steiness;

Knudsen Number

- = ratio of mean free path to characteristic length scale

$$\mathrm{Kn} = \frac{\lambda}{L}$$

- Needed for hydro to be valid $\mathrm{Kn} \to 0$
- Example ideal gas: $\mathrm{Kn}_{\text{ideal gas}} = \dfrac{k_B T}{\sqrt{2}(4\pi r^2)pL}$
  - N_2 molecules at STP: Kn ~ 30

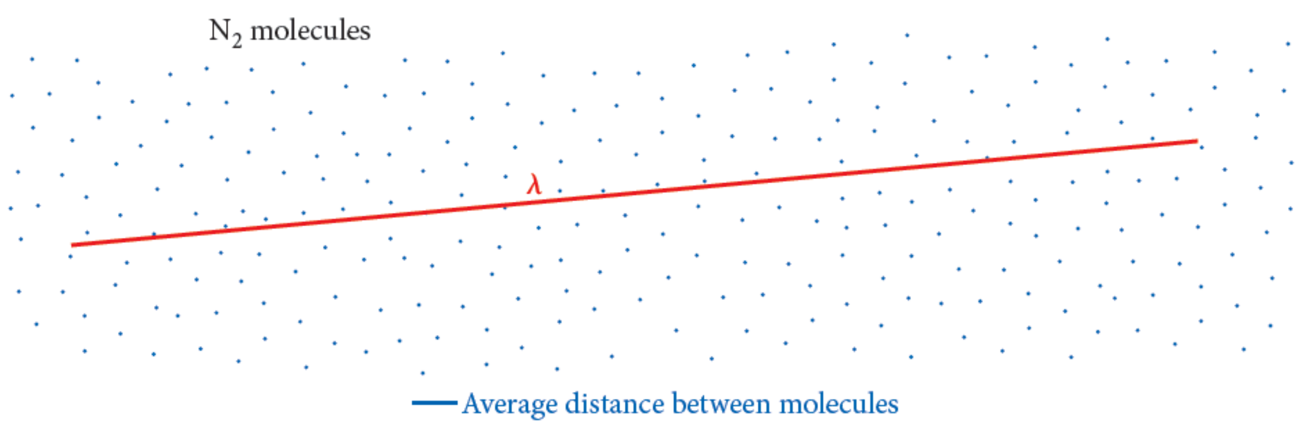


FIGURE 19.27 Scale drawing showing the size of nitrogen molecules, the mean free path of the molecules in the gas, and the average distance between molecules.

Credit: Bauer & Westfall 2013

Can be derived …

Navier-Stokes Equation by Stochastic Variational Method

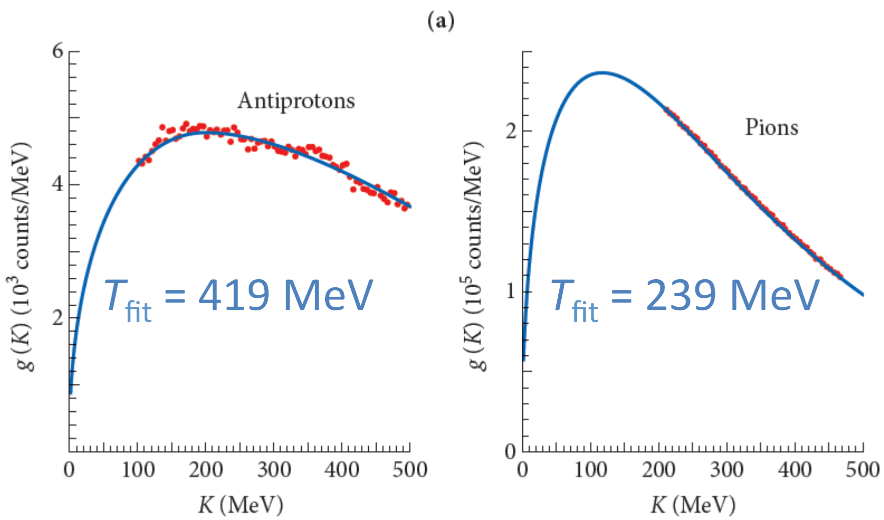
T. Koide and T. Kodama
Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, 21941-972, Rio de Janeiro, Brazil

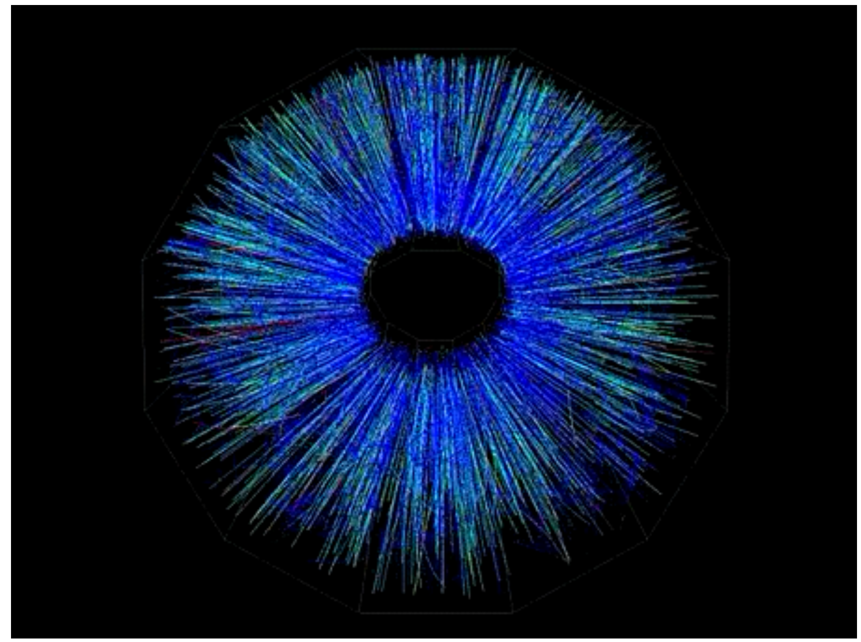
We show for the first time that the stochastic variational method can naturally derive the Navier-Stokes equation starting from the action of ideal fluid. In the frame work of the stochastic variational method, the dynamical variables are extended to stochastic quantities. Then the effect of dissipation is realized as the direct consequence of the fluctuation-dissipation theorem. The present result reveals the potential availability of this approach to describe more general dissipative processes.

arXiv.org > cond-mat > arXiv:1105.6256

Applications of Hydro (1)

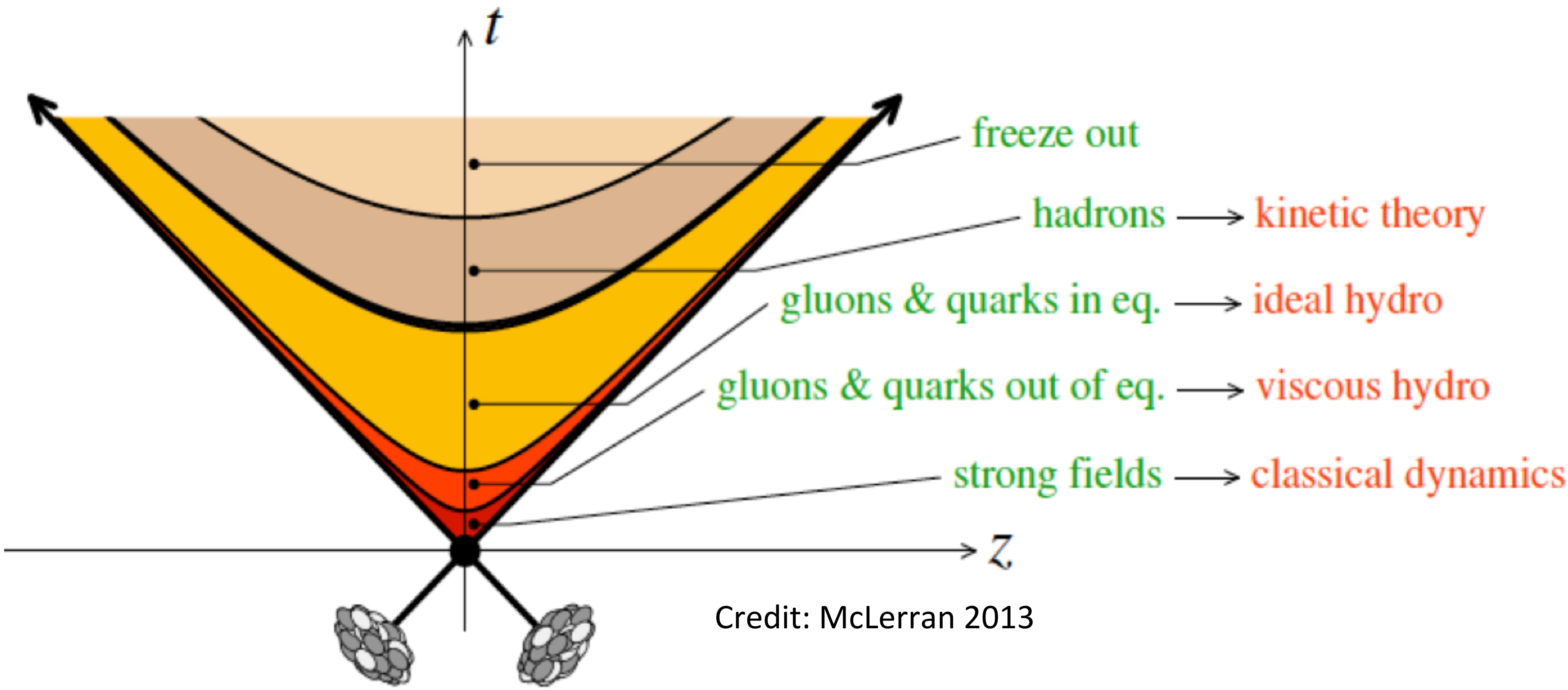
100 *A*Gev Au + Au 100 *A*Gev





Relativistic Heavy Ion Collider (RHIC).
Data: STAR Collaboration

Applications of Hydro (1)

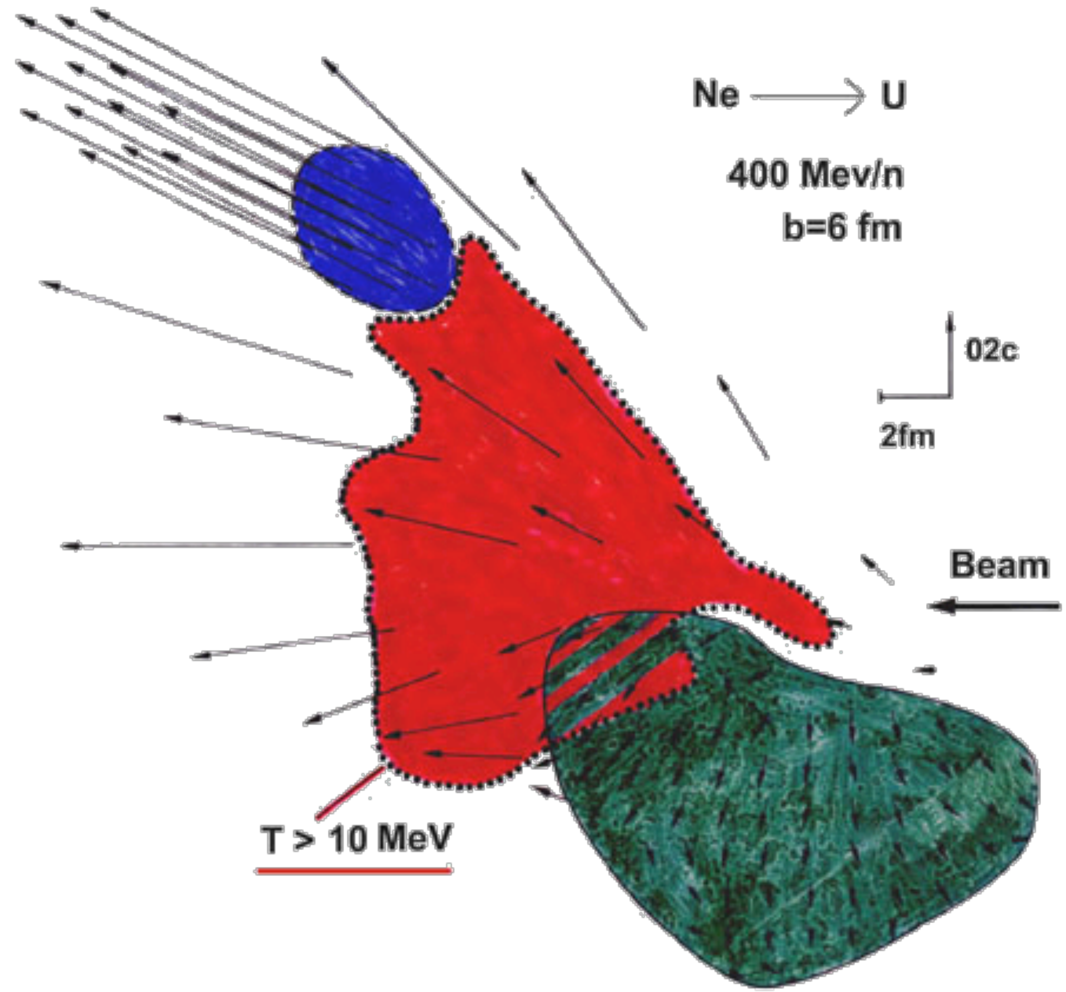


Credit: McLerran 2013

Cartoon of the time evolution of an ultra-relativistic heavy ion collision

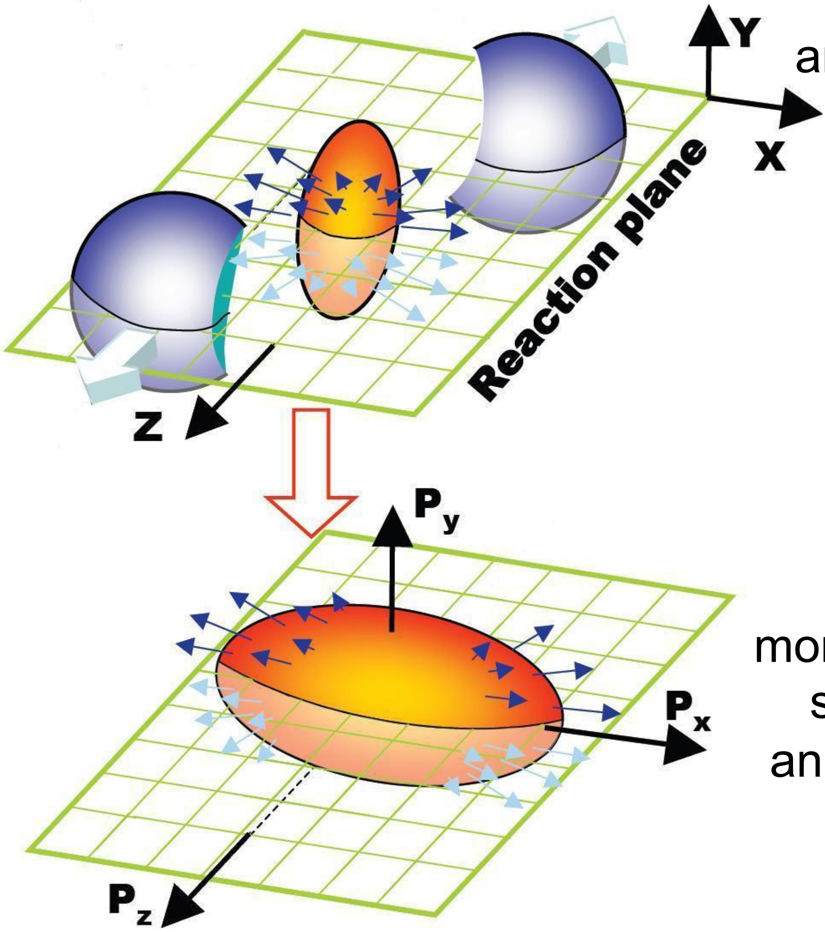
Applications of Hydro (1)

- Relativistic heavy ion collisions
- Scale 10^{-15} m
- Shock wave (?)
- Successful @RHIC
  - v_2 ✓
  - η/s small ✓



H. Stöcker, J.A. Maruhn, and W. Greiner, PRL **44**, 725 (1980)

Applications of Hydro (1)



coordinate space anisotropy

momentum space anisotropy

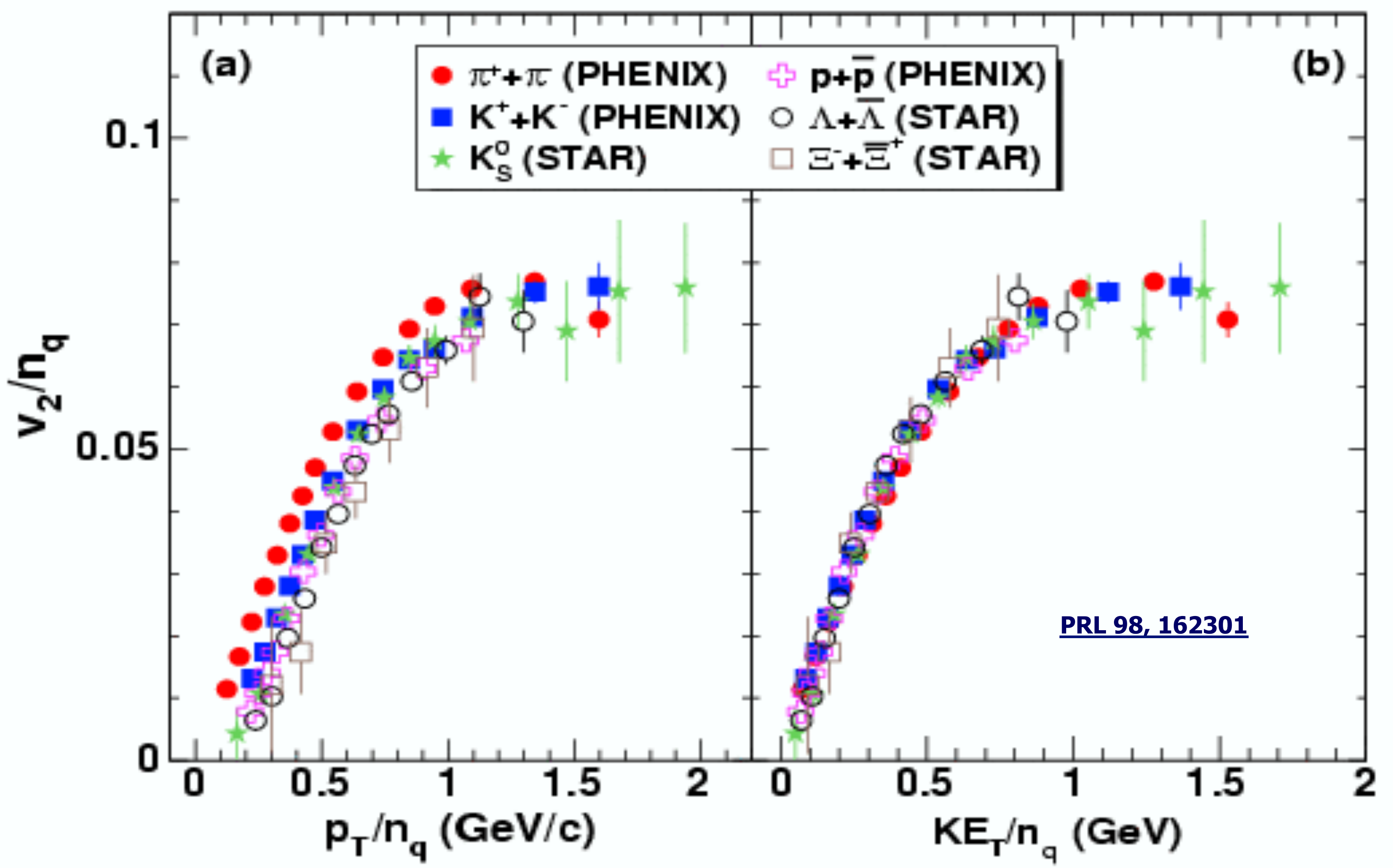
Credit: Na Li, 25th WWND, 2009

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_T, y)\cos\left(n\left(\phi - \Psi_r\right)\right)\right)$$

$$v_n = \left\langle \cos\left(n\left(\phi - \Psi_r\right)\right)\right\rangle$$

- **Azimuthal correlation with the reaction plane.**
- **Built up in the early stage, therefore supplies the early information of matter generated in the collision.**

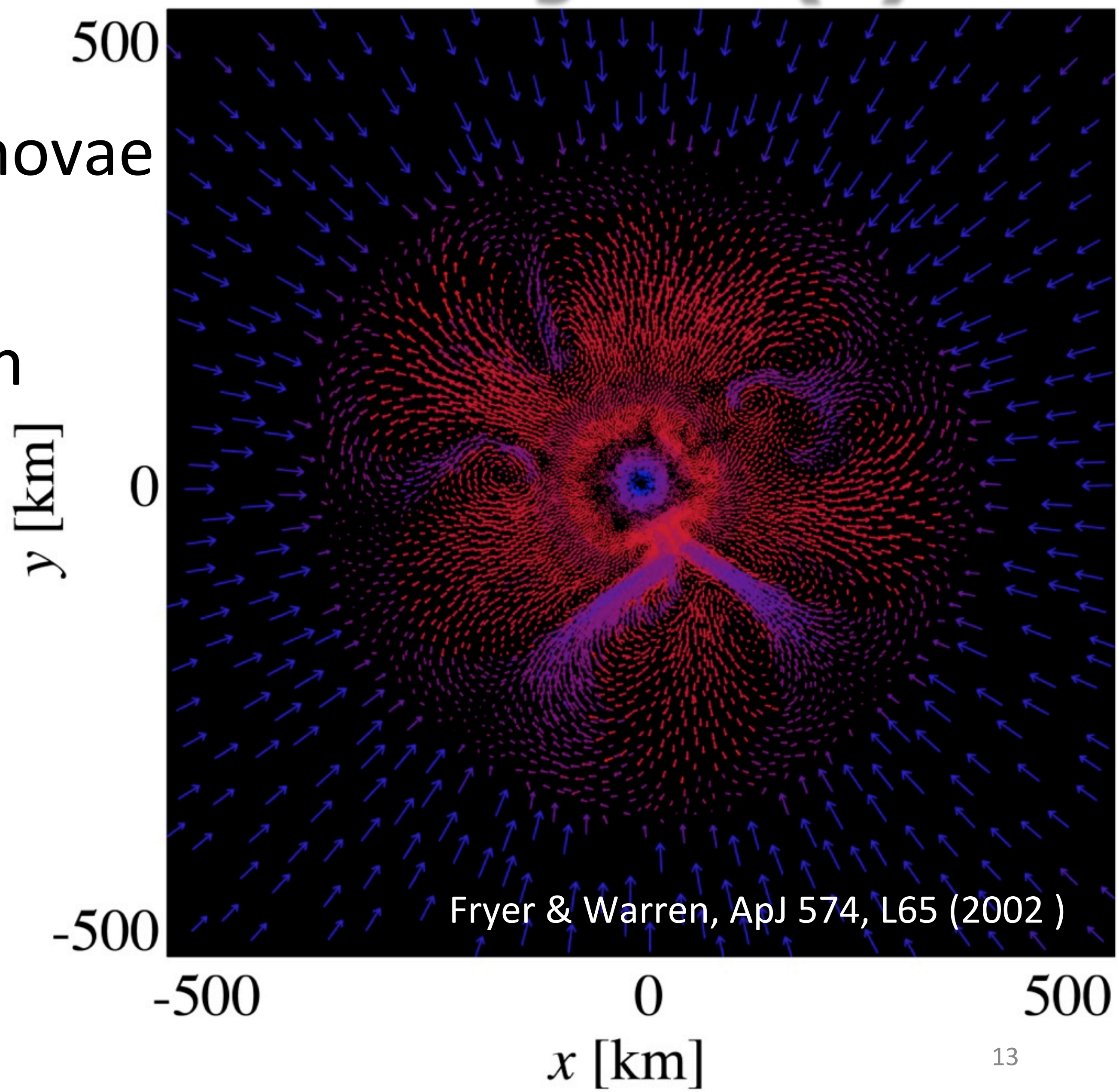
Applications of Hydro (1)



Strong indication for hydrodynamic flow!

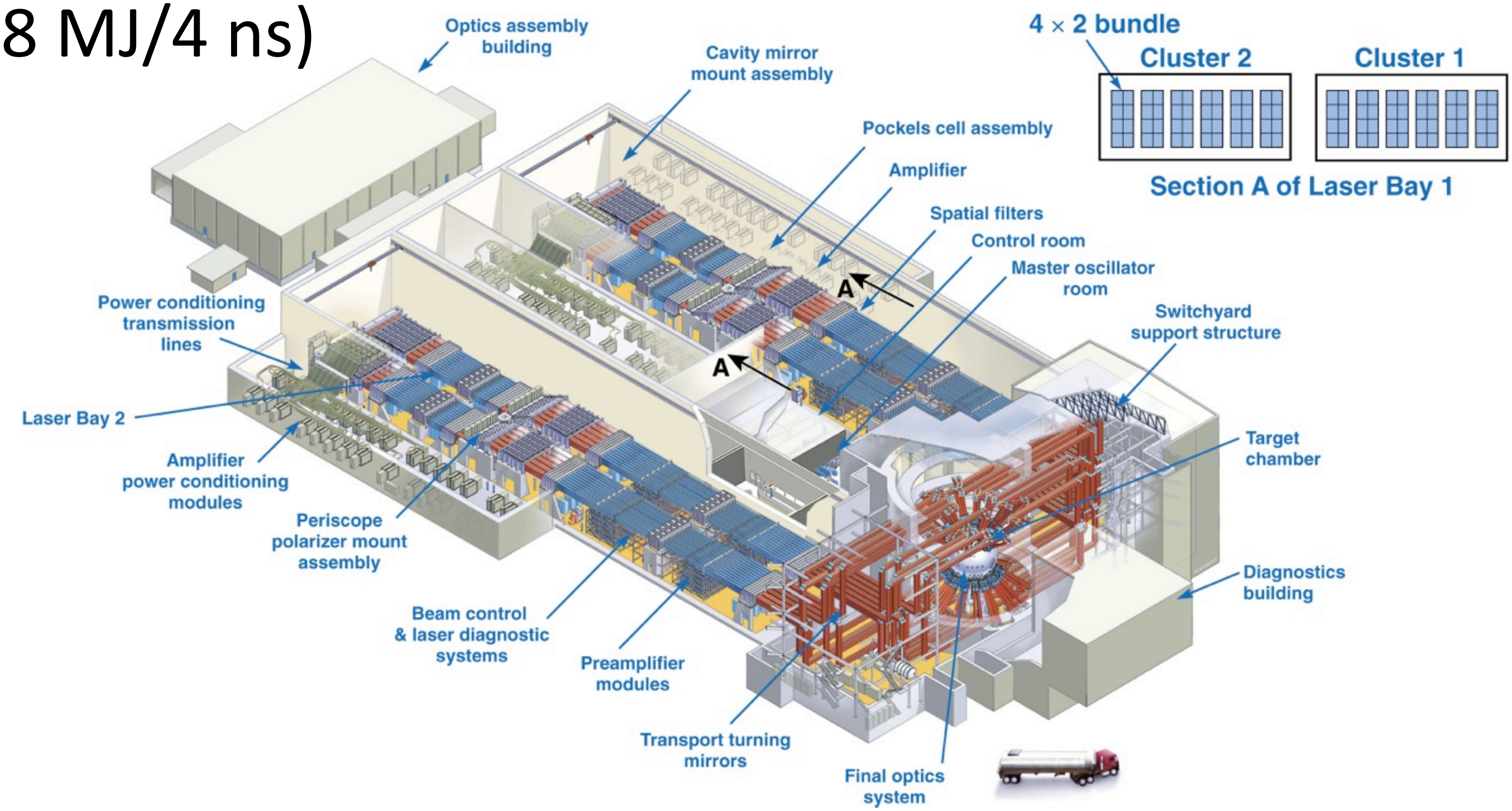
Applications of Hydro (2)

- Type II core collapse supernovae
- Scale 10^7 m
- Neutrino-driven dynamics
- Stalled shock wave



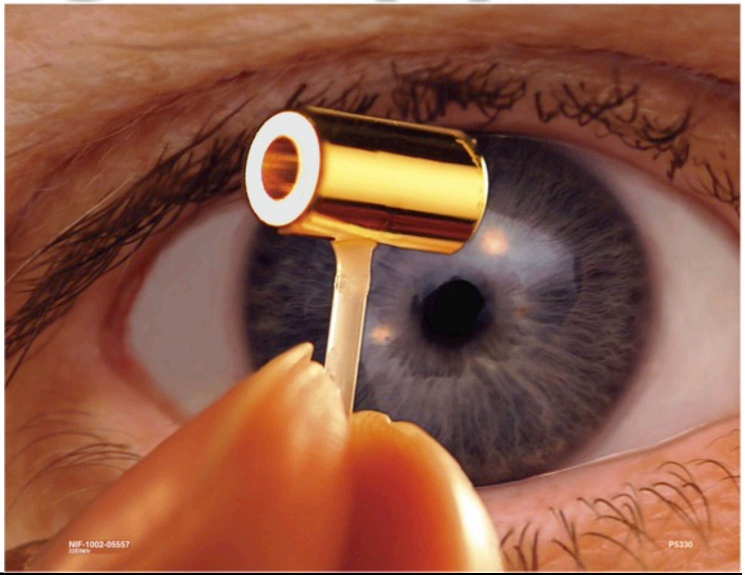
Applications of Hydro (3)

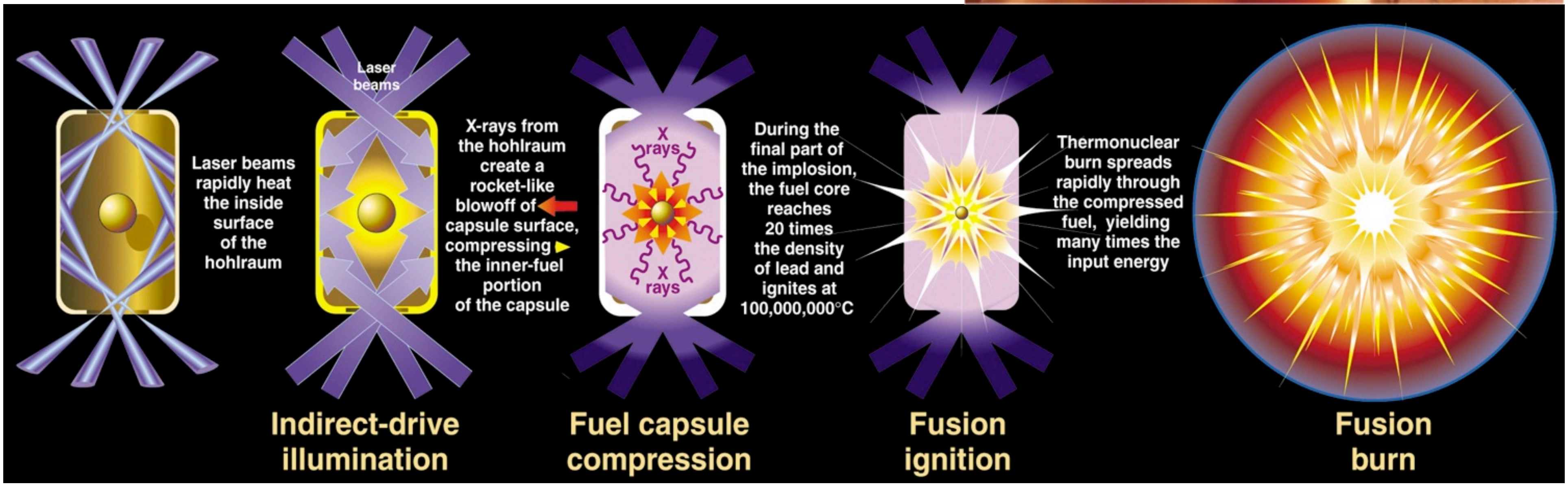
- National Ignition Facility
- Most powerful laser in the world: 0.5 PW (1.8 MJ/4 ns)



Applications of Hydro (3)

- ICF capsule
- Scale 10^{-5} m
- Livermore hydro codes fail
  - Ignition predicted, but not achieved





Kinetic Theory 1
(more motivation than derivation)

- Start from many-body Hamiltonian

$$i\hbar\partial_t|\Psi_N\rangle = \hat{H}|\Psi_N\rangle$$

- Construct density matrix for many particle wave function $\rho_N = |\Psi_N\rangle\langle\Psi_N| \Rightarrow i\hbar\partial_t\rho_N = [\hat{H},\rho_N]$

- BBGKY (Bogoliubov–Born–Green–Kirkwood–Yvon) Hierarchy

$$\partial_t\rho_n = F(\rho_n,\rho_{n+1})$$

- Truncate at some level n: $\rho_{n+1} = G(\rho_n)$
  - Here: truncate at 3-body level; 3-body matix = product of 2-body density matrices

Kinetic Theory 2
(more motivation than derivation)

- Introduce Wigner transform:

$$f(x,p,t) \equiv \frac{1}{\pi\hbar}\int \left|\psi(x+y)\right\rangle\left\langle\psi(x-y)\right| e^{2ipy/\hbar}dy$$

- Final result: time evolution equation for 1-body Wigner-transform, which contains two-body correlations

$$\begin{aligned}\frac{\partial}{\partial t}f(\vec{r},\vec{p},t) &+ \frac{\vec{p}}{m}\vec{\nabla}_r\, f(\vec{r},\vec{p},t) - \vec{\nabla}_r U\,\vec{\nabla}_p f(\vec{r},\vec{p},t)\\ &= \frac{g}{2\pi^3\,m^2}\int d^3q_{1'}\,d^3q_2\,d^3q_{2'}\\ &\quad \delta\left(\frac{1}{2m}(p^2+q_2^2-q_{1'}^2-q_{2'}^2)\right)\cdot\delta^3(\vec{p}+\vec{q}_2-\vec{q}_{1'}-\vec{q}_{2'})\cdot\frac{d\sigma}{d\Omega}\\ &\quad \cdot\Big\{f(\vec{r},\vec{q}_{1'},t)\,f(\vec{r},\vec{q}_{2'},t)\left(1-f(\vec{r},\vec{p},t)\right)\left(1-f(\vec{r},\vec{q}_2,t)\right)\\ &\quad -f(\vec{r},\vec{p},t)\,f(\vec{r},\vec{q}_2,t)\left(1-f(\vec{r},\vec{q}_{1'},t)\right)\left(1-f(\vec{r},\vec{q}_{2'},t)\right)\Big\}\end{aligned}$$

Kinetic Theory 3
(more motivation than derivation)

- Approximate f by a sum of delta functions in phase space:

$$f(\vec{r},\vec{p},t) = \int d^3r_0\, d^3p_0\, \delta^3(\vec{r}-\vec{R}(\vec{r}_0,\vec{p}_0,t_0))\, \delta^3(\vec{p}-\vec{P}(\vec{r}_0,\vec{p}_0,t_0))\, f(\vec{r}_0,\vec{p}_0,t_0)$$

- Insert this into integral transport equation to obtain equations of motion for 6 coordinates of each test particle

$$\frac{d}{dt}\vec{p}_i = -\vec{\nabla}U_{EOS}(\vec{r}_i) - \vec{\nabla}U_C(q_i,\vec{r}_i) + \vec{C}(\vec{p}_i)$$

$$\frac{d}{dt}\vec{r}_i = \frac{\vec{p}_i}{m_i}$$

$$i = 1, \ldots, N$$

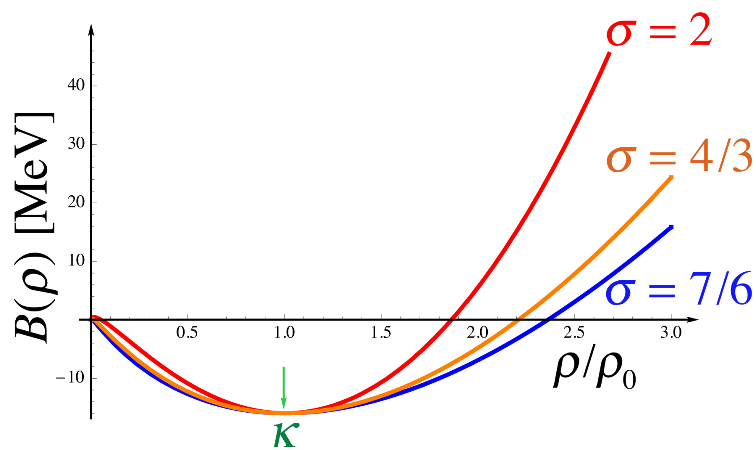
Coulomb

Nuclear EOS

2-body scattering

Nuclear Equation of State

- Energy functional as a function of density temperature, momentum, isospin, …



- Not easy!

Collision Integral

- Two-body cross sections from experiment
- Most accurate method: **Distance of closest approach**

$$\chi = (\vec{r}_{\rm rel}(t) \cdot \vec{v}_{\rm rel}(t))(\vec{r}_{\rm rel}(t+\Delta t) \cdot \vec{v}_{\rm rel}(t+\Delta t))$$

  - CPU time $O(N^2)$
  - Arbitrarily precise shock wave localization
    [J. Cugnon et al. NPA352, 505 (1981)]
- Fastest method: **Direct Simulation Monte Carlo**
  - Scattering grid
  - CPU time $O(N \log N)$
  - Causality violations and shock wave diffusion unavoidable
    [F.J. Alexander, A.L. Garcia, B.J. Alder, PRL 74, 5212 (1995),
    G. Kortemeyer, F. Daffin,WB, PLB 374, 25 (1996)]
- **Best of both Worlds?**
  [I. Sagert et al, sub. Physics of Fluids (2012)]

HI Collisions

- Point 1: Kinetic theory without collisions (= Vlasov) reproduces mean field theory (= TDHF)

JOSEPH J. MOLITORIS, DETLEV HAHN, HORST STÖCKER,
Prog. Nuc. Part. Phys. 15, 239 (1985)

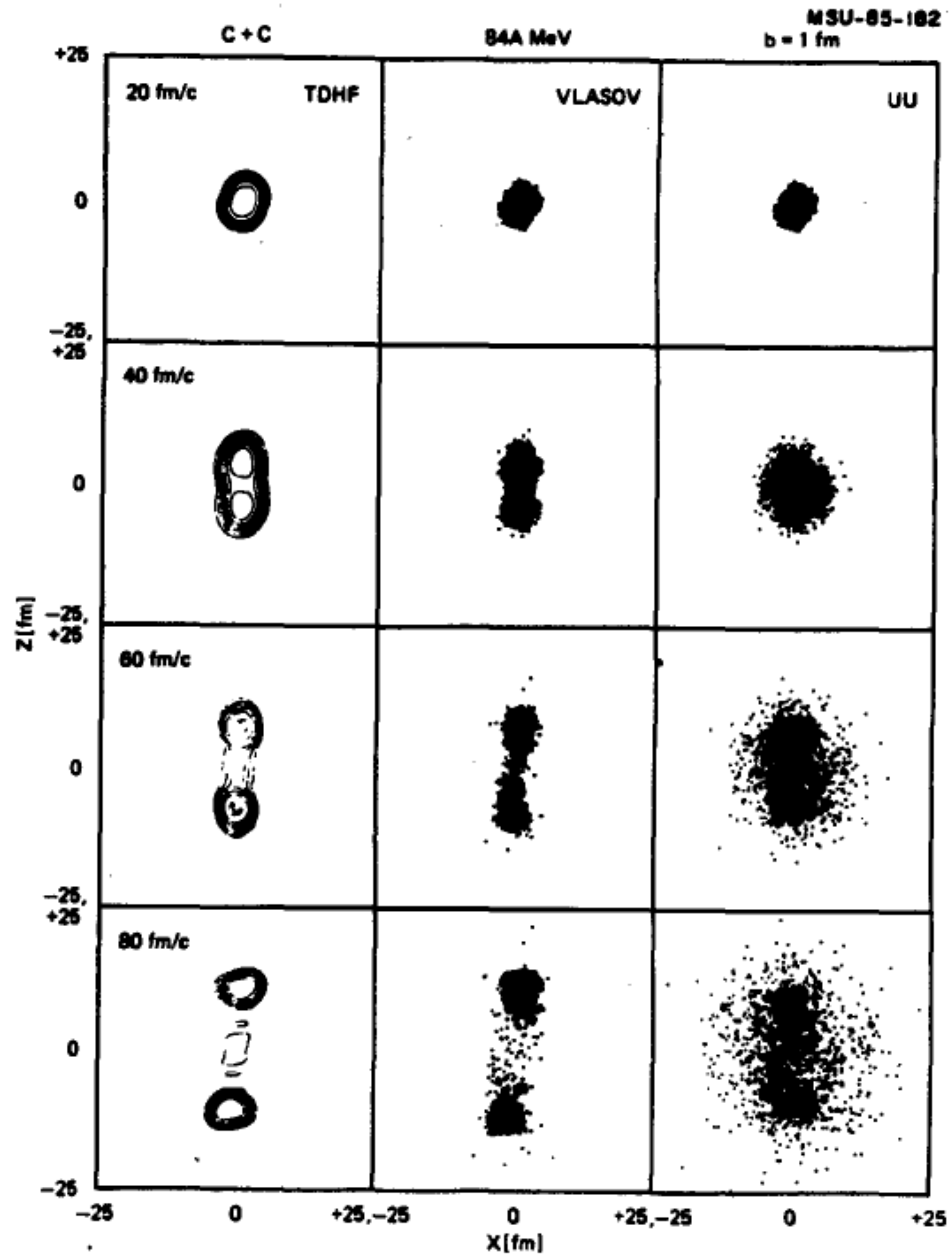
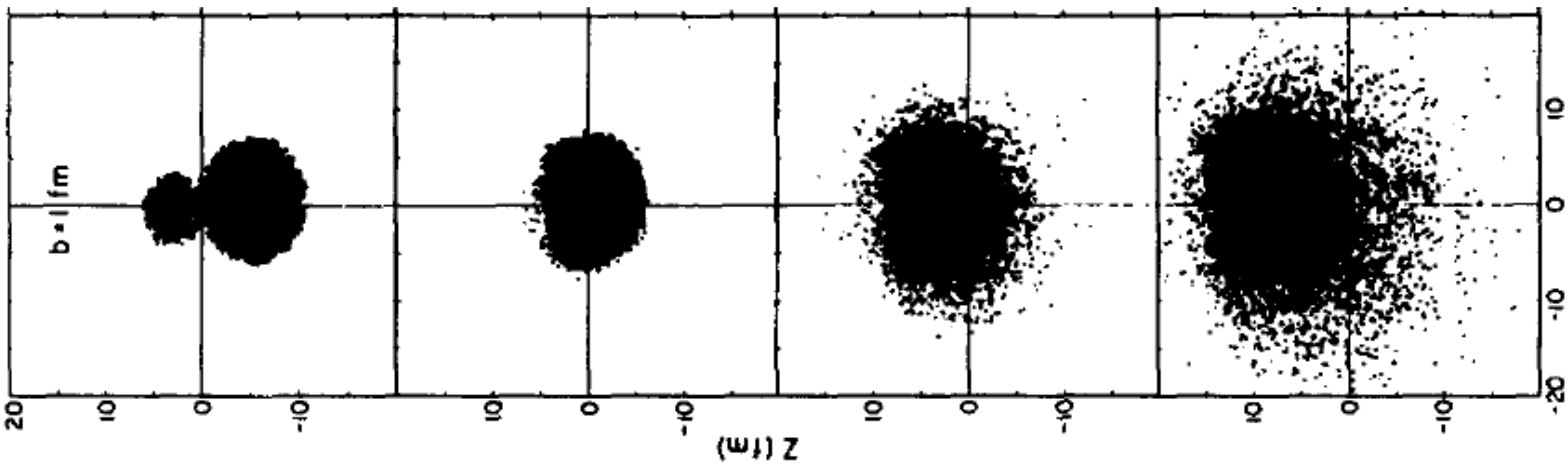


Fig. II.2 Time evolution in configuration and momentum space for C (85 MeV/nucleon) + C at b=1 fm for TDHF, the Vlasov equation, and the Vlasov-Uehling-Uhlenbeck theory. Transparency occurs in both cases with a mean field only.

HI Collisions

- Point 2: Kinetic theory with collisions (= VUU, BUU, …) reproduces hydro!

JOSEPH J. MOLITORIS, DETLEV HAHN, HORST STÖCKER,
Prog. Nuc. Part. Phys. 15, 239 (1985)



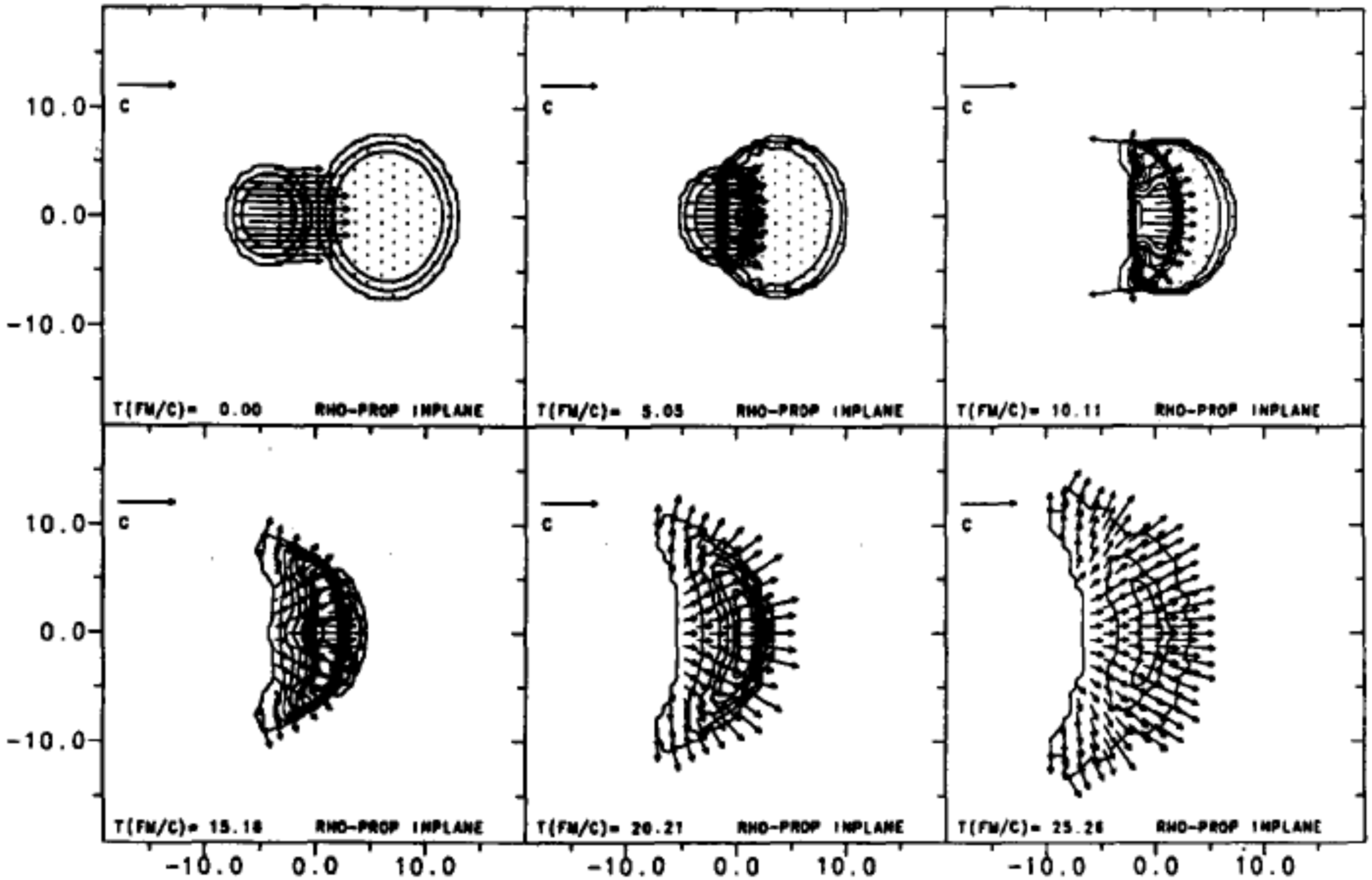
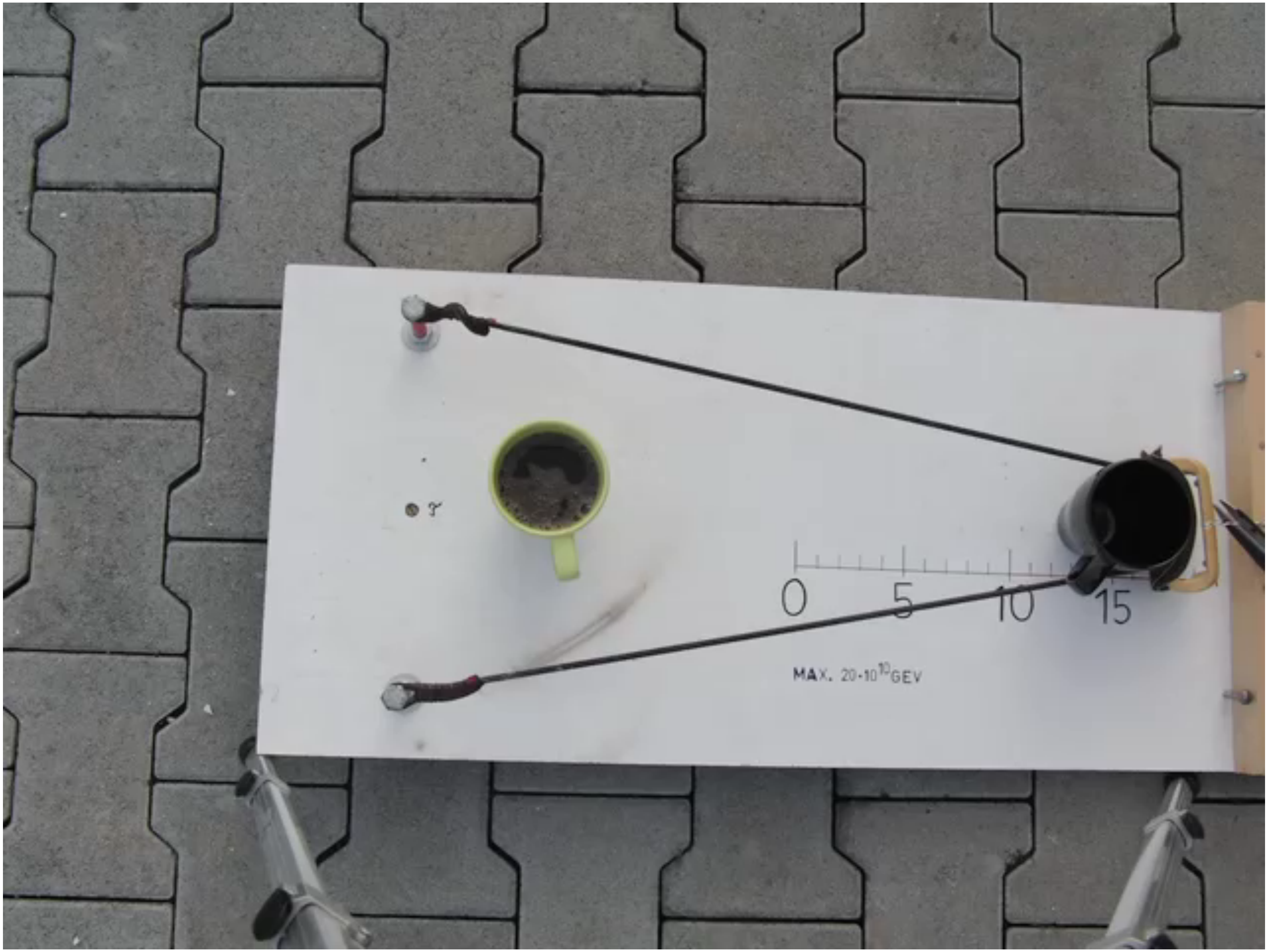
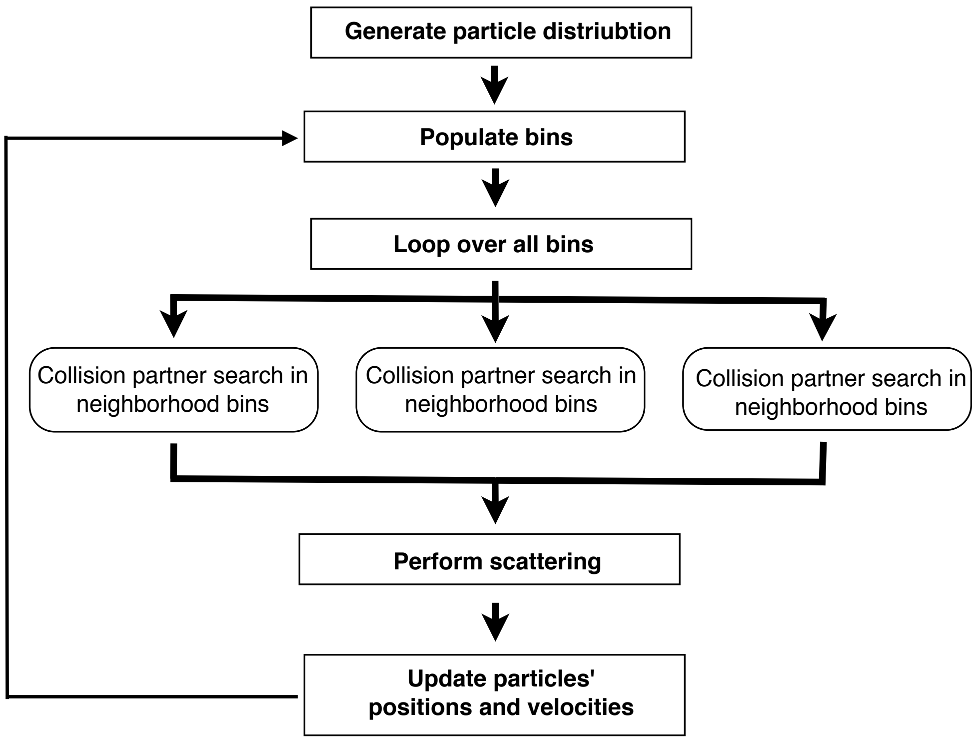


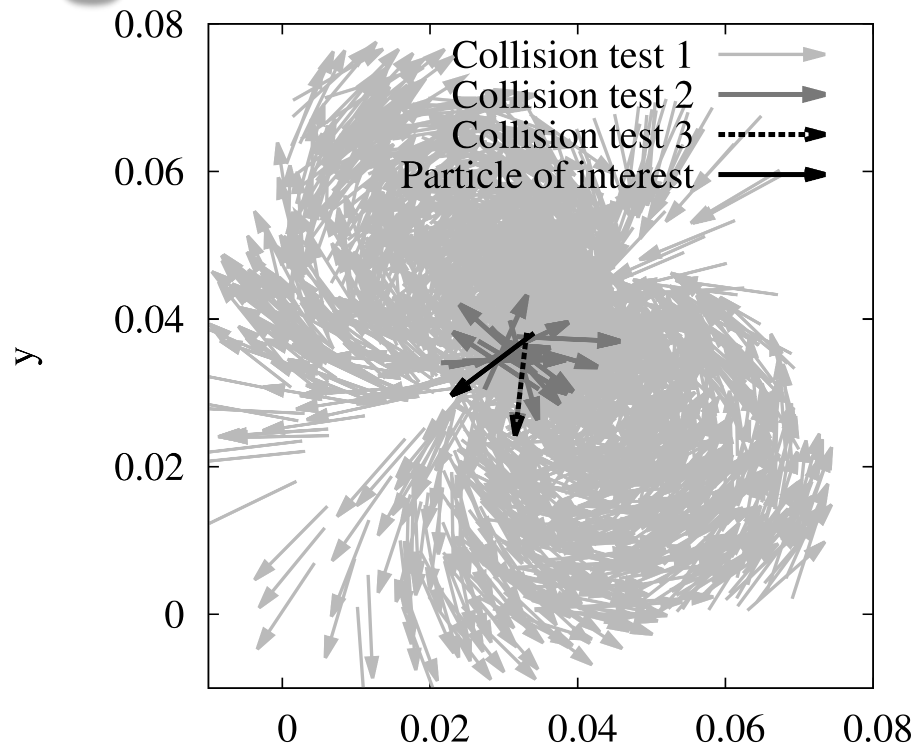
Fig. II.12 Collision of Ar (770 MeV/nucleon) + Pb at b = 0 fm in the Nuclear Fluid Dynamic model. Note the remarkable similarity to the VUU theory.

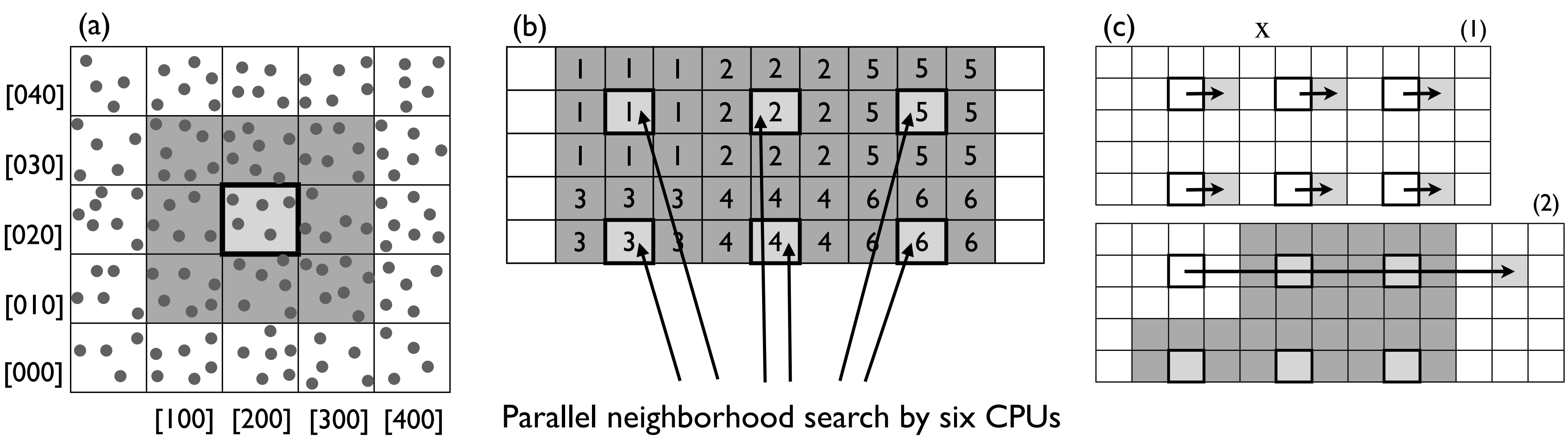
JOSEPH J. MOLITORIS, DETLEV HAHN, HORST STÖCKER,
Prog. Nuc. Part. Phys. 15, 239 (1985)


0 5 10 15
MAX. 20·10^10 GEV

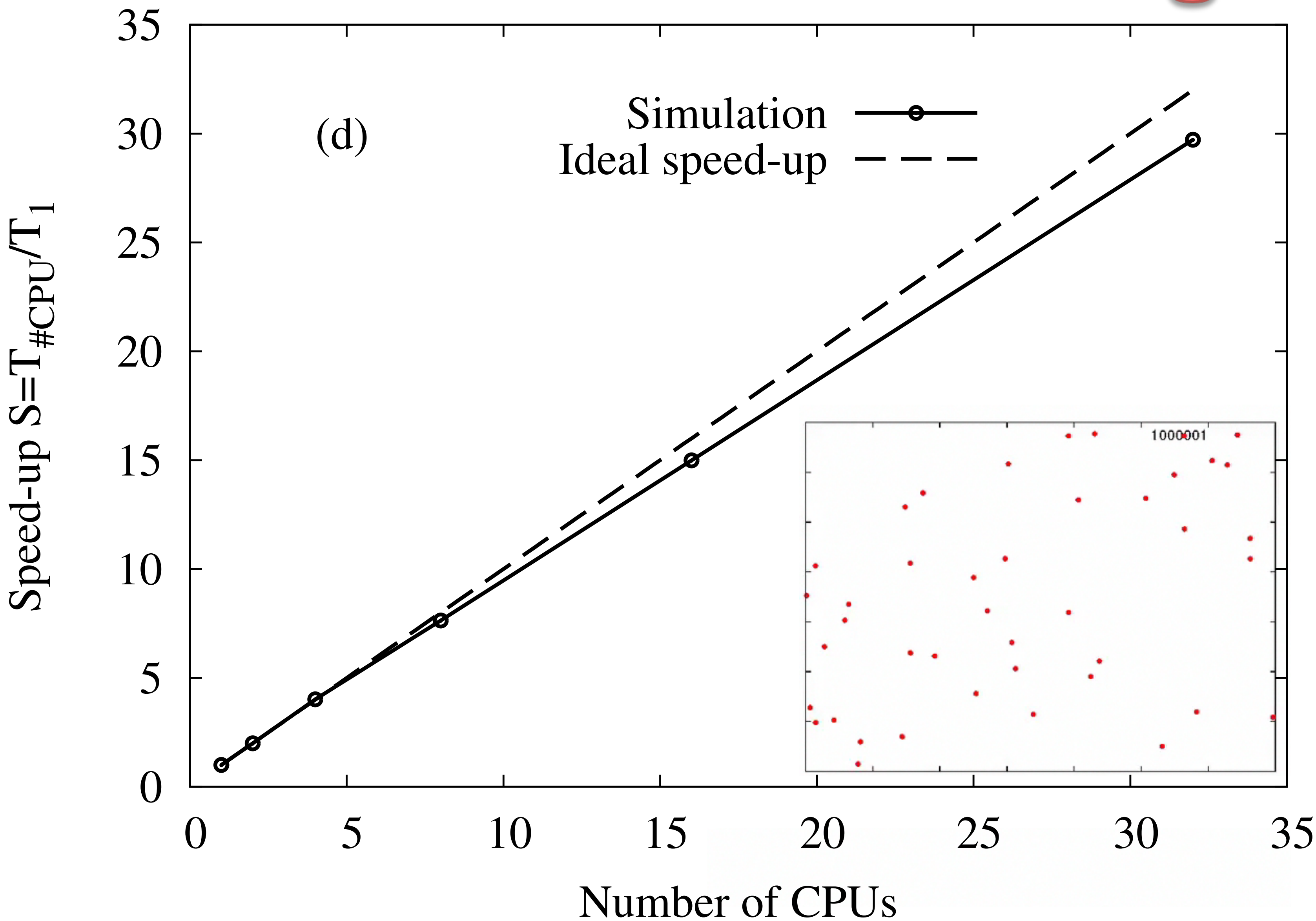
Collision Algorithm







Multi-Processor Scaling



Tests

- Observables
  - Bulk velocity

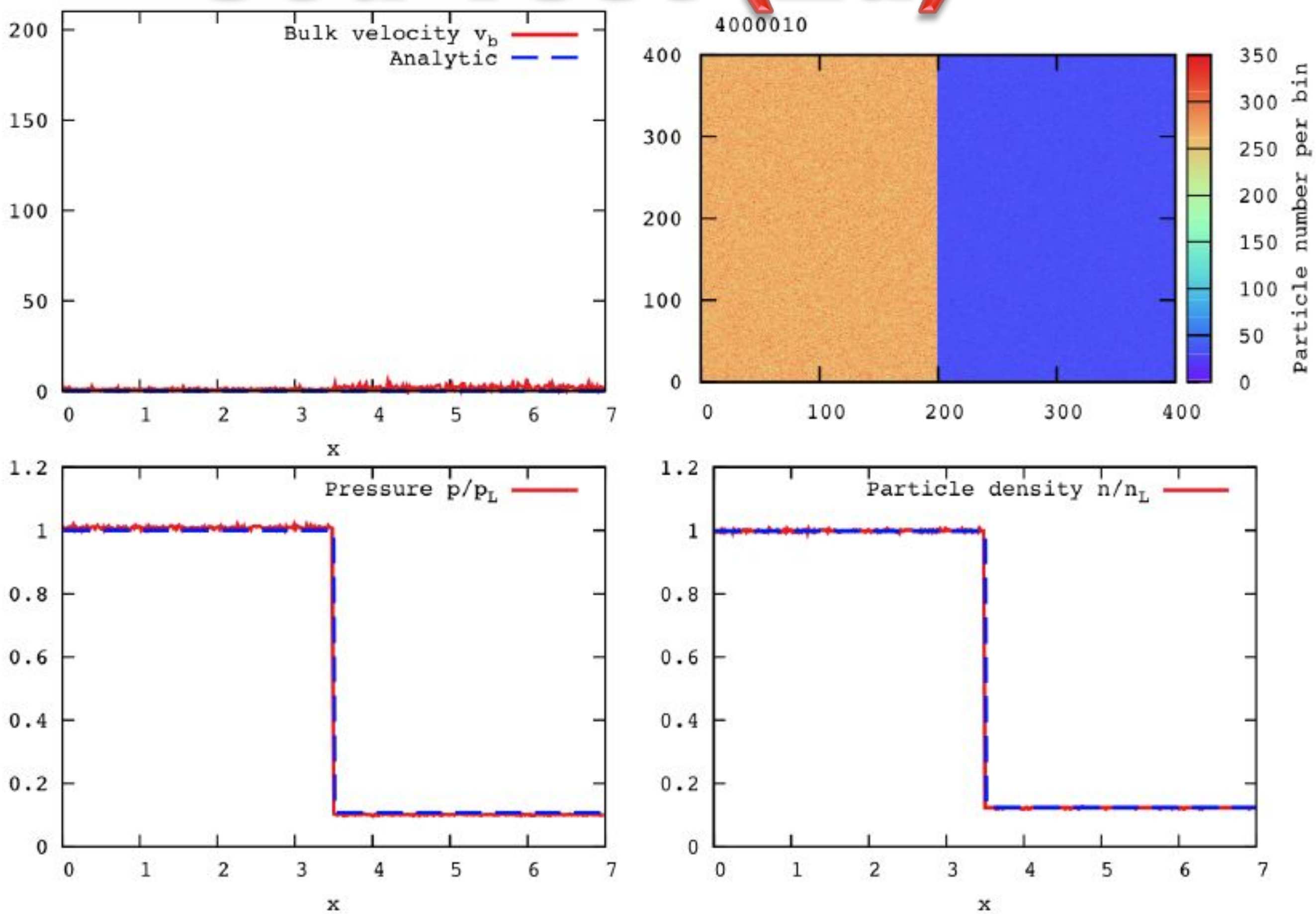
$$v_b = \frac{1}{N_V}\sqrt{v_{b,x}^2 + v_{b,y}^2 + v_{b,z}^2}, \quad v_{b,\alpha} = \sum_{i=1}^{M} v_{i,\alpha}$$

  - Pressure (= average of diagonal elements of stress tensor per volume)

$$\mathbf{P}_{\alpha\beta} = -\left(\sum_i m\left(v_{i,\alpha} - v_{b,\alpha}\right)\left(v_{i,\beta} - v_{b,\beta}\right) + \frac{1}{2}\frac{1}{\Delta t}\sum_i \sum_{i \neq j} r_{ij,\alpha}\,\Delta p_{i,\beta}\right)$$

  - Density

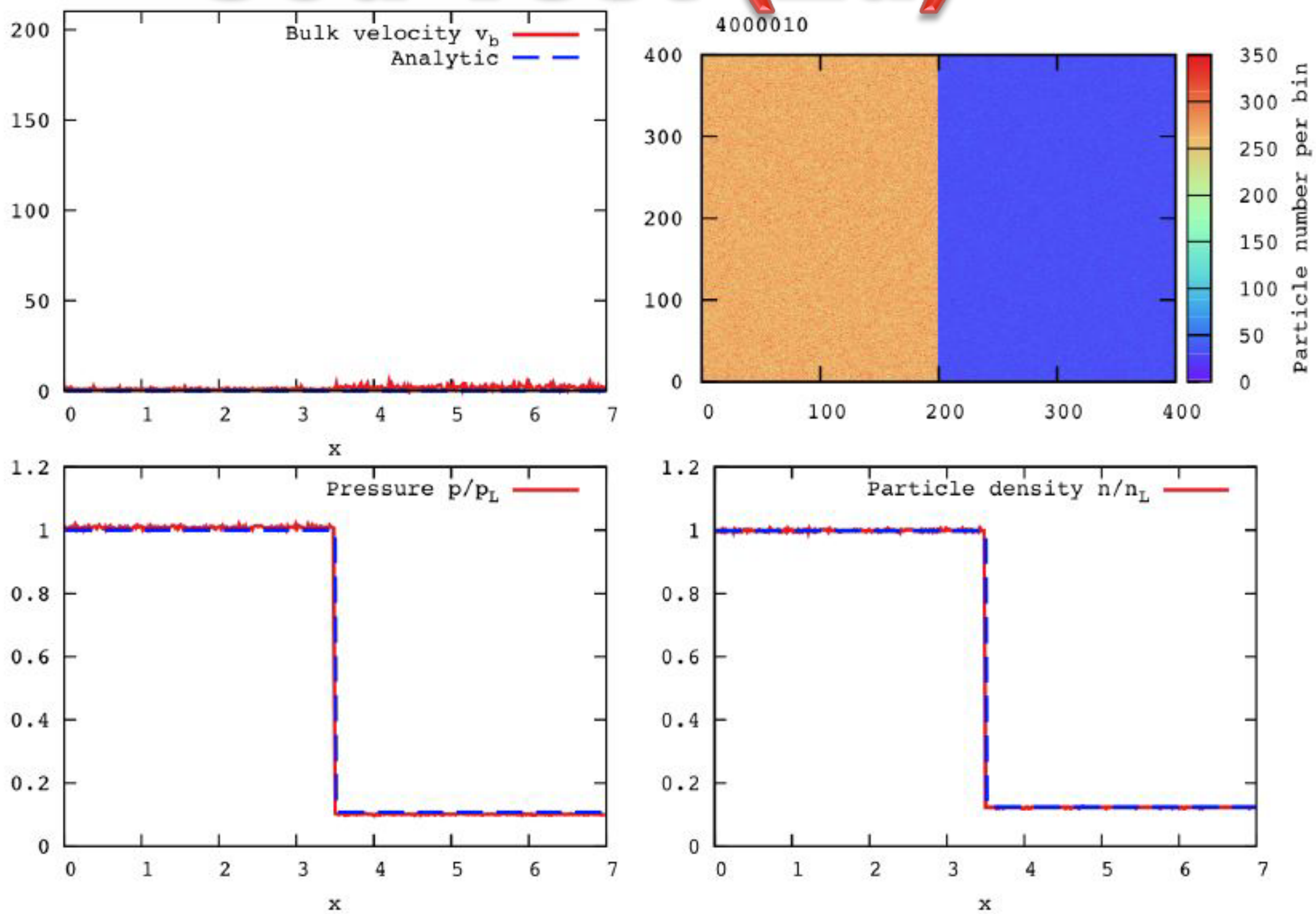
Sod Test (2d)



Initial conditions:

$$(n, p, v_b)_L = (1, 1, 0), \quad (n, p, v_b)_R = (0.125, 0.1, 0)$$

Sod Test (2d)



Initial conditions:

$$(n, p, v_b)_L = (1, 1, 0), \quad (n, p, v_b)_R = (0.125, 0.1, 0)$$

Sod Test (3d)

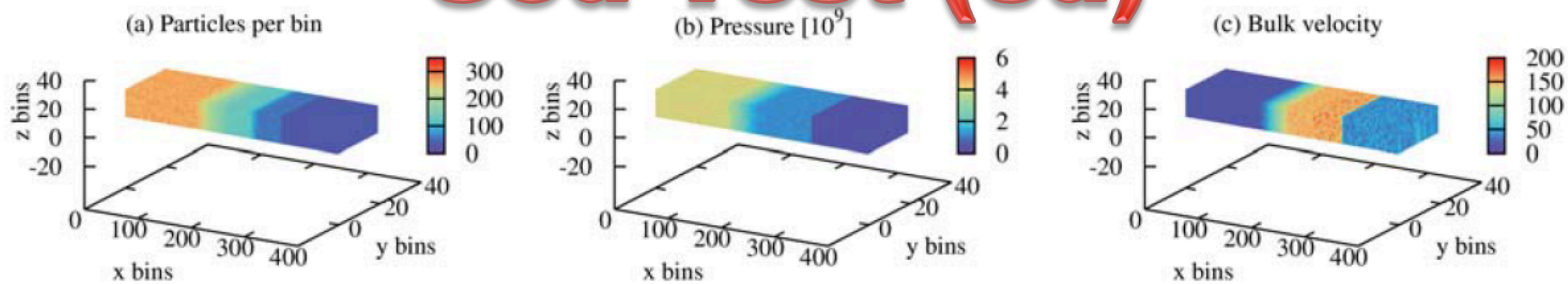
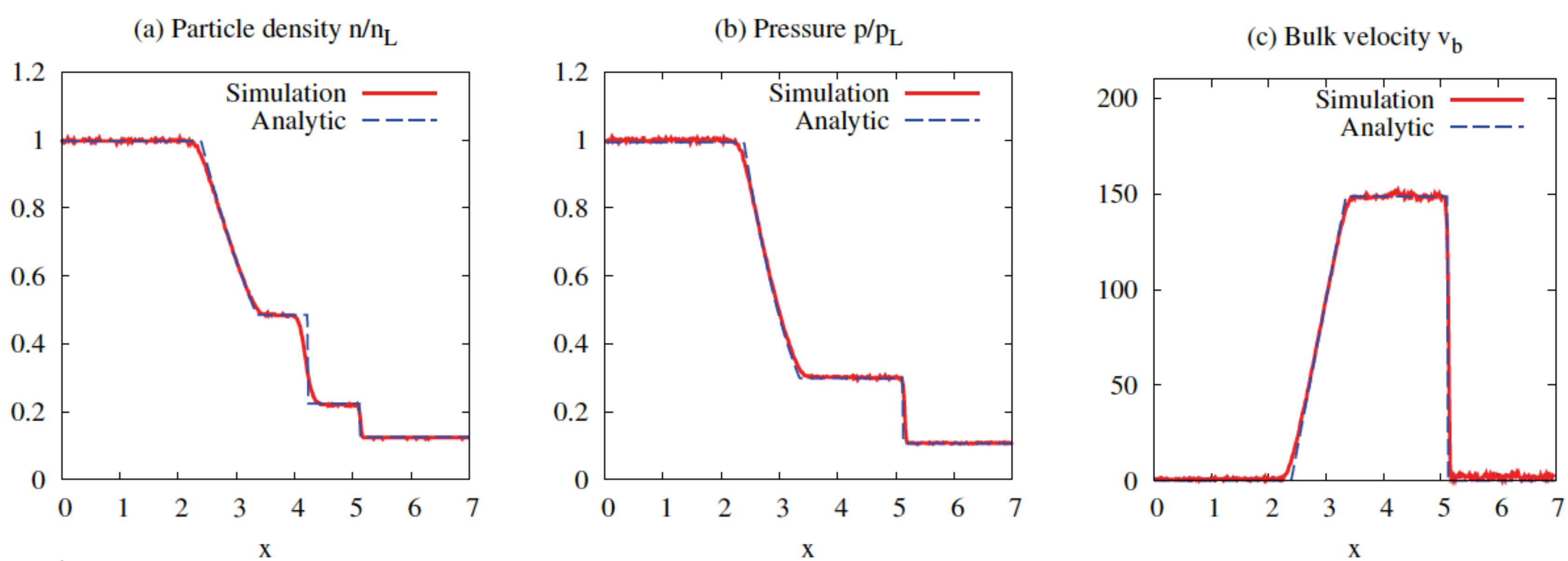


FIG. 9: 3D Sod shock test: $N = 2.4 \times 10^7$ test-particles are distributed over $400 \times 20 \times 20$ bins with $\lambda = 0.01\,\Delta x$ and $\Delta t = 0.25\,\Delta x/v_R$. (a) Particle number per bin, (b) pressure, and (c) bulk velocity with developed shock profiles at timestep $t = 350\,\Delta t$.



Cylindrical Noh-Test

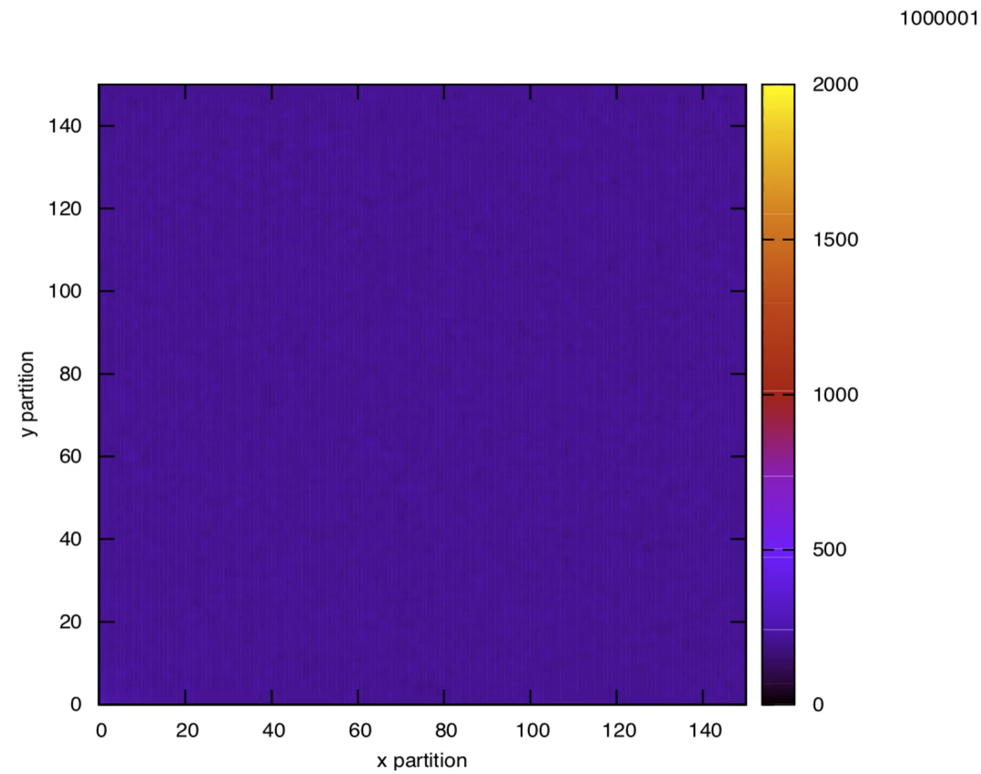
Homogeneous gas with uniform radial inward speed $v_{\rm in}$

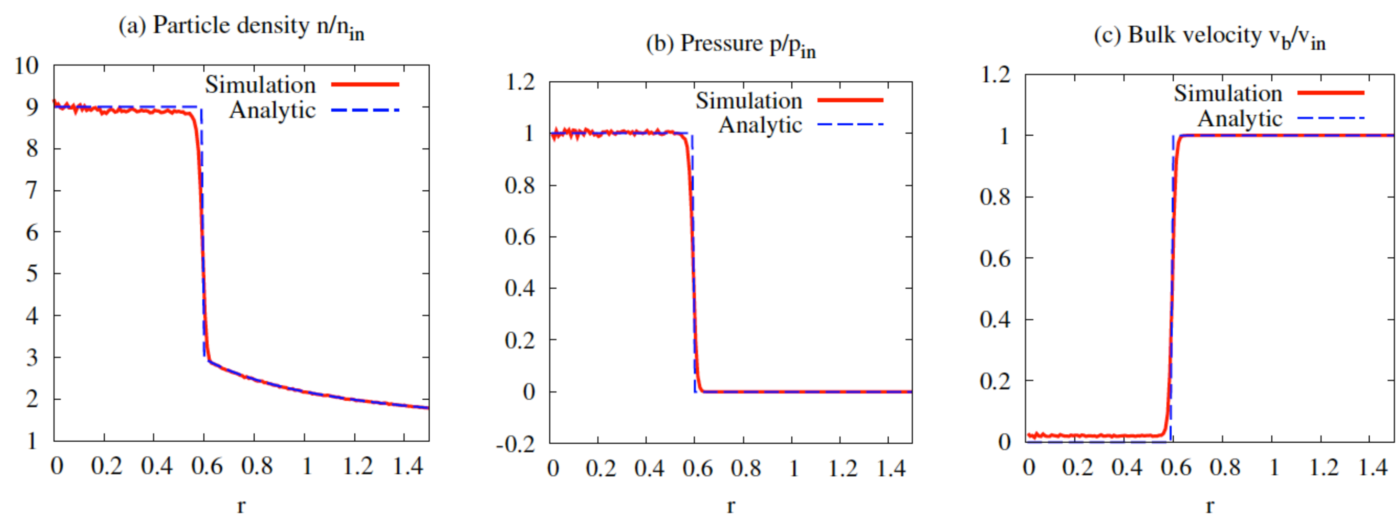
$$r_{\rm shock}(t) = \frac{1}{2}(\gamma - 1)\, v_{\rm in} t$$

$$n(r) \;=\; n_0 \left(\frac{\gamma+1}{\gamma-1}\right)^d, \quad r < r_{\rm shock}$$

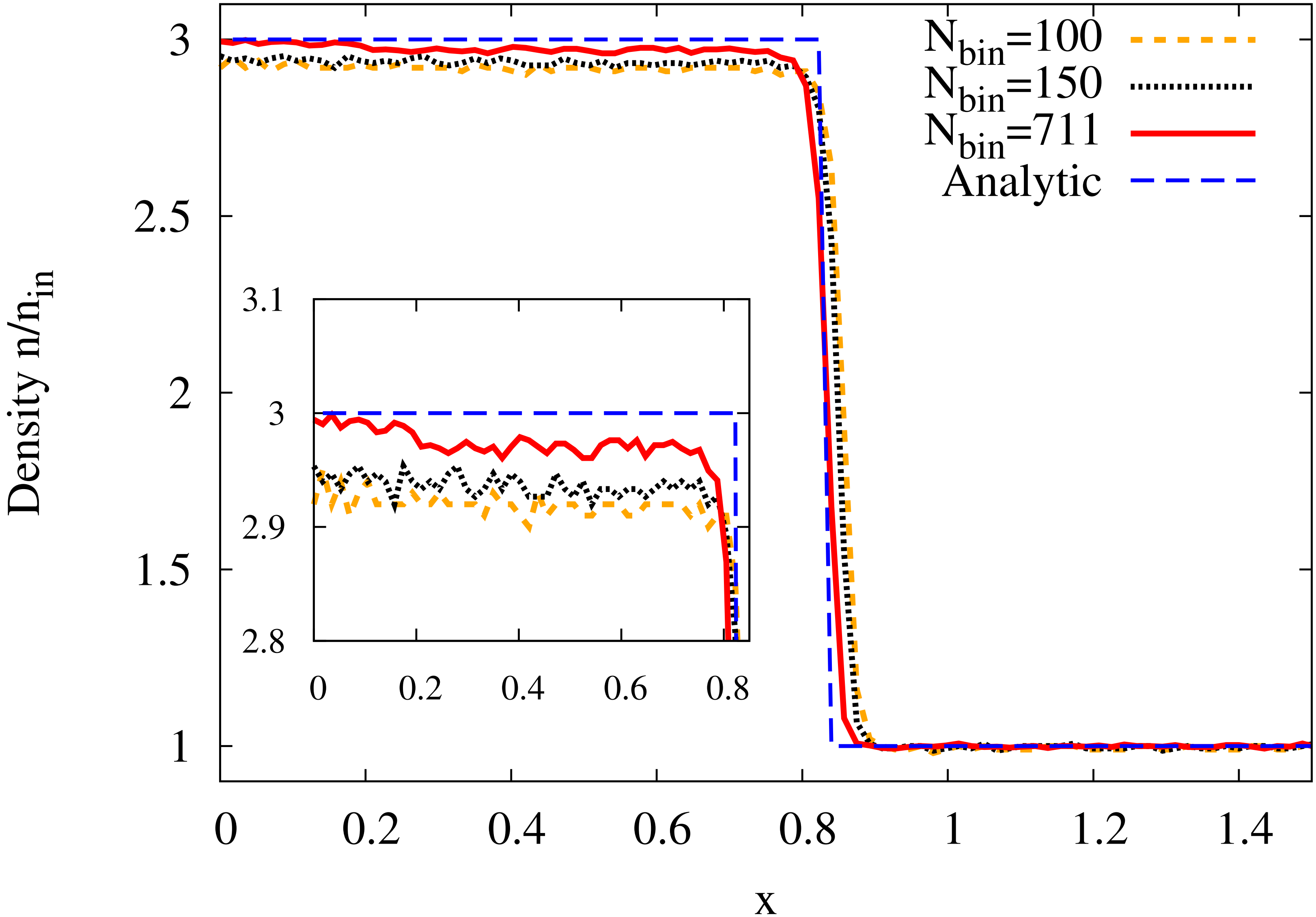
$$n(r) \;=\; n_0 \left(1 + \frac{v_{\rm in} t}{r}\right)^{d-1}, \quad r \geq r_{\rm shock}$$

$(\gamma = 1 + 1/\text{dof})$

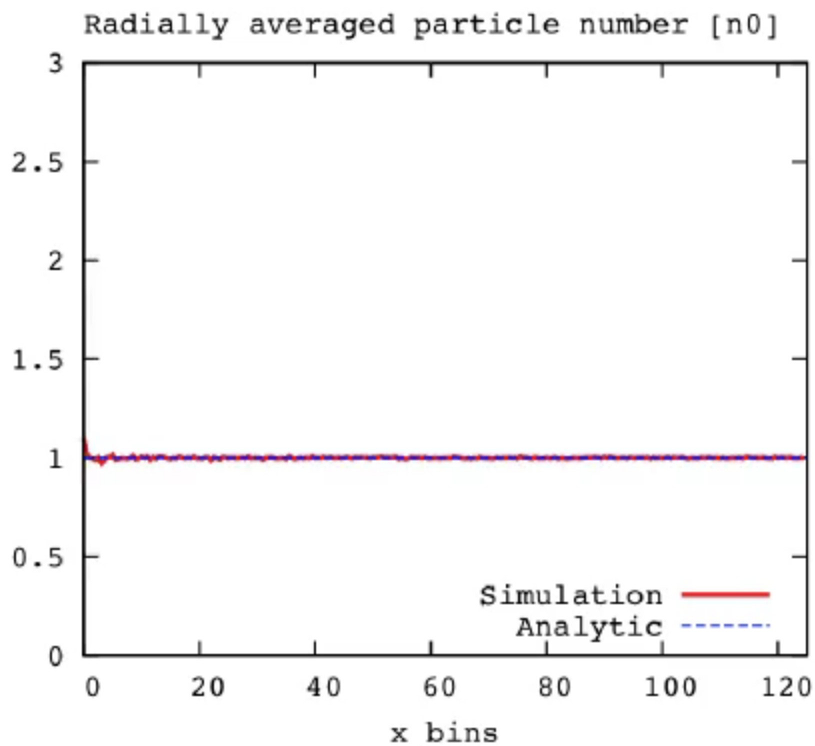


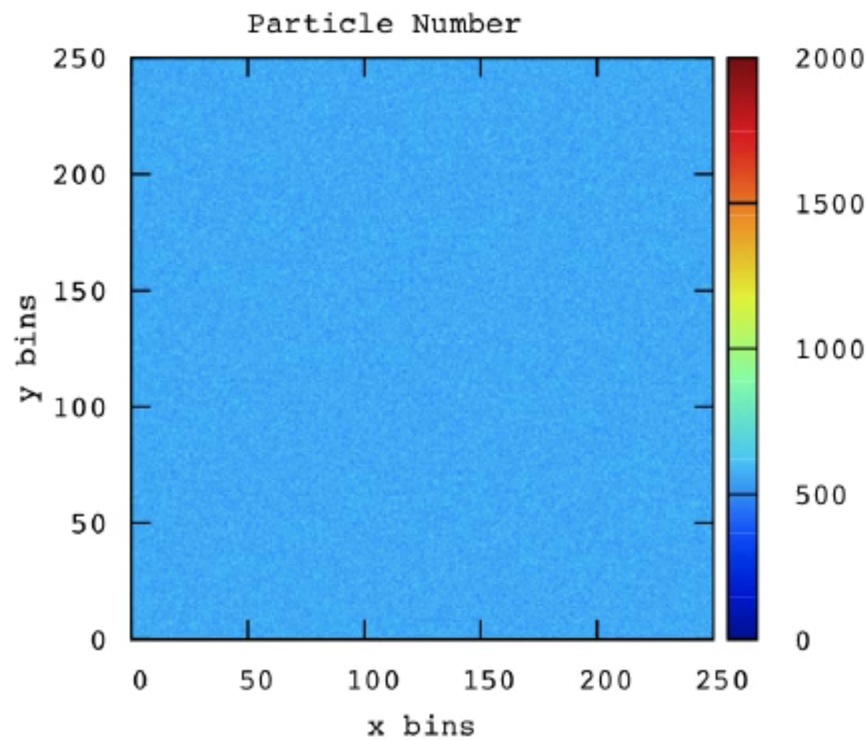


Planar Noh-Test: Convergence



Sedov Test

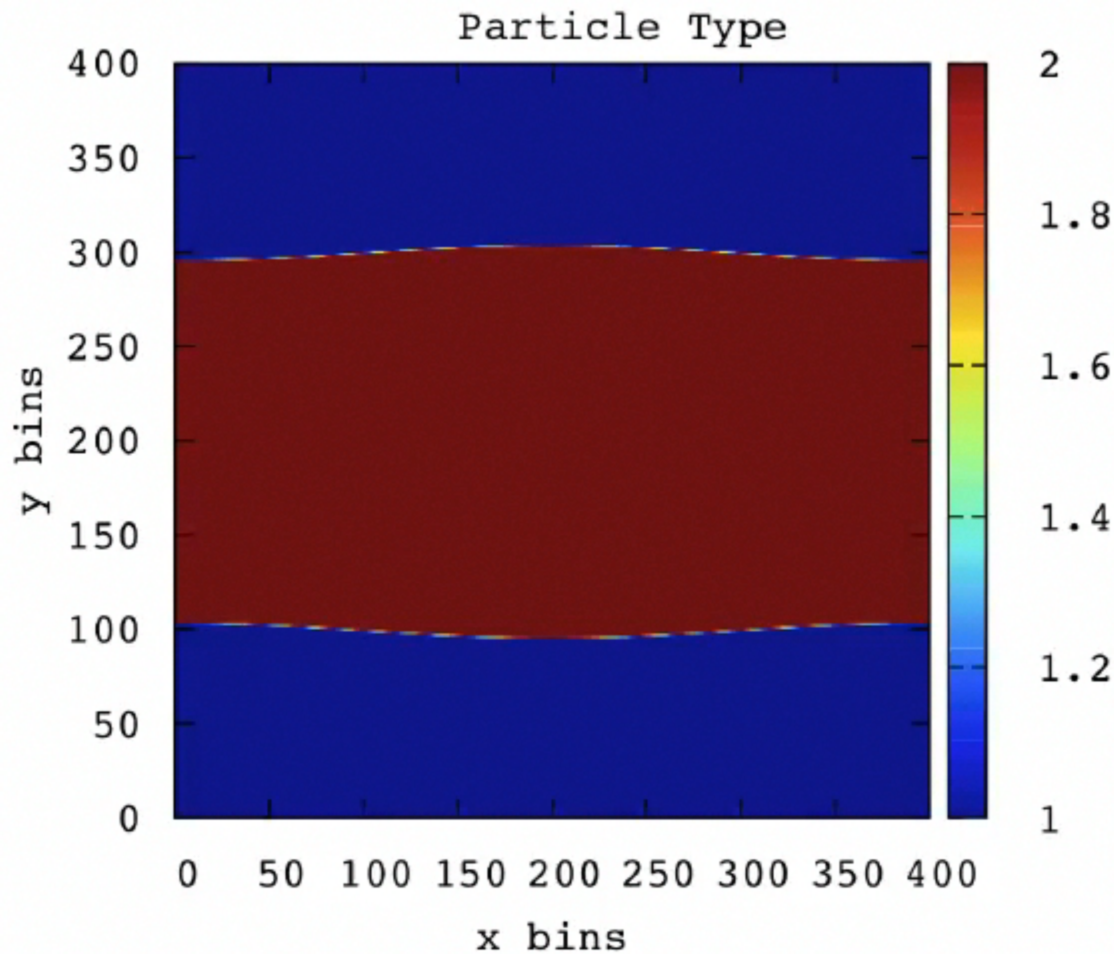




- No wall heating
- No causality violations

- No shock wave diffusion
- No "running ahead" of shock front

Kelvin-Helmholtz Instability



Turbulent flow

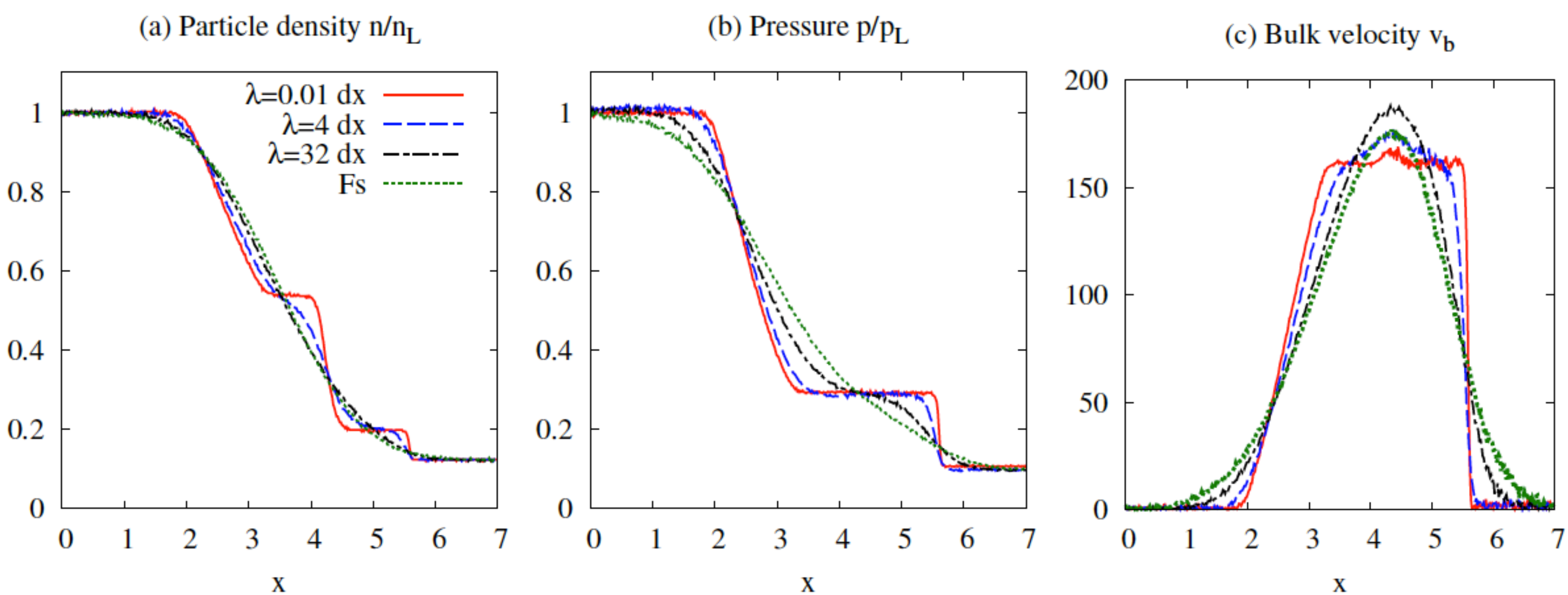
Summary of Tests

- Code passes all standard hydrodynamic tests
- (Slow) convergence to analytic results with increasing test particle number
  - Typical number of test particles used in 3d tests: 10 million – 100 million
- No physical limit to precision of shock wave localization

Beyond the Standard Tests

What Hydro Cannot Do

- Large mean free path, large Knudsen number
- Sod shock test:



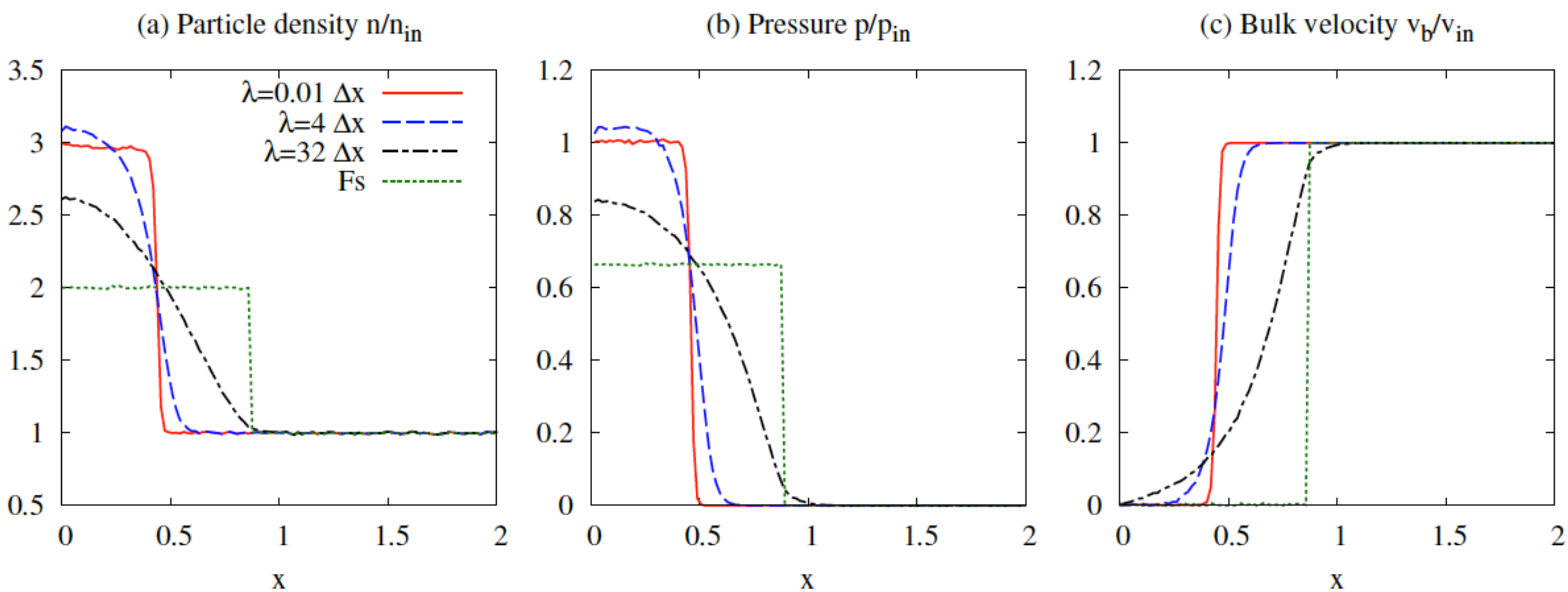
λ = mean free path

dx = box size

Fs = "free streaming", λ infinite

What Hydro Cannot Do

- Large mean free path, large Knudsen number
- 2d Noh shock test:



λ = mean free path
dx = box size
Fs = “free streaming”, λ infinite

What about neutrinos?

- Supernova explosion driven by neutrino shock (?)
- Neutrinos cannot be modeled by hydro
  - Extremely small cross sections
  - Very large Knudsen number
- Kinetic theory: no problem
  - Can be calculated in the same framework

Coupled transport equations

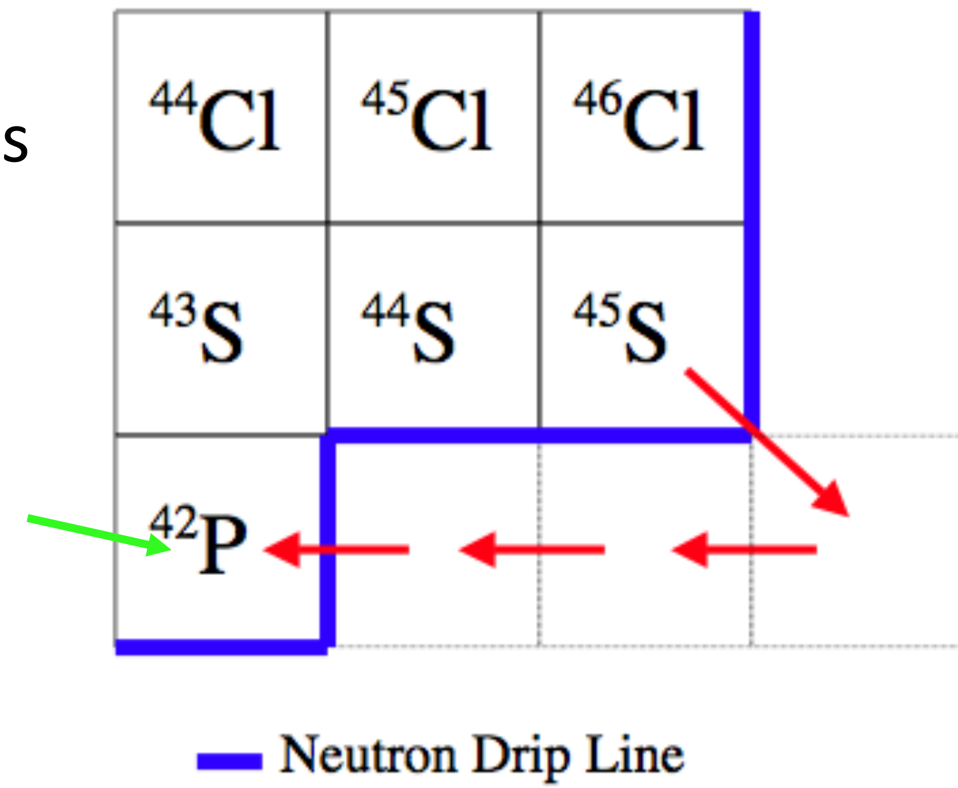
$$\frac{\partial f_b(xp)}{\partial t} + \frac{\Pi^i}{E_b^*(p)} \nabla_i^x f_b(xp) - \frac{\Pi^\mu}{E_b^*(p)} \nabla_i^x U_\mu(x) \nabla_p^i f_b(xp) + \frac{M_b^*}{E_b^*(p)} \nabla_i^x U_s \nabla_p^i f_b(xp)$$
$$= I_{bb}^b(xp) + \boxed{I_{b\nu}^b(xp)}$$
$$\frac{\partial f_\nu(xk)}{\partial t} + \frac{k \cdot \nabla^x}{E_\nu(k)} f_\nu(xk) = \boxed{I_{b\nu}^\nu(xk)}$$

- 2-body collision terms structurally identical to BUU source term
  - Couples transport equations of baryons and neutrinos
  - Essential input: neutrino-nucleus cross sections (Nakamura et al, ApJ 1999; K. Sumiyoshi et al, NPA 2001, Fröhlich et al, PRL 2006, B.A. Brown, …)

Matter Test Particle Properties

- Explicitly represent all nuclei
  - Many hundreds of isotopes
  - Lots of work: reaction network, weak interaction cross sections
  - All Z,A between drip lines
  - Ensemble propagation
  - "Coupled channels" in reaction network
  - Free baryons

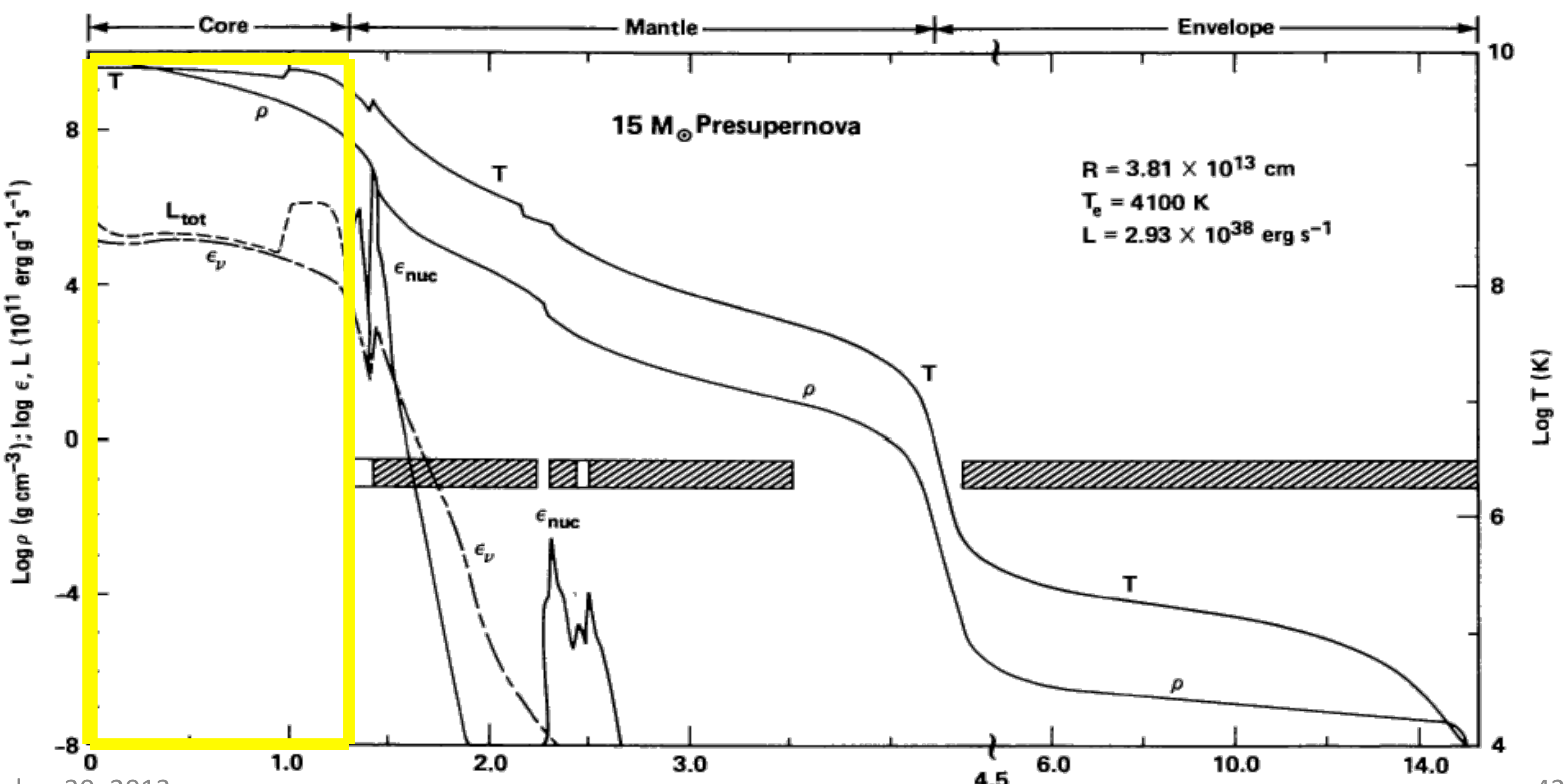
3 free neutrons



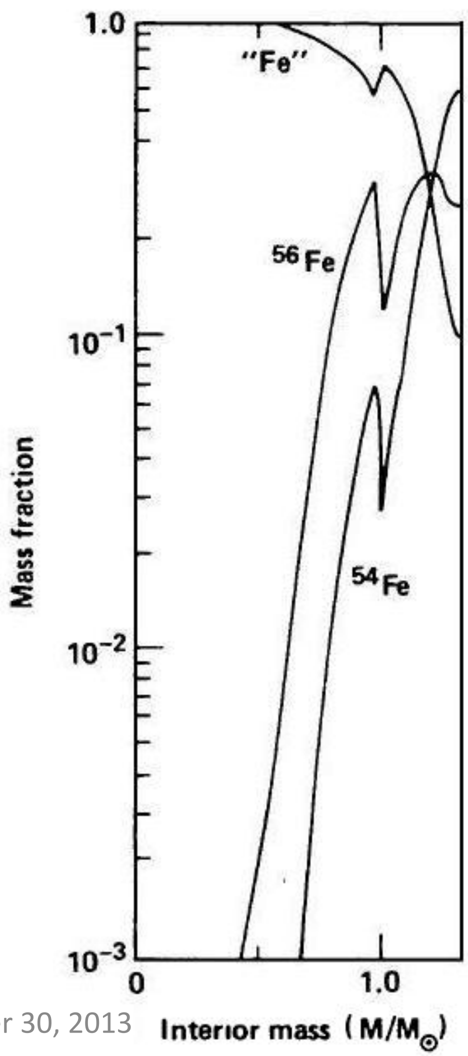
Initial Conditions

Use for the first 10^7 years
Concentrate on last 0.3 seconds

- Start with Woosley & Weaver's 15 $M_\odot$ progenitor



Core Modeled: Initial Conditions

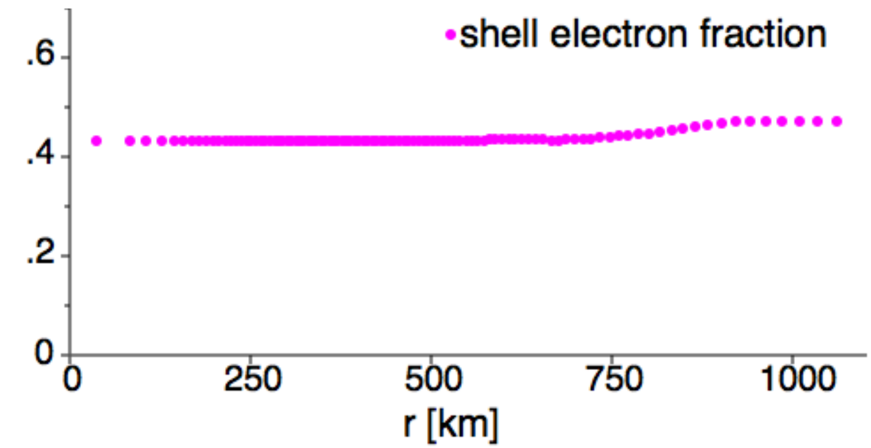


- $M_{core} = 1.33\ M_\odot$
- Spherically symmetric
- Radius ≈ 1000 km

Some Results

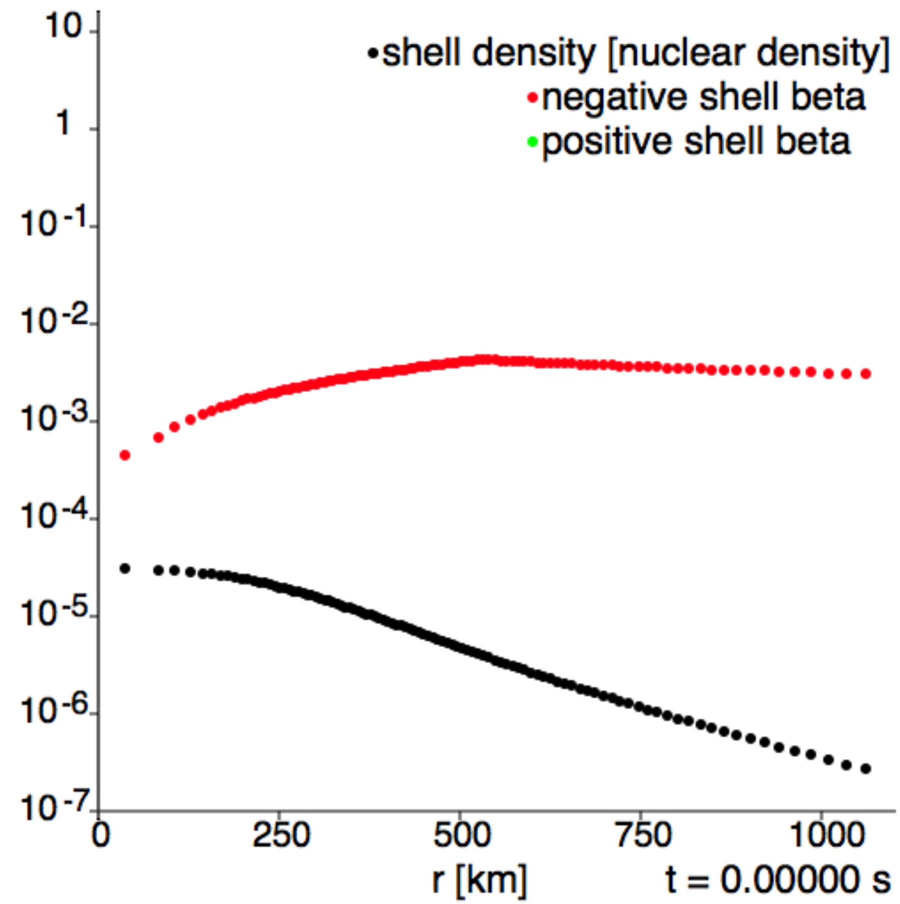
- **Single** processor (spherical symmetry)
- 1 million matter test particles
  - 385 nuclei + free baryons
- Cold soft BKD nuclear EOS
- Weak interaction network
  - Electron capture (reduced FFN rates)
  - Neutrino-matter interactions
  - Neutrino oscillations a la "MSW"
- No fusion or photo-disintegration channels included

Time evolution

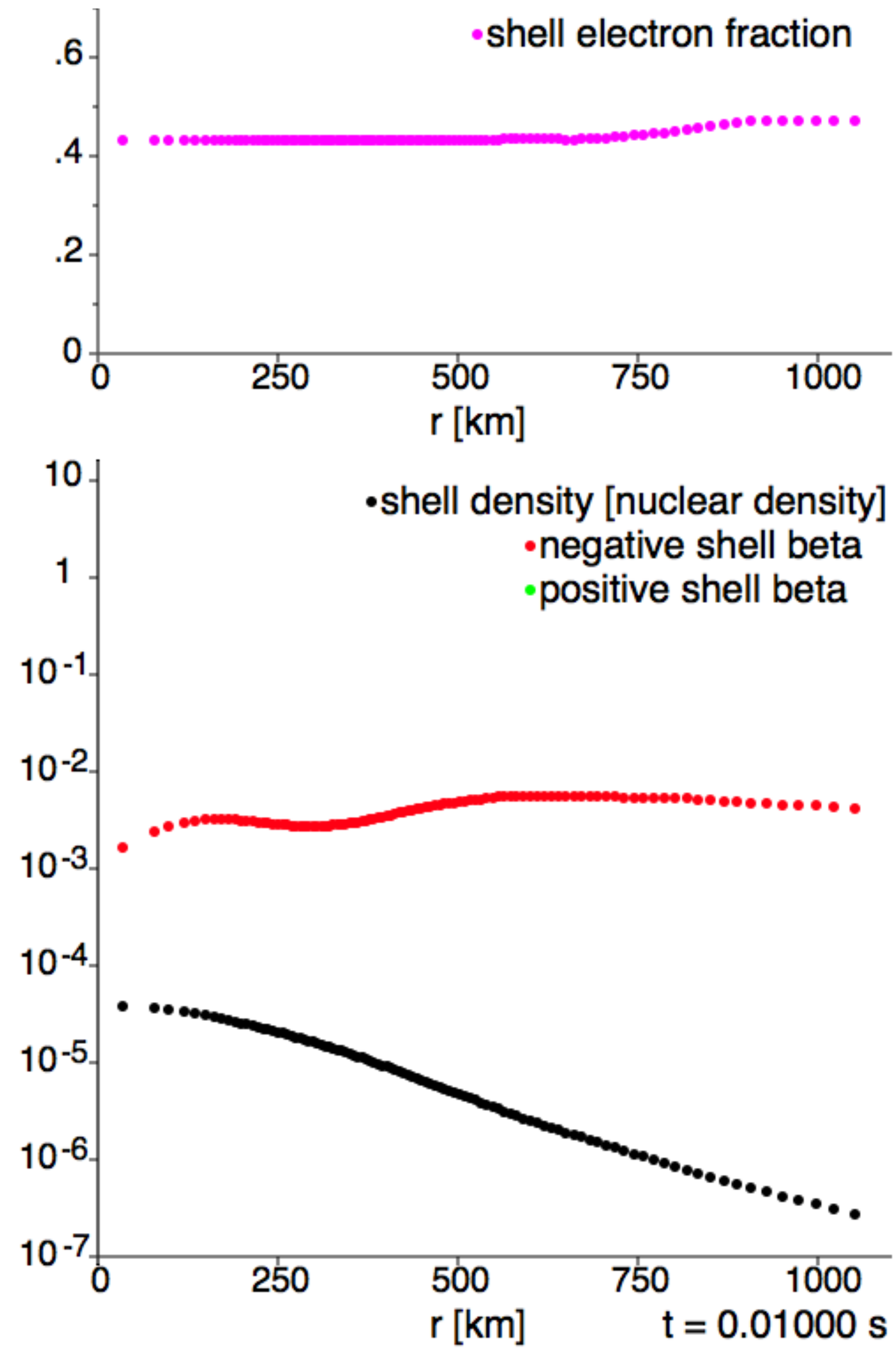


Interplay of macro- and micro-scales forces very large number of comparatively small time steps
(c ~ 1 ft/ns)
$\Delta t = 10^{-5}$ s

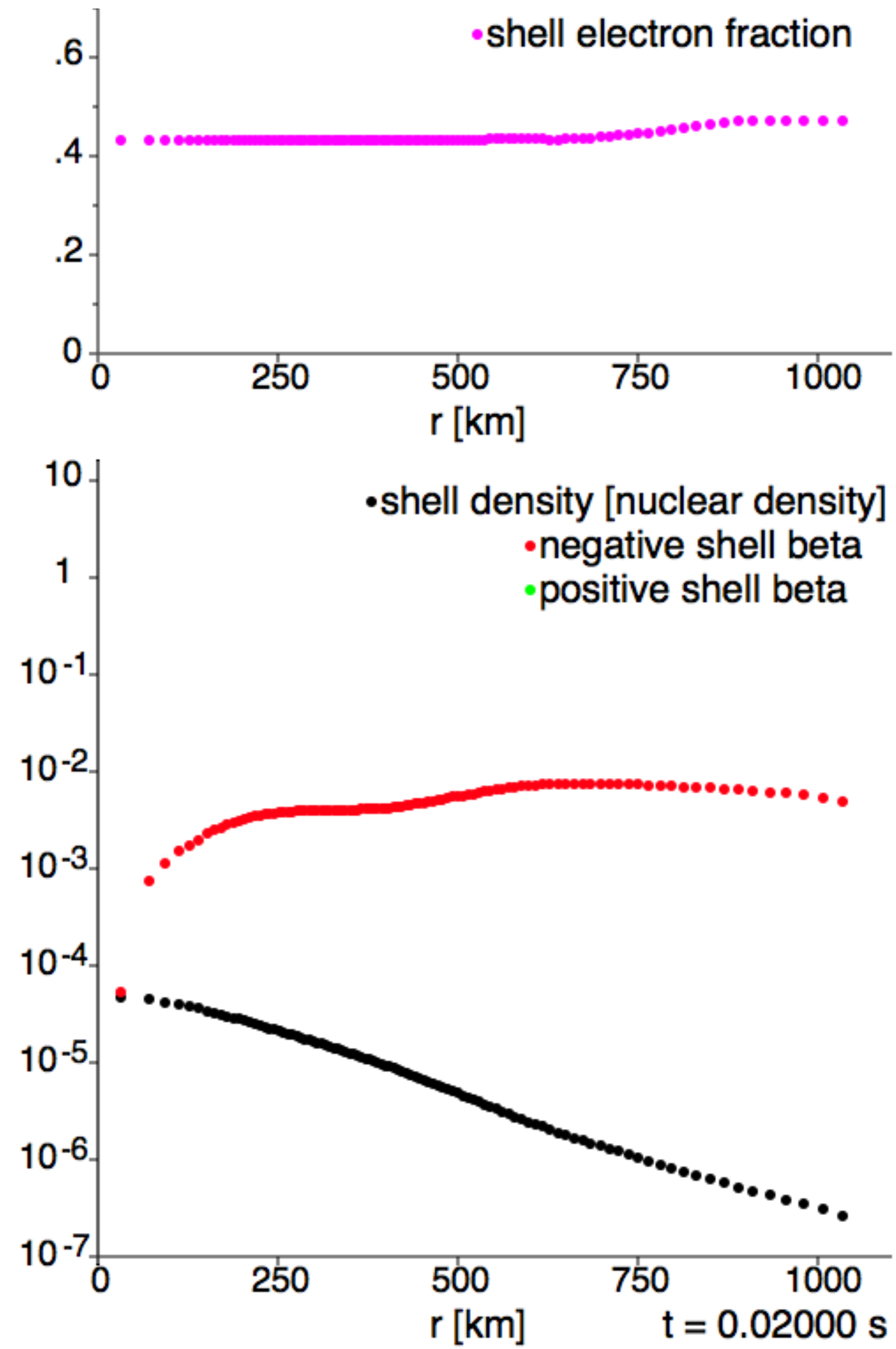
=> Mostly boring initial time evolution (take 1000 steps between frames)



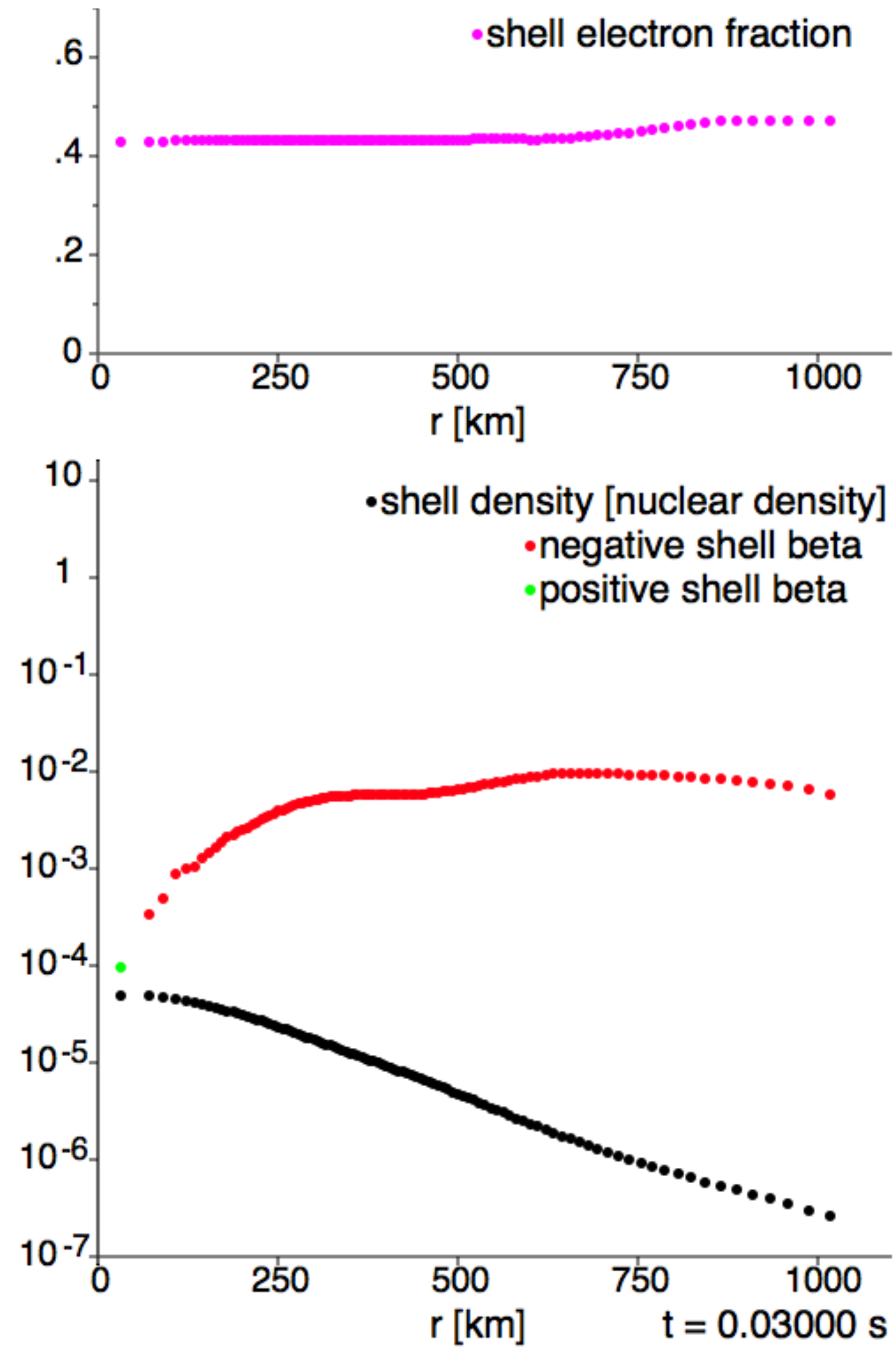
Time evolution



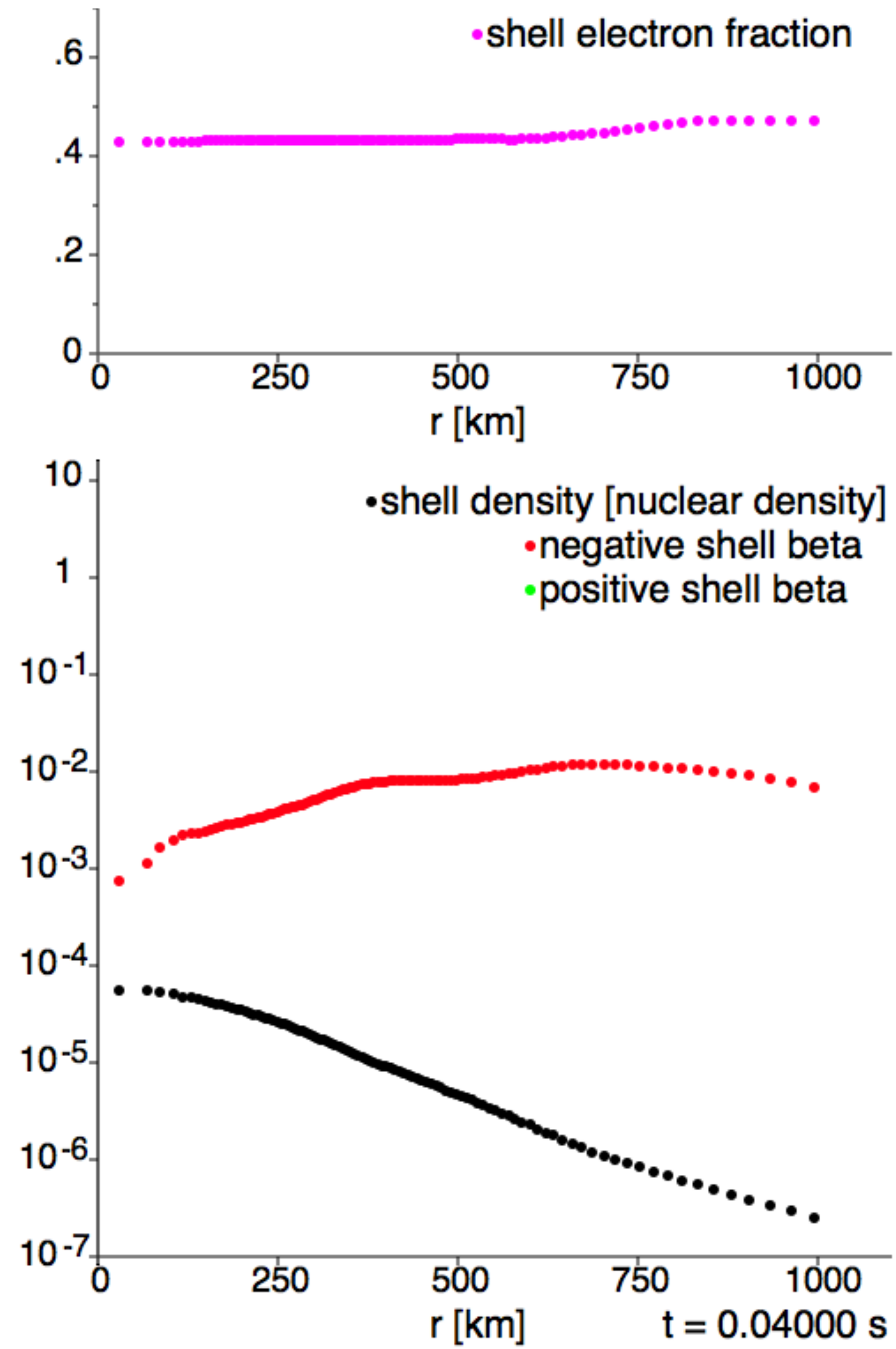
Time evolution



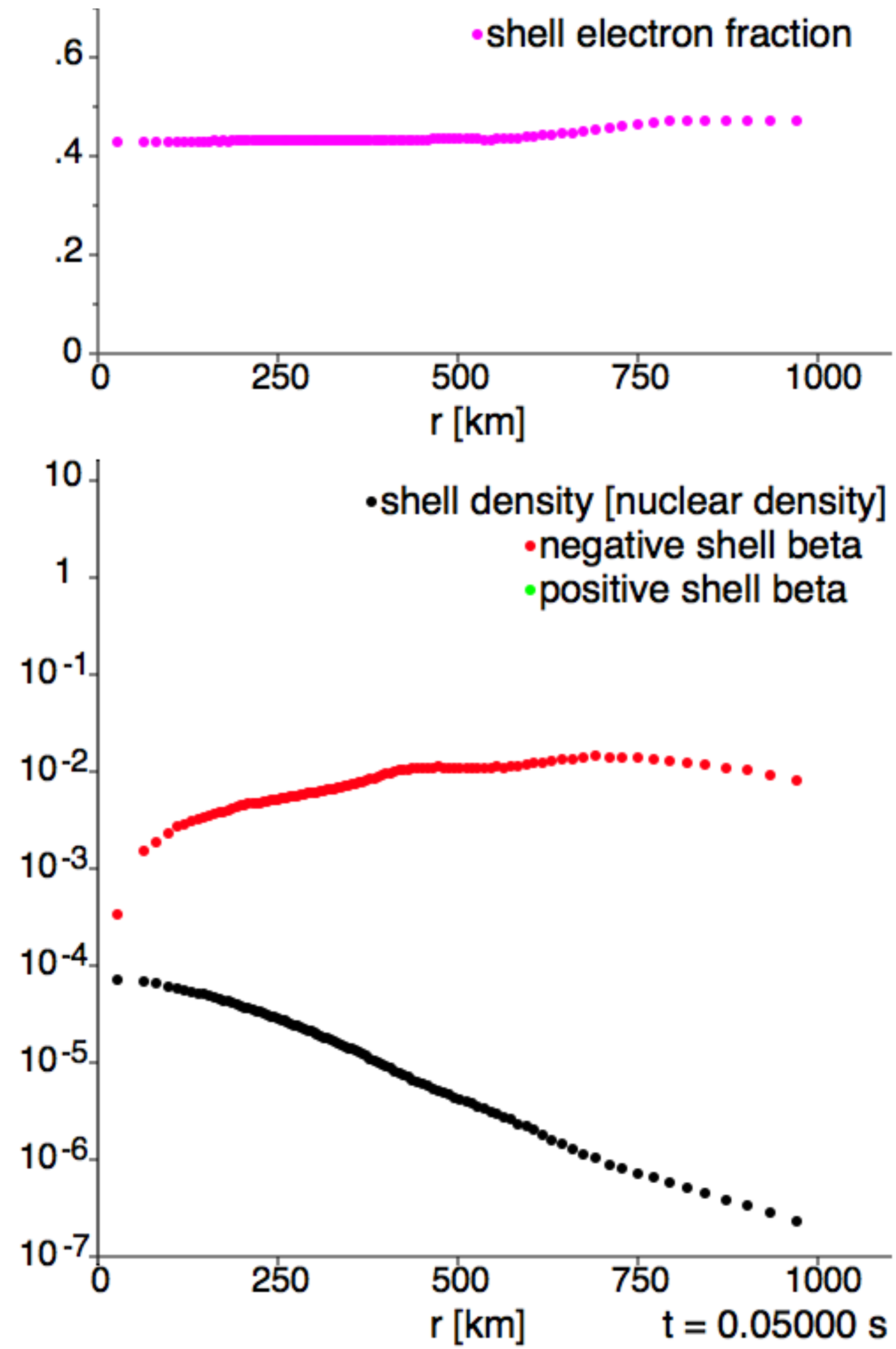
Time evolution



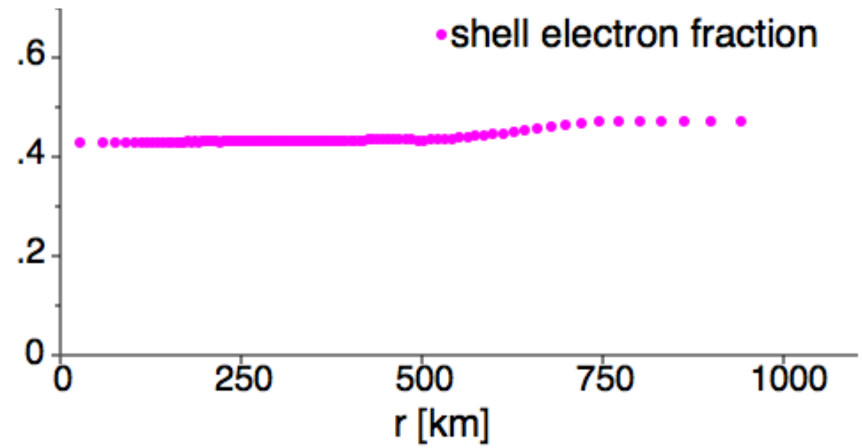
Time evolution

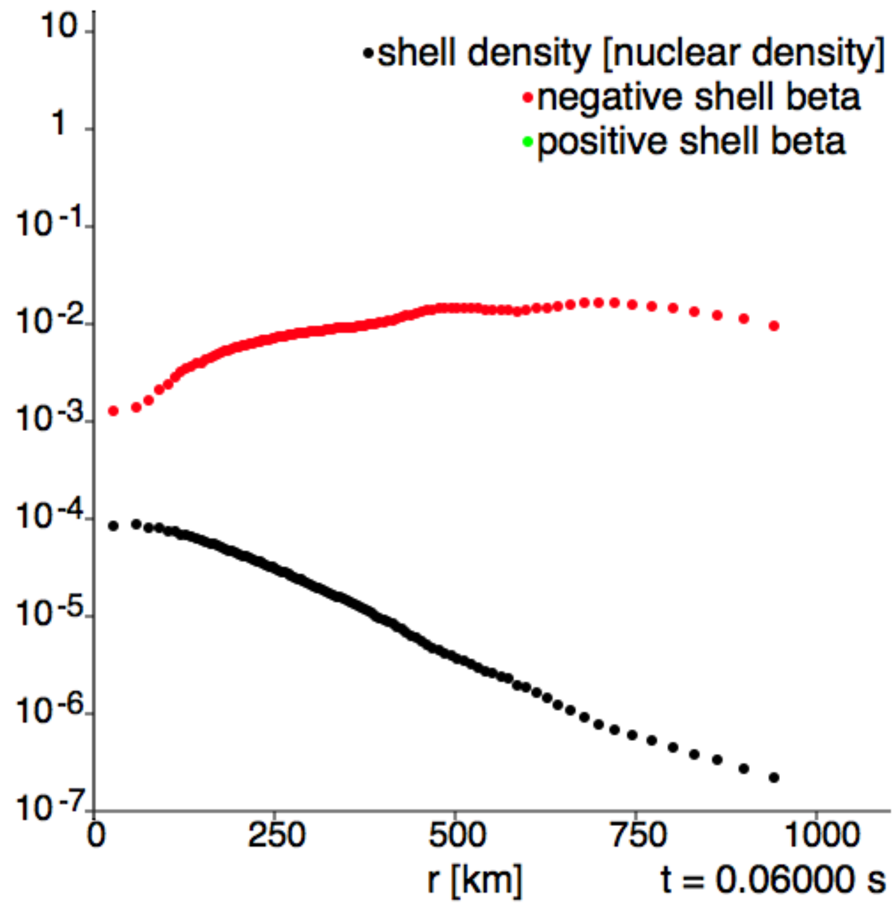


Time evolution

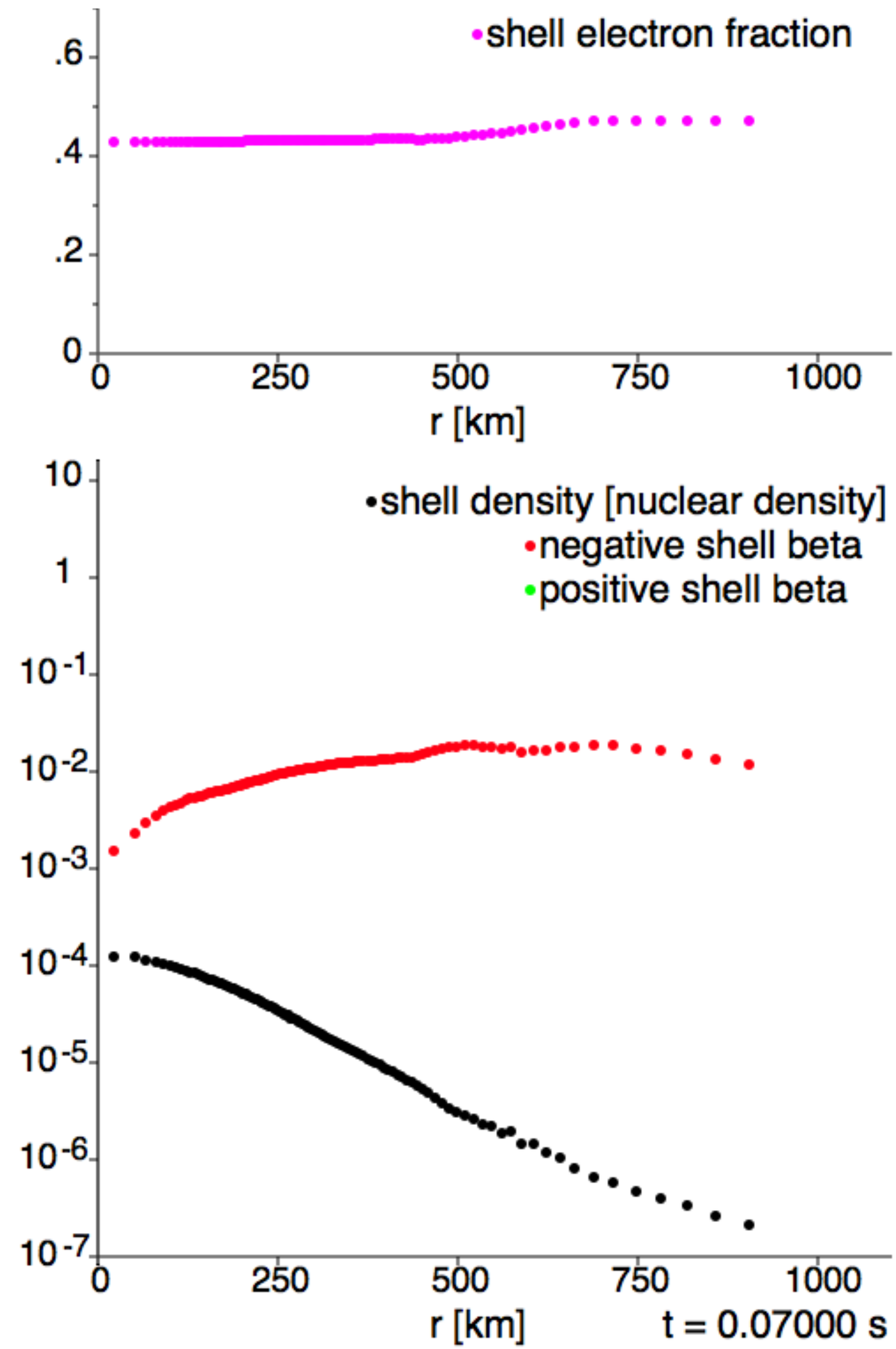


Time evolution

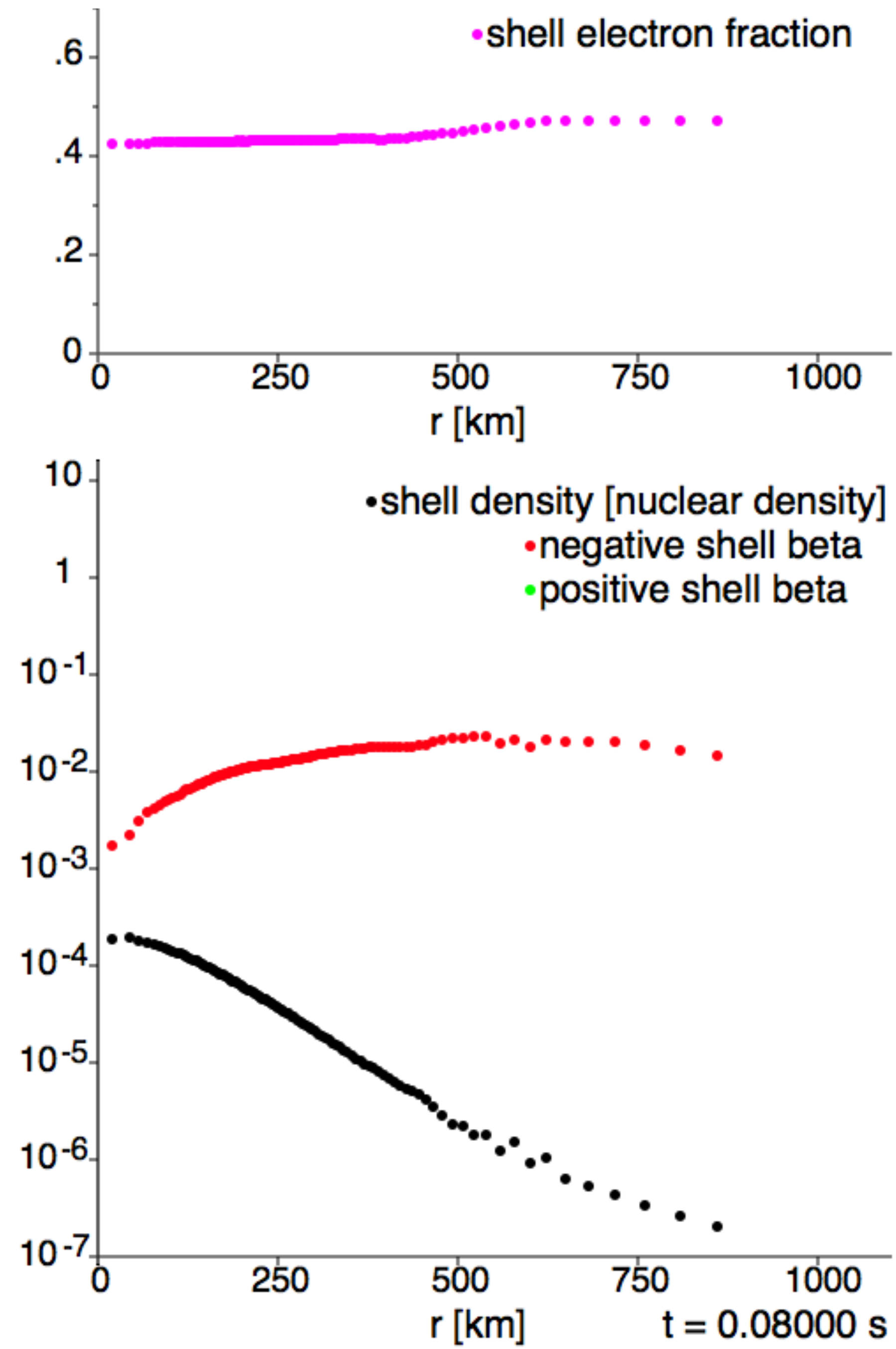




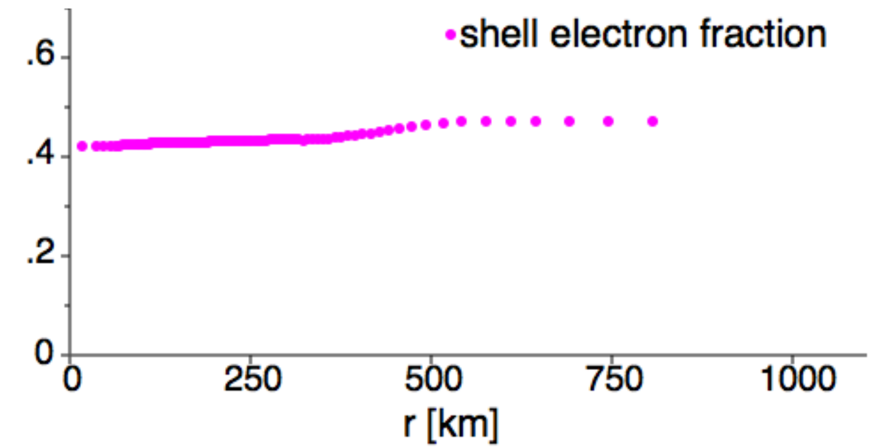
Time evolution

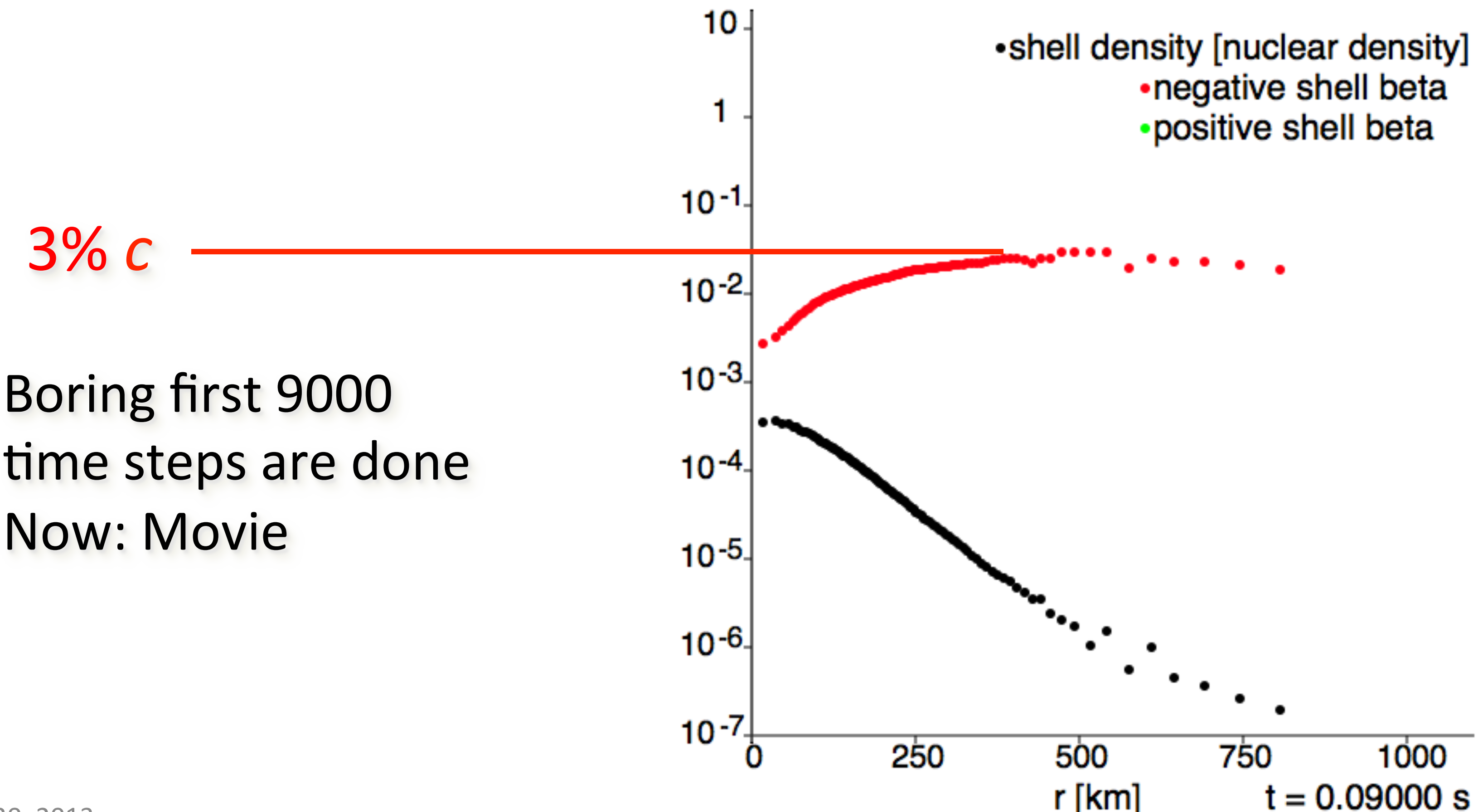


Time evolution



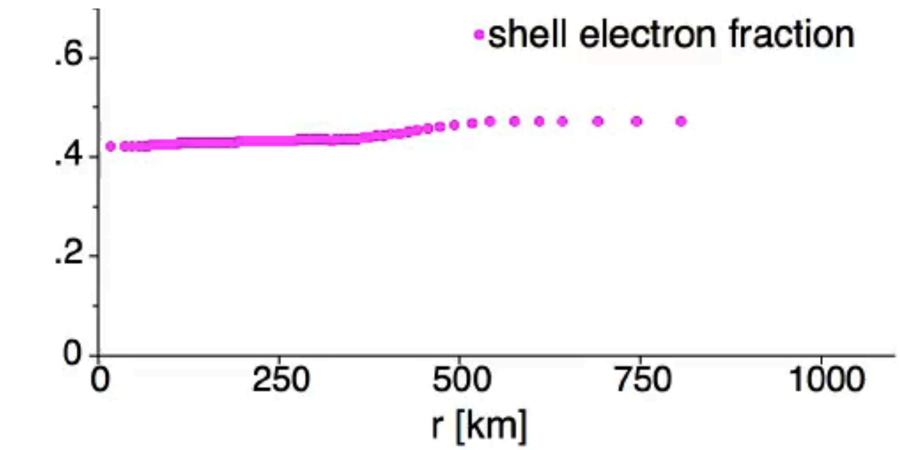
Time evolution

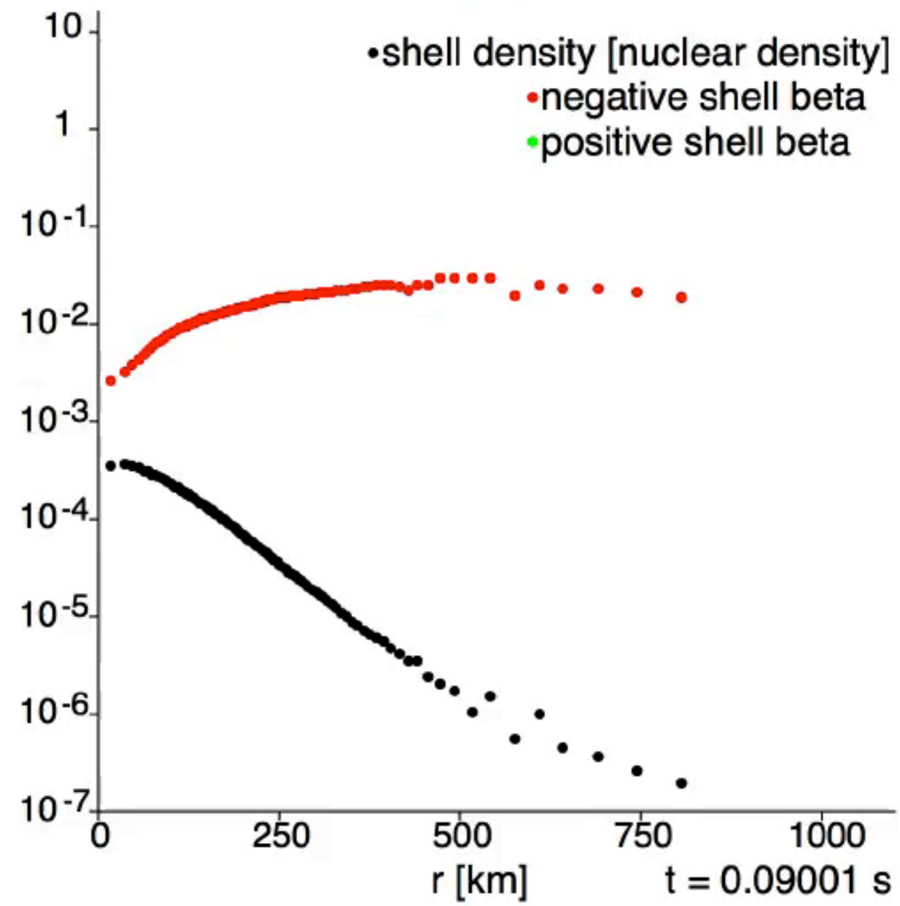




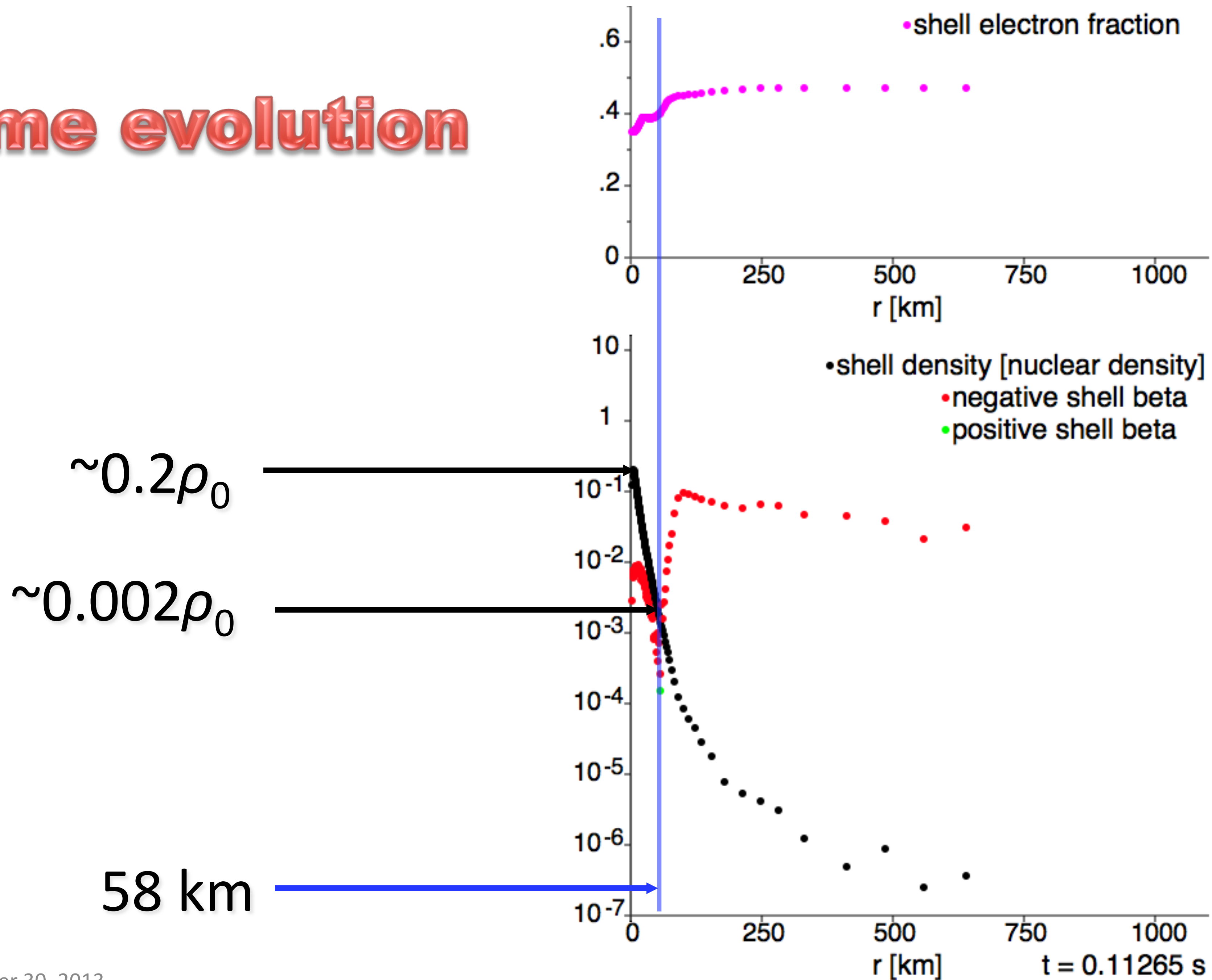
Boring first 9000
time steps are done
Now: Movie

Time evolution

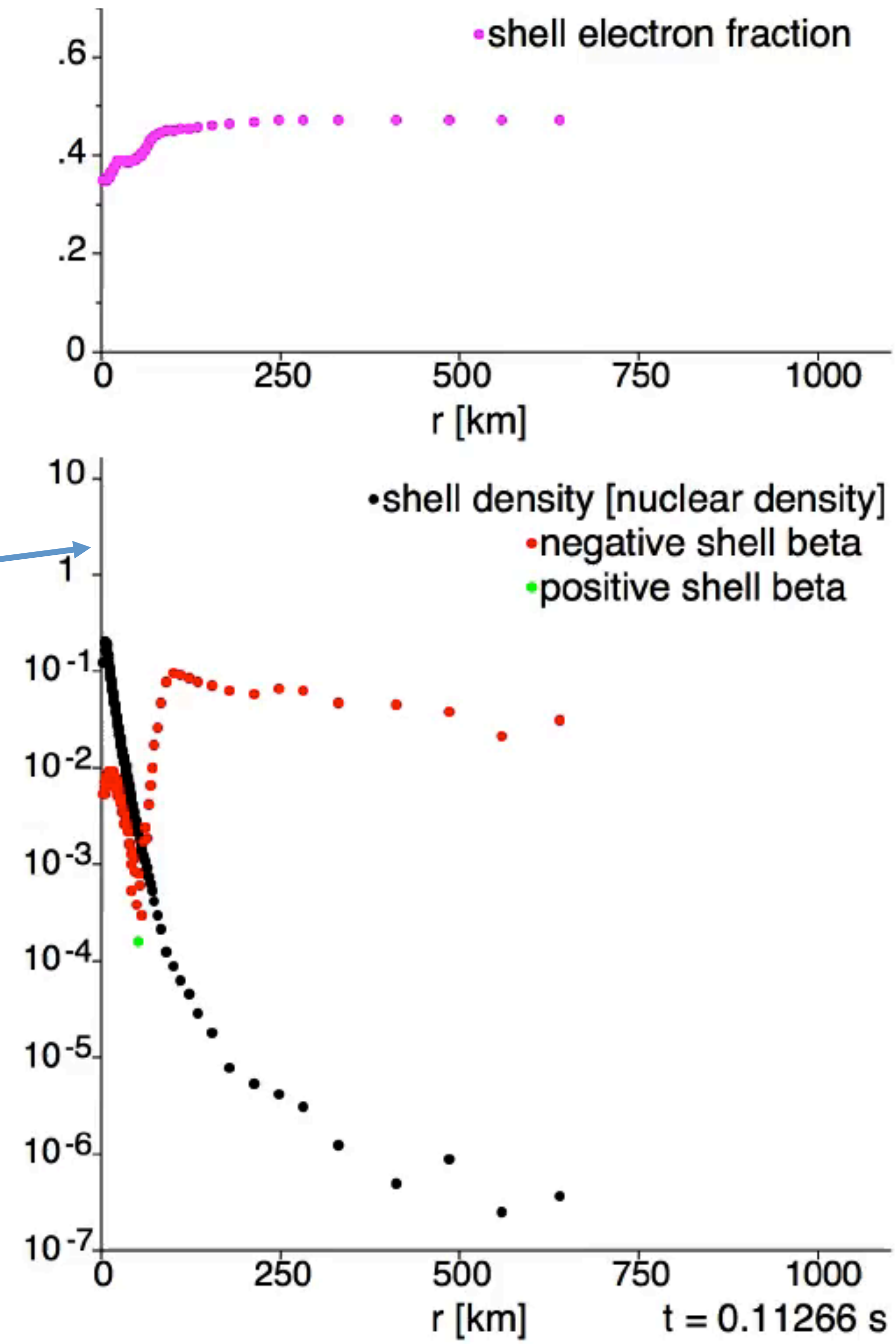




Time evolution



Time evolution



Remnant

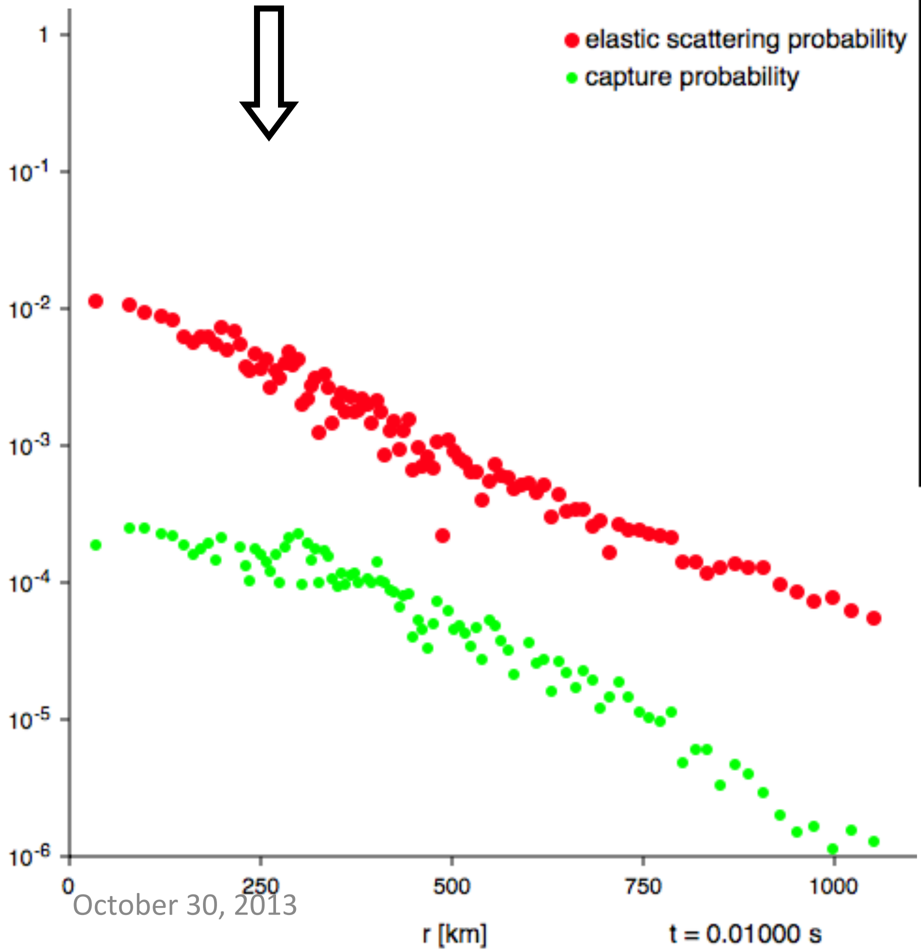
- M=0.25 $M_{\odot}$
- R=7.3 km
- $\rho_{central} = 1.7\rho_0$
- $\eta_{central} = 0.27$

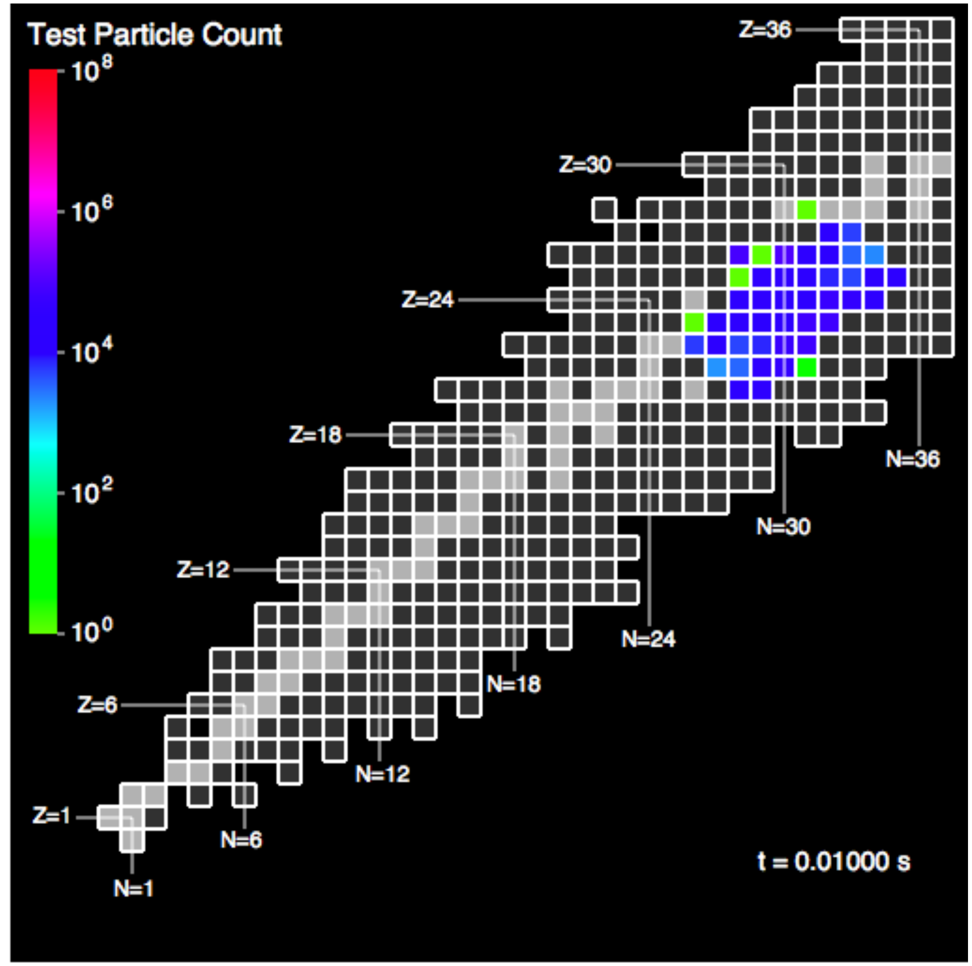
Why?

- Electron fraction spike "cuts" the core in two
  - Proto-remnant "gently" assumes ideal configuration
  - Role of nuclear EOS totally different
- How does the spike form?
  - $\rho(r_{\mathrm{exp}}) \sim 0.002\rho_0$
  - Study neutrino-matter interaction probabilities
    - Nuclear structure
    - Relativistic electron gas statistical mechanics
    - Essential input: neutrino cross sections & nuclear structure (weak neutral current ~ A^2)

t = 0.01000 s

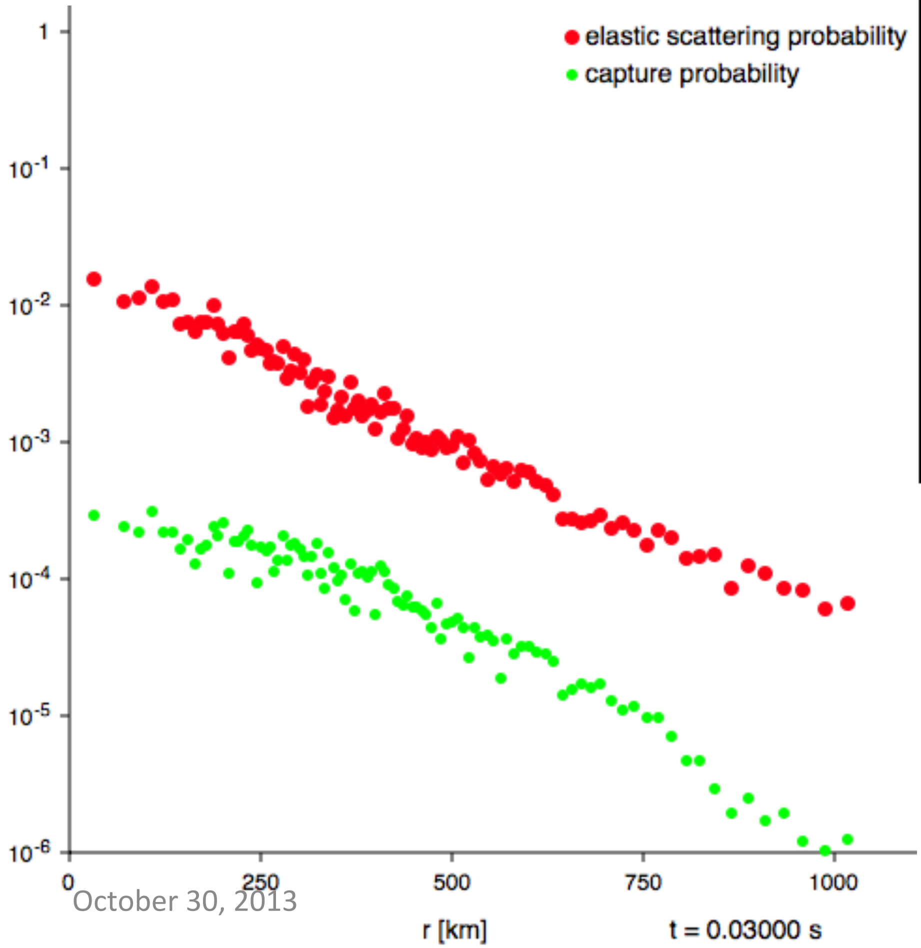
Average neutrino interaction probability

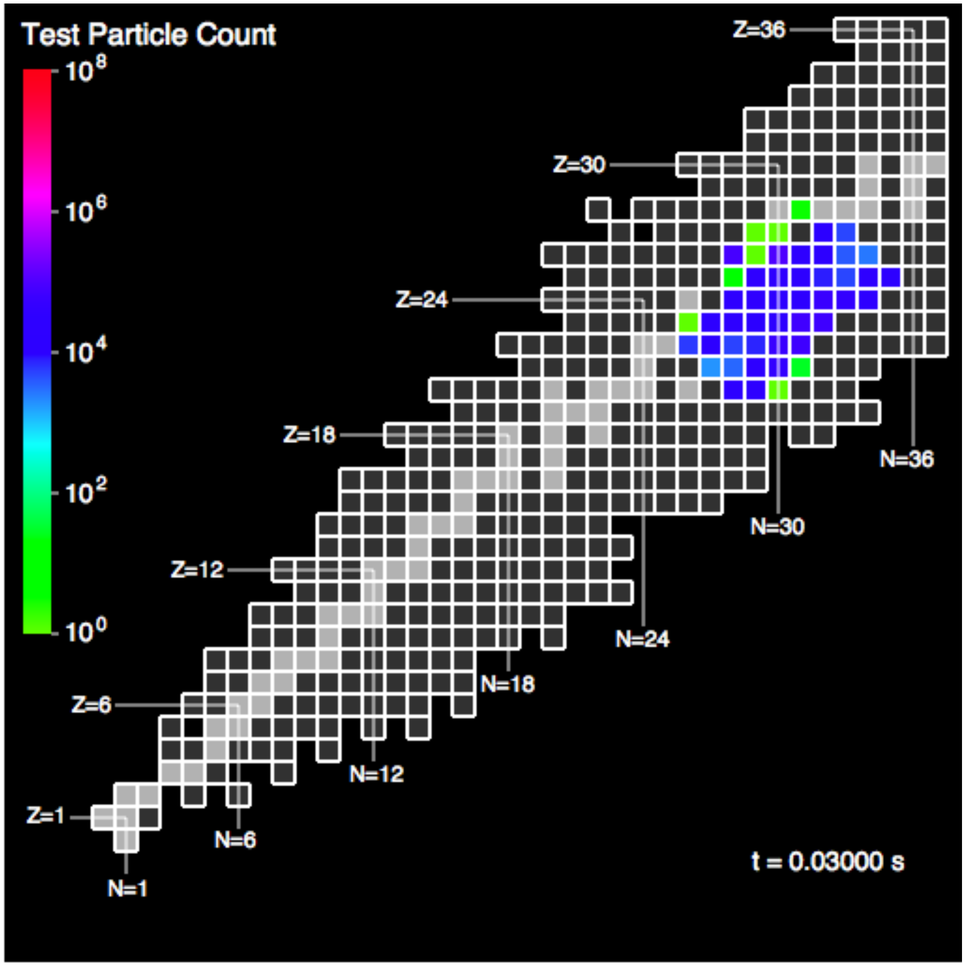




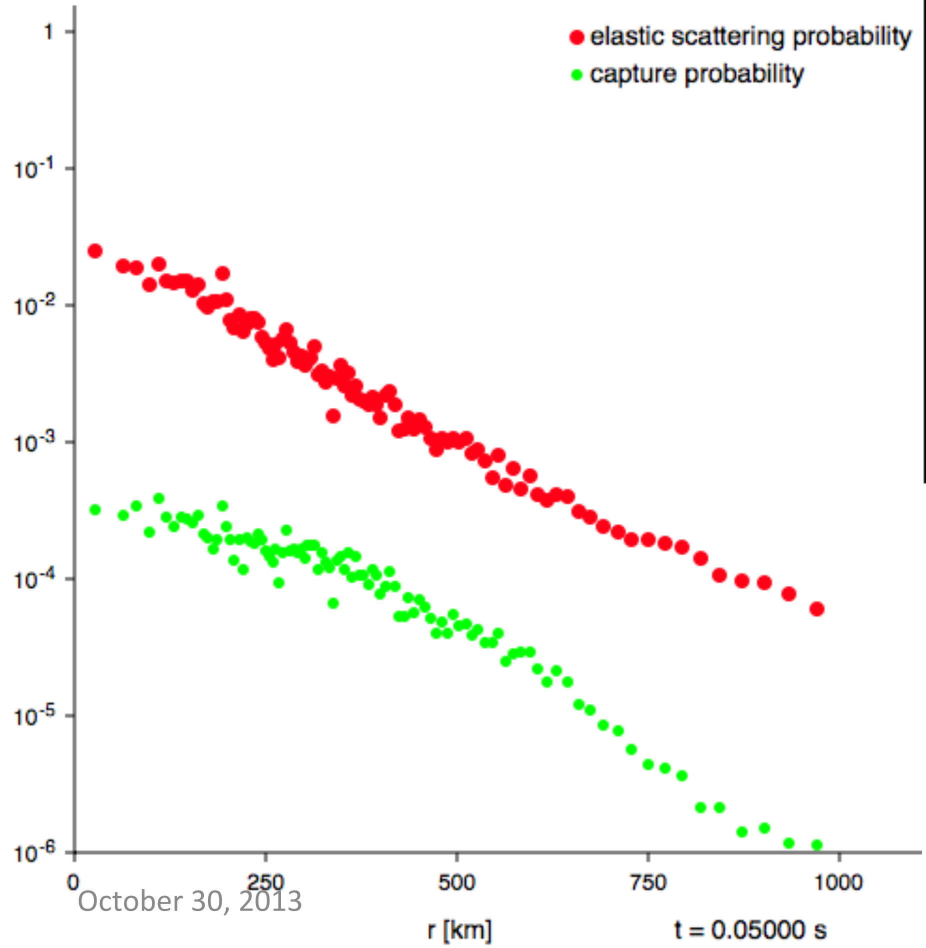
Isotope composition

t = 0.03000 s

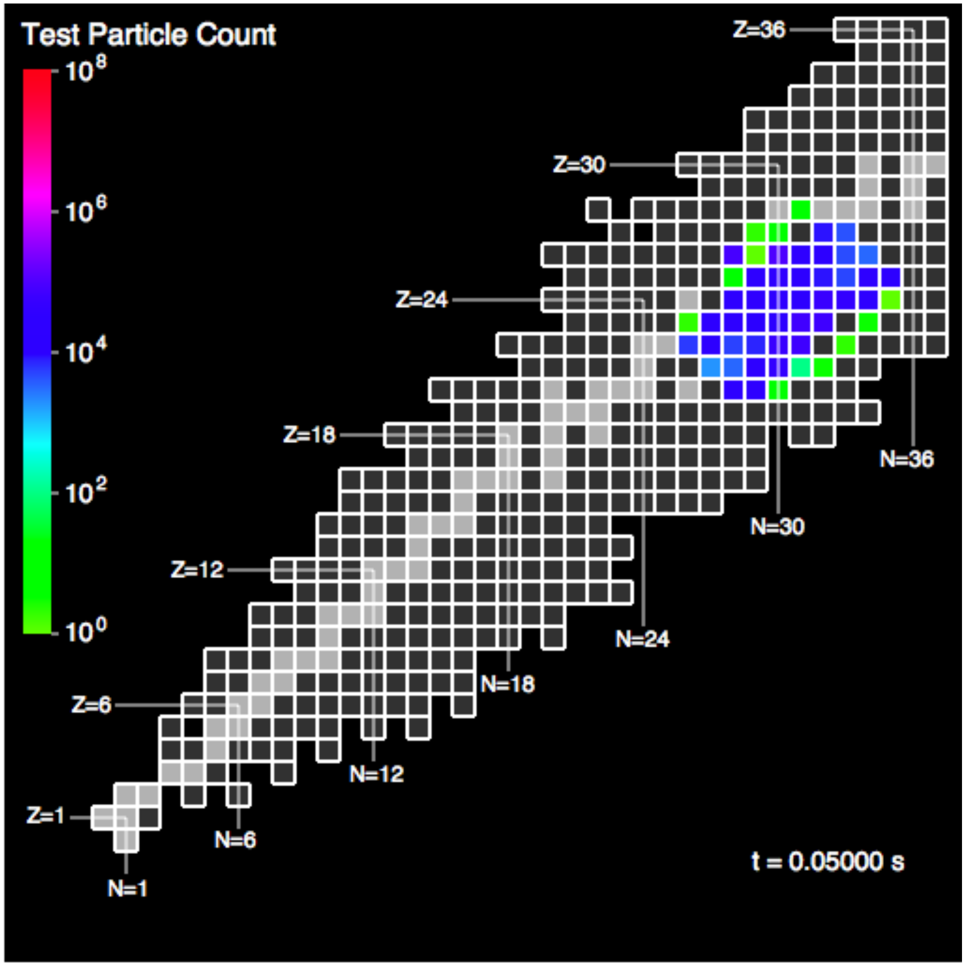

elastic scattering probability
capture probability
1
10-1
10-2
10-3
10-4
10-5
10-6
0
250
500
750
1000
r [km]
t = 0.03000 s


Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.03000 s

t = 0.05000 s

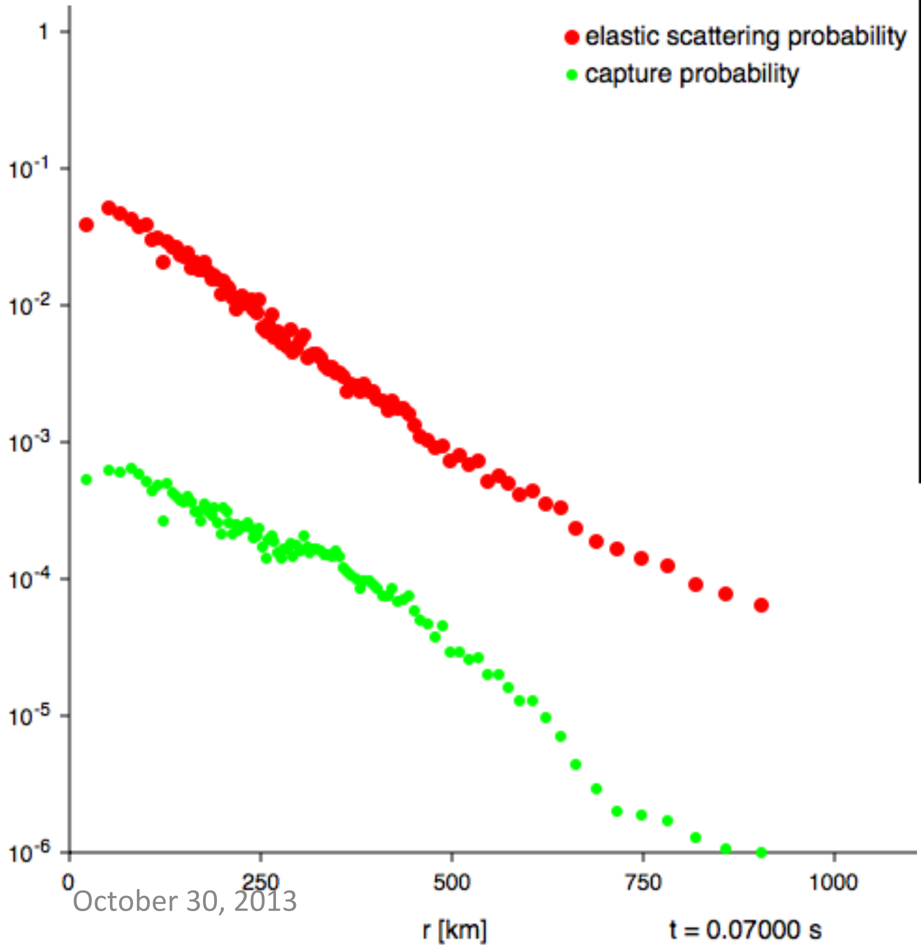


elastic scattering probability
capture probability
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
250
500
750
1000
r [km]
t = 0.05000 s

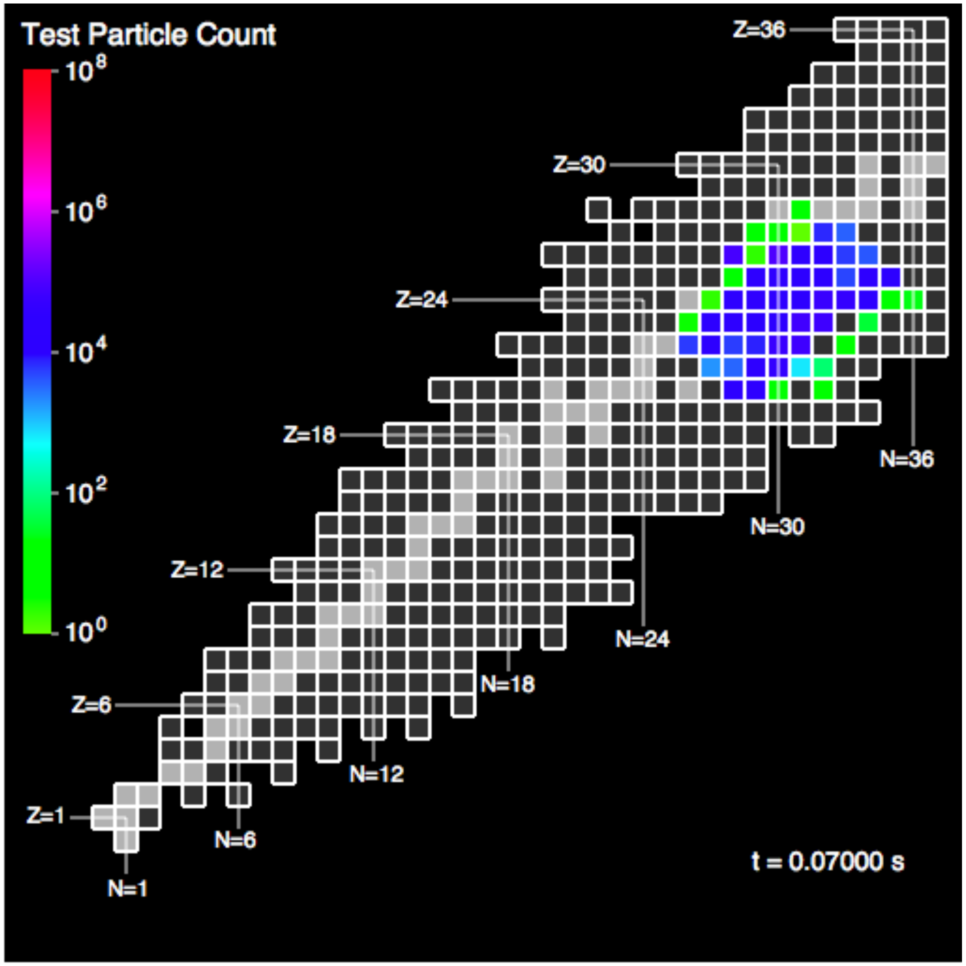


Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.05000 s

t = 0.07000 s

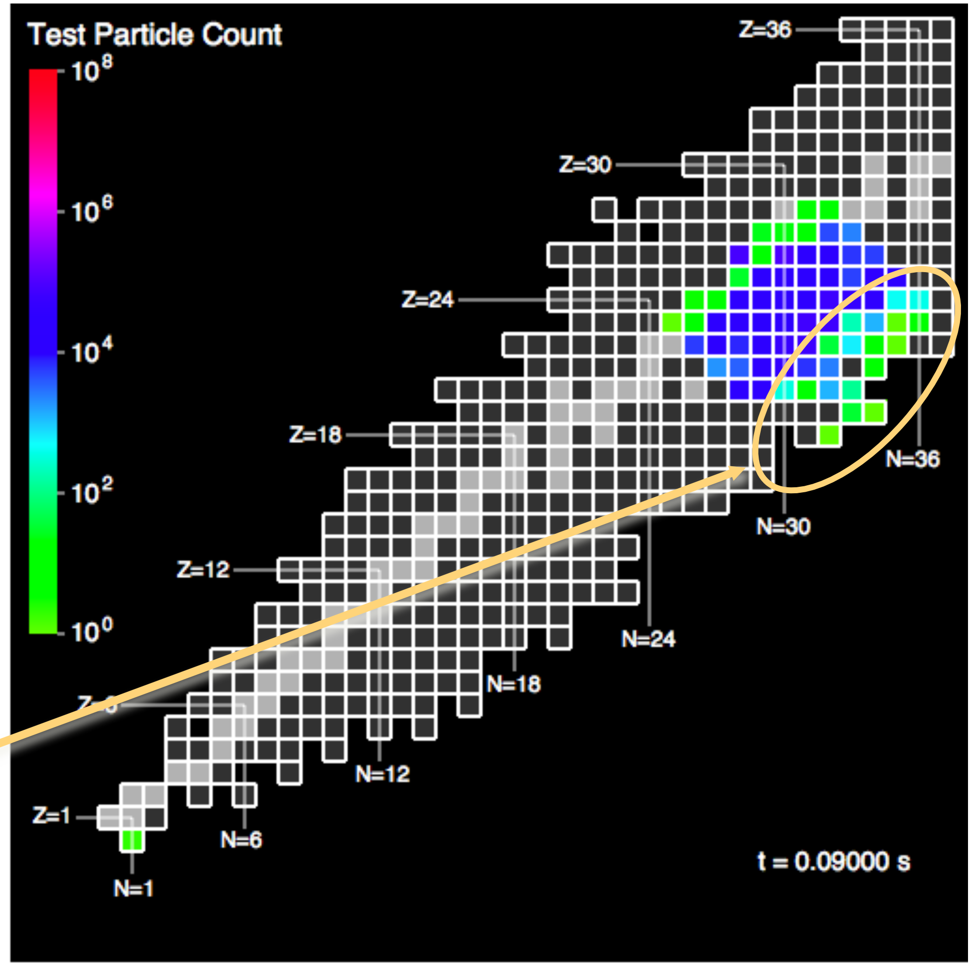


elastic scattering probability
capture probability
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
250
500
750
1000
r [km]
t = 0.07000 s

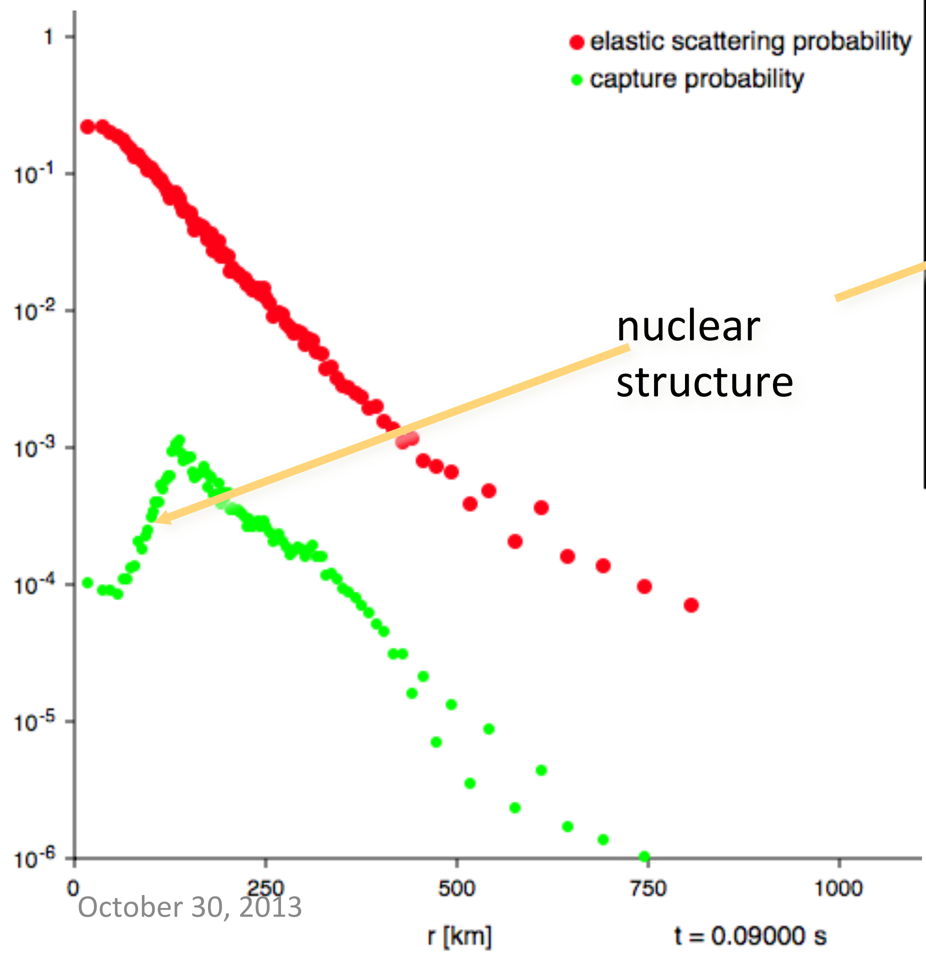


Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.07000 s

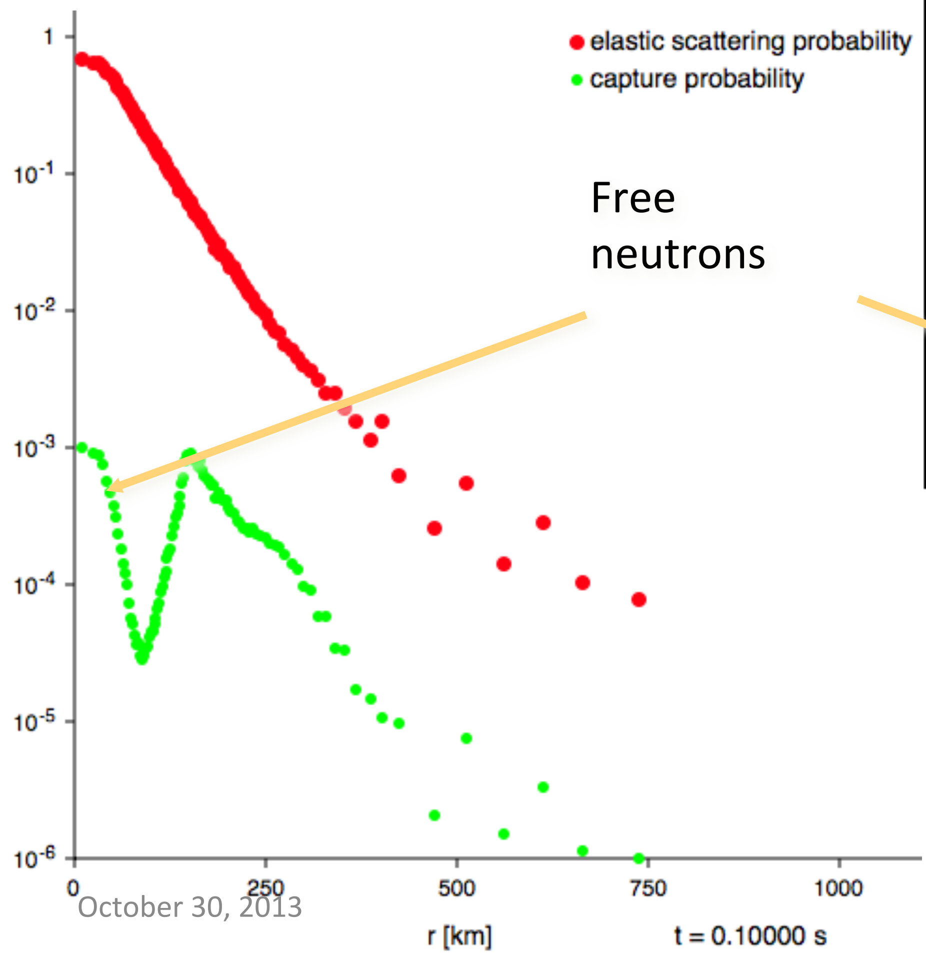
t = 0.09000 s



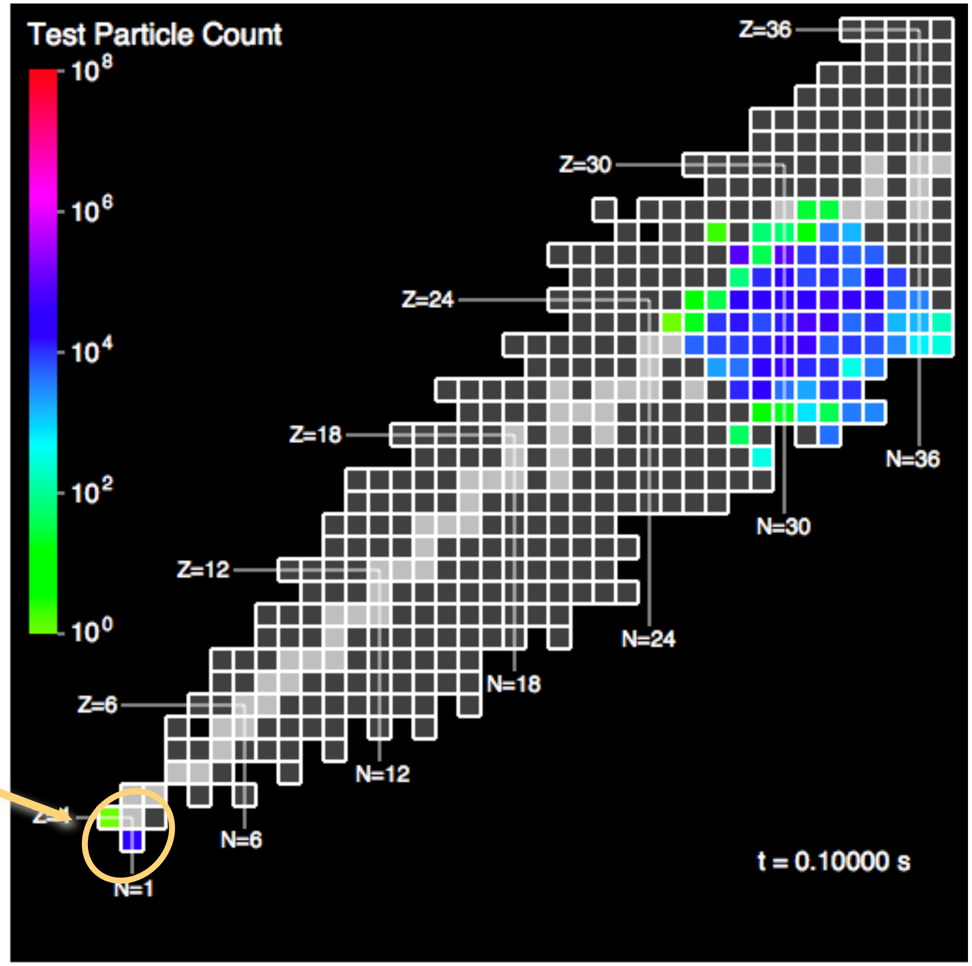
nuclear structure



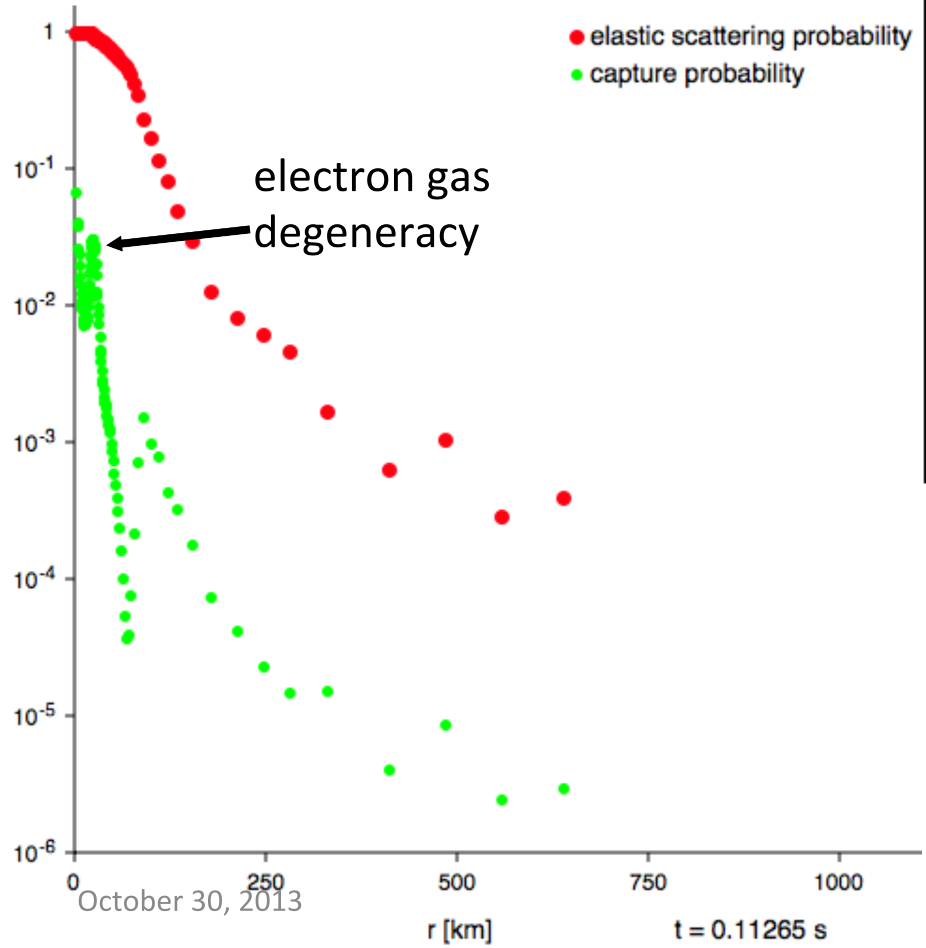
t = 0.10000 s



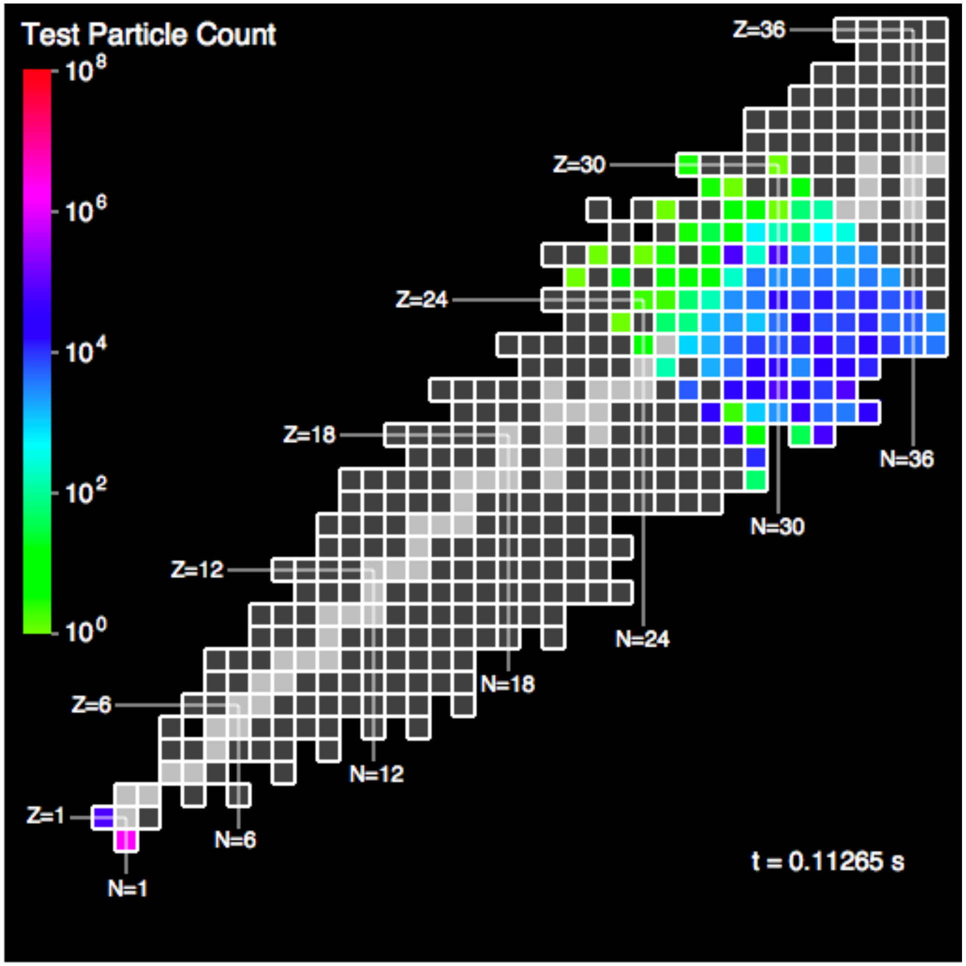
Free neutrons



t = 0.11265 s

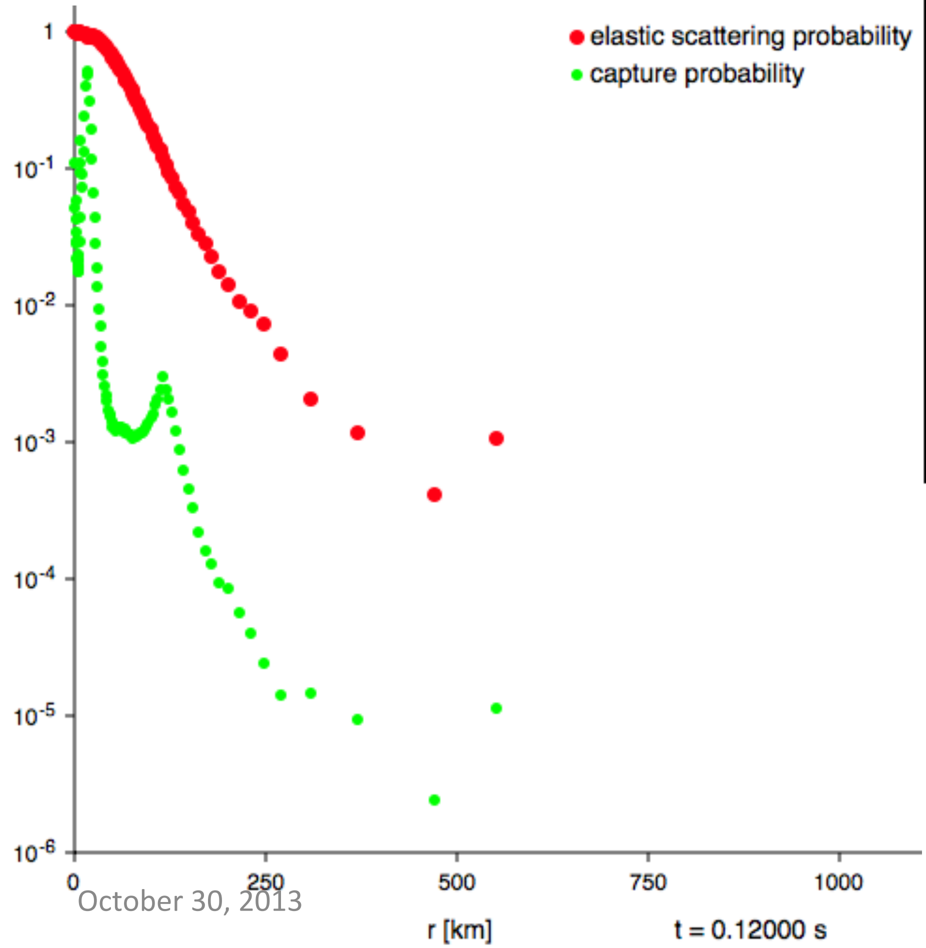


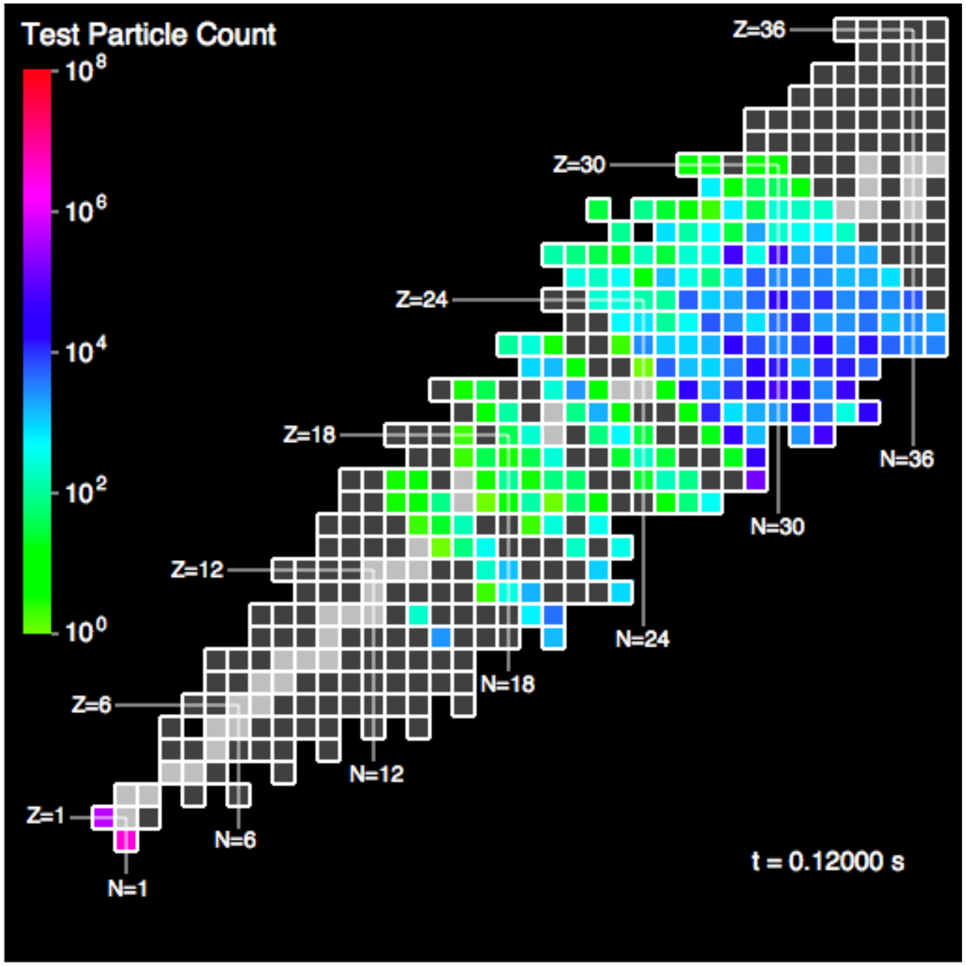
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
250
500
750
1000
r [km]
t = 0.11265 s
elastic scattering probability
capture probability
electron gas degeneracy



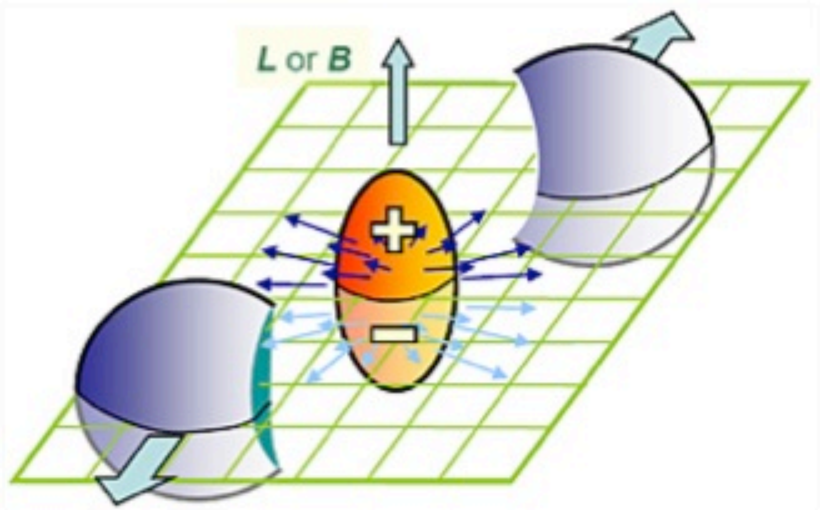
Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.11265 s

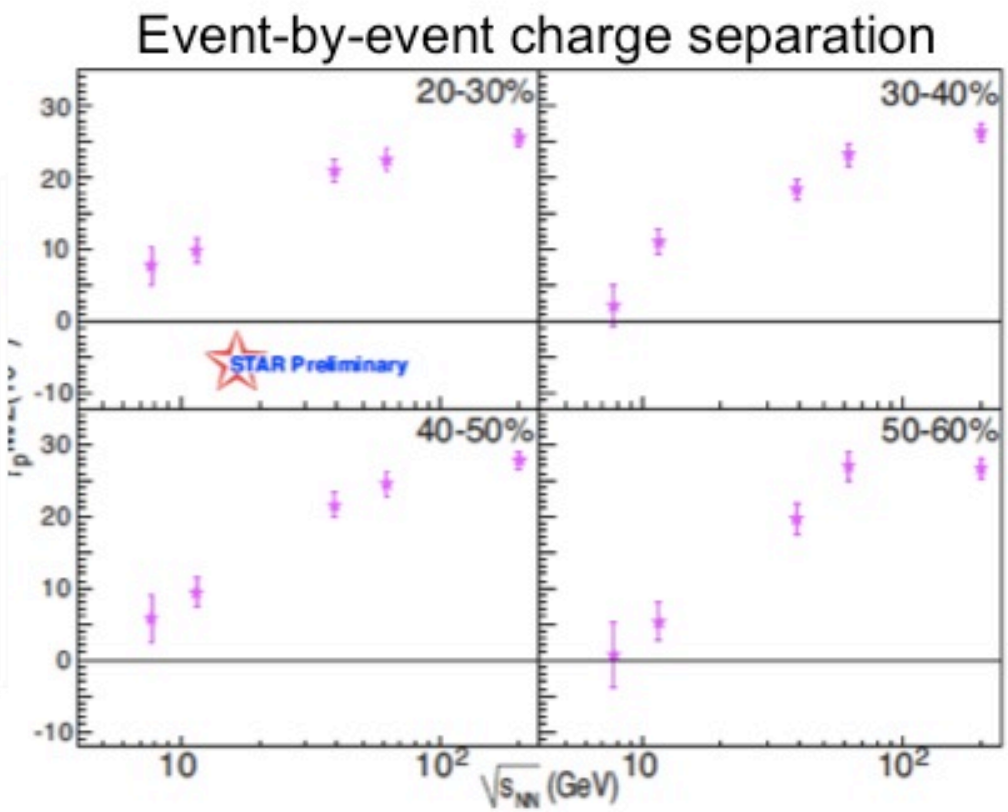
t = 0.12000 s


elastic scattering probability
capture probability
1
10-1
10-2
10-3
10-4
10-5
10-6
0
250
500
750
1000
r [km]
t = 0.12000 s


Test Particle Count
10^8
10^6
10^4
10^2
10^0
Z=36
Z=30
Z=24
Z=18
Z=12
Z=6
Z=1
N=1
N=6
N=12
N=18
N=24
N=30
N=36
t = 0.12000 s

Disappearence of QGP? LPV



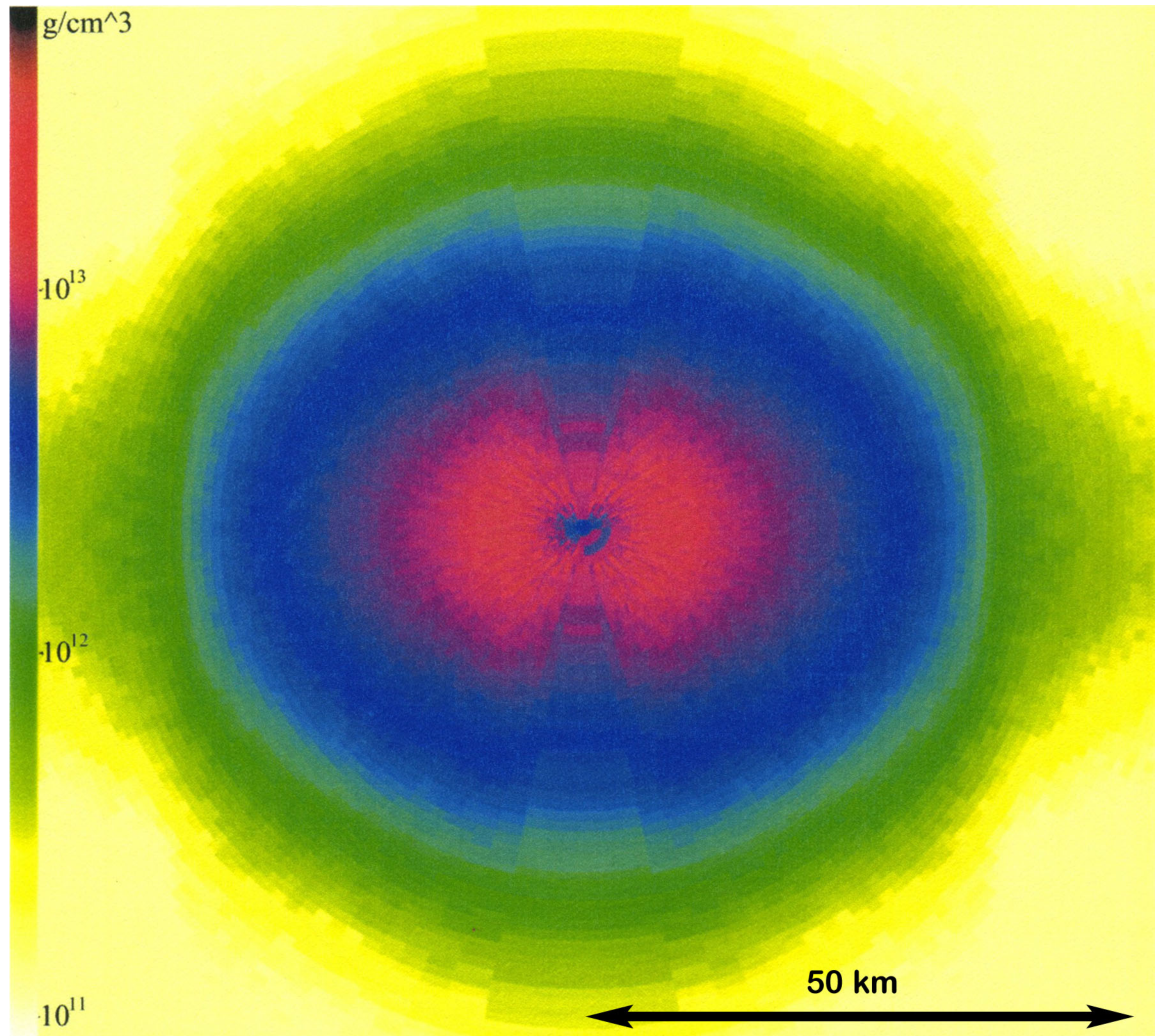


P. Sorensen (this meeting)

Charge seperation (thought to be related to parity violating regions in a QGP) "disappears" below ~19.6 GeV

Supernova Precursor Rotation

- Angular momentum conservation
- Baryons fall in on equator; neutrinos escape along poles
- Macroscopic parity violation
- Finite recoil of neutron start

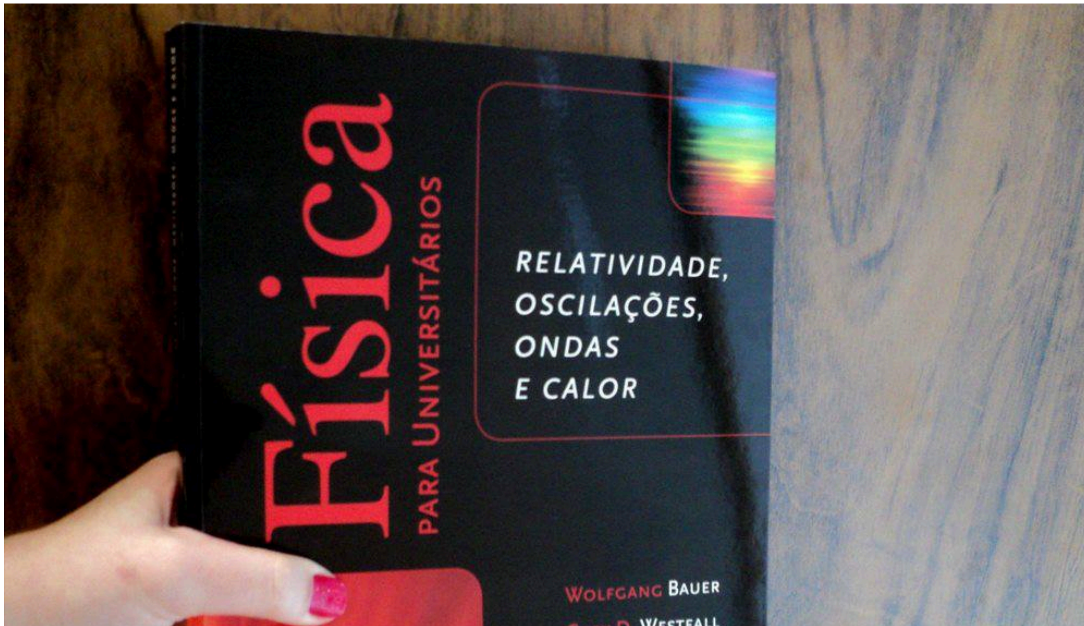


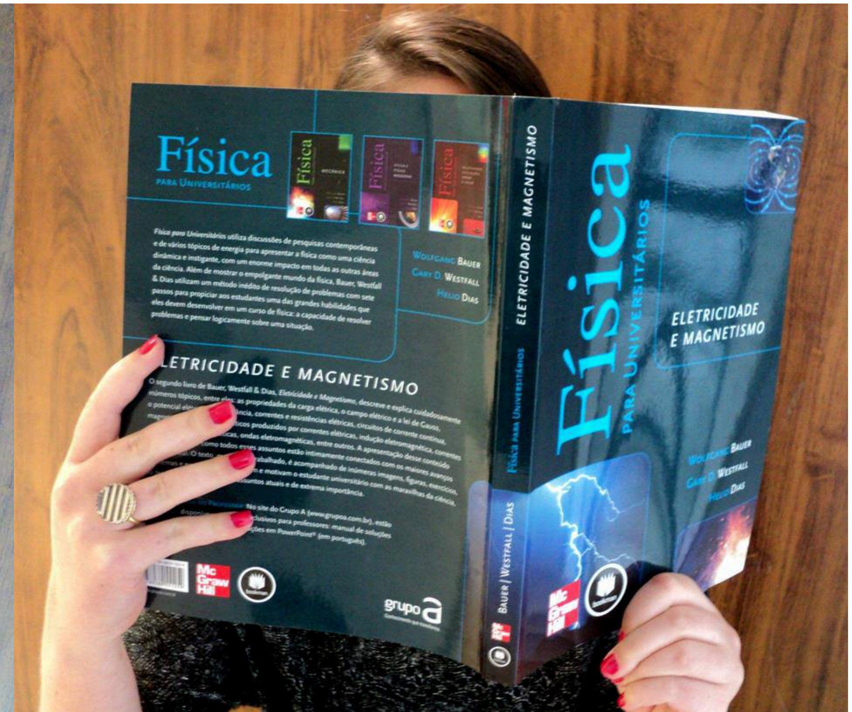
Summary

- New solution method for supernova dynamics
  - Test particle method
  - Link between nuclear dynamics and astrophysics
  - Passes all standard hydrodynamic verification tests
- New explosion mechanism
  - Shockwave originates ~ 50 km above neutron star surface
  - Due to neutrino heating / opacity change
  - VERY dependent on nuclear structure and neutrino cross sections

Mike Lisa:

What enters the textbooks?


Física
PARA UNIVERSITÁRIOS
RELATIVIDADE, OSCILAÇÕES, ONDAS E CALOR
WOLFGANG BAUER


Física
PARA UNIVERSITÁRIOS
ELETRICIDADE E MAGNETISMO
WOLFGANG BAUER
GARY D. WESTFALL
HELIO DIAS


Física
PARA UNIVERSITÁRIOS
MECÂNICA
WOLFGANG BAUER
GARY D. WESTFALL
HELIO DIAS
bookman


Física
PARA UNIVERSITÁRIOS
ÓPTICA E FÍSICA MODERNA

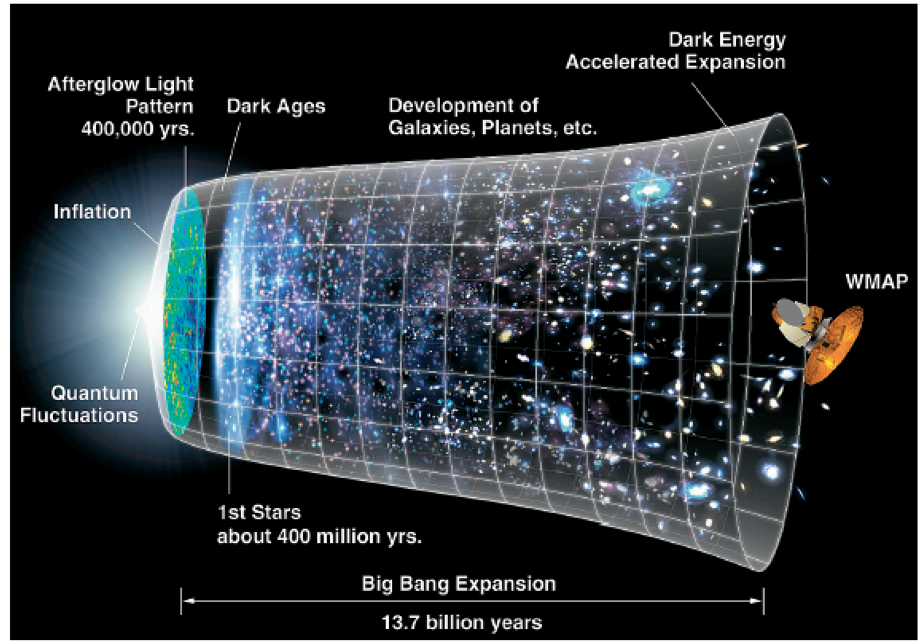


FIGURE 39.44 Illustration of the concept of inflation of the early universe.

of spontaneous symmetry breaking, to illustrate the concept: In liquid water, the water molecules can have any arrangement and orientation. However, in the process of freezing, this symmetry has to be broken to force the molecules into the crystalline structure of ice.)

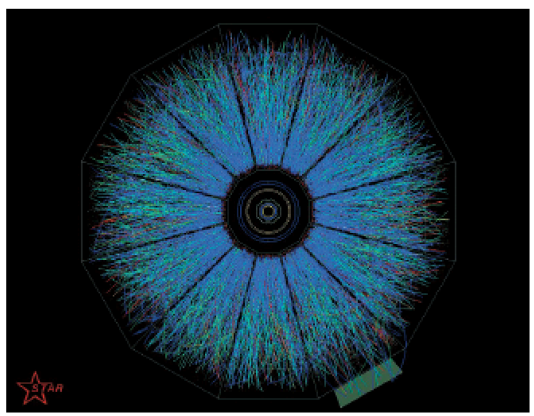
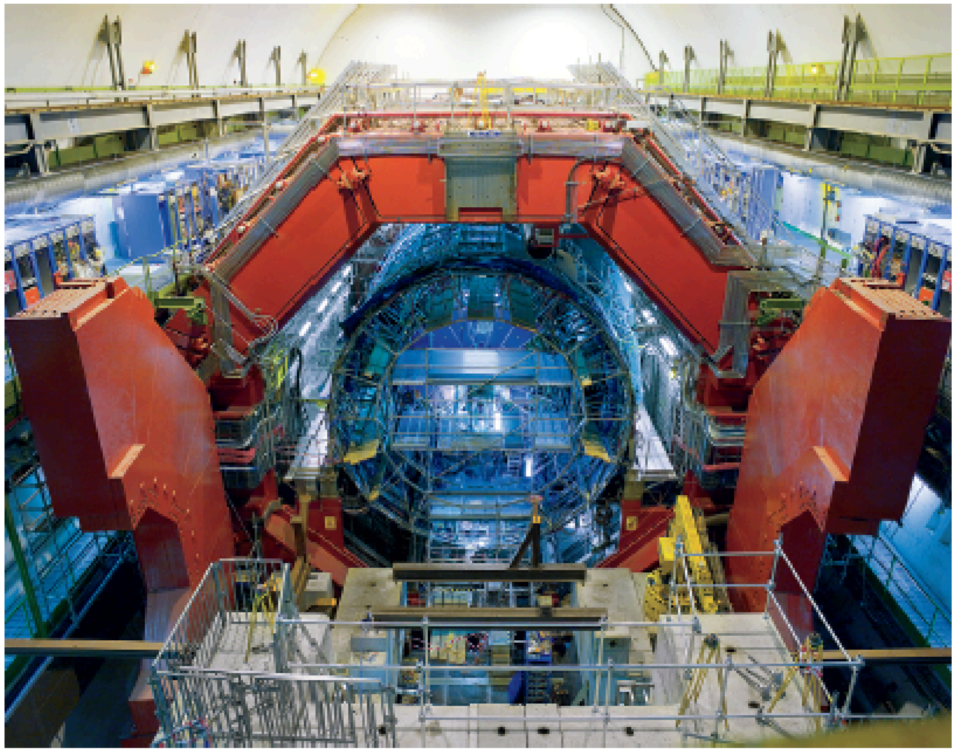


FIGURE 39.45 Event display from a collision of gold nuclei the Relativistic Heavy Ion Collider. Each line represents the track that one of the over 5000 particles resulting from the collision produced in the STAR detector. From these tracks, scientists are trying to determine the properties of the quark-gluon state of matter that was created transiently during the nuclear collision and learn about the history of the universe.

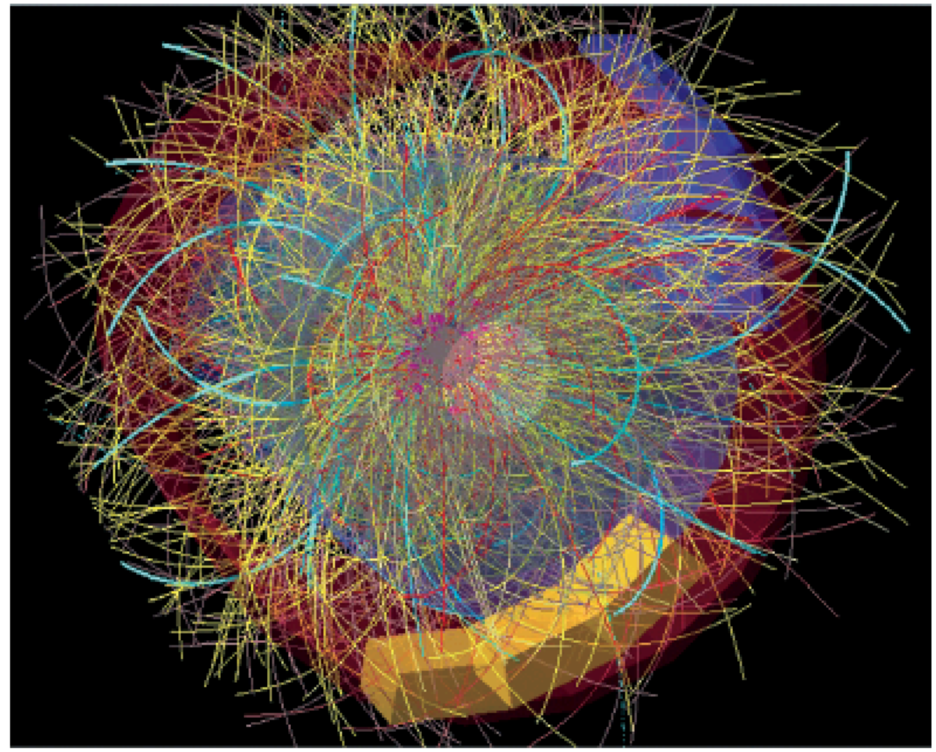
Quark-Gluon Plasma

After 10^{-11} s, and lasting until about 10^{-4} s, the universe was a mixture of quarks, gluons, and leptons, forming a plasma. Despite carrying color charges, the quarks and gluons were asymptotically free due to the high temperature. At about 10^{-4} s, the universe had cooled to a temperature of approximately $2.1 \cdot 10^{12}$ K ($k_B T = 180$ MeV). Lattice-QCD calculations indicate that at this temperature the quarks and gluons coalesced into color singlets.

Amazingly, the quark-gluon state of matter that dominated the early universe from 10^{-11} s to 10^{-4} s after the Big Bang can be recreated in accelerator-based experiments today. Experiments at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven, Long Island, use colliding beams of gold nuclei with total kinetic energy of up to 20 TeV each to probe the conditions during the early time evolution of the universe and to produce small volumes of the quark-gluon matter. However, this state of matter lasts less than 10^{-23} s in the laboratory before it explodes into more than 5000 particles, mainly pions. A collision event recorded at RHIC is displayed in Figure 39.45. Since 2010, nuclear physicists also have the Large Hadron Collider (LHC) at CERN at their disposal. It can collide lead ions at an energy of 2.76 TeV per nucleon pair, which is 13 times higher than the energy achieved by the RHIC.



(a)



(b)

FIGURE 8 (a) The ALICE detector at the Large Hadron Collider during its construction. (b) Electronically reconstructed tracks of thousands of charged subatomic particles produced inside the ALICE detector by a high-energy collision of two lead nuclei.

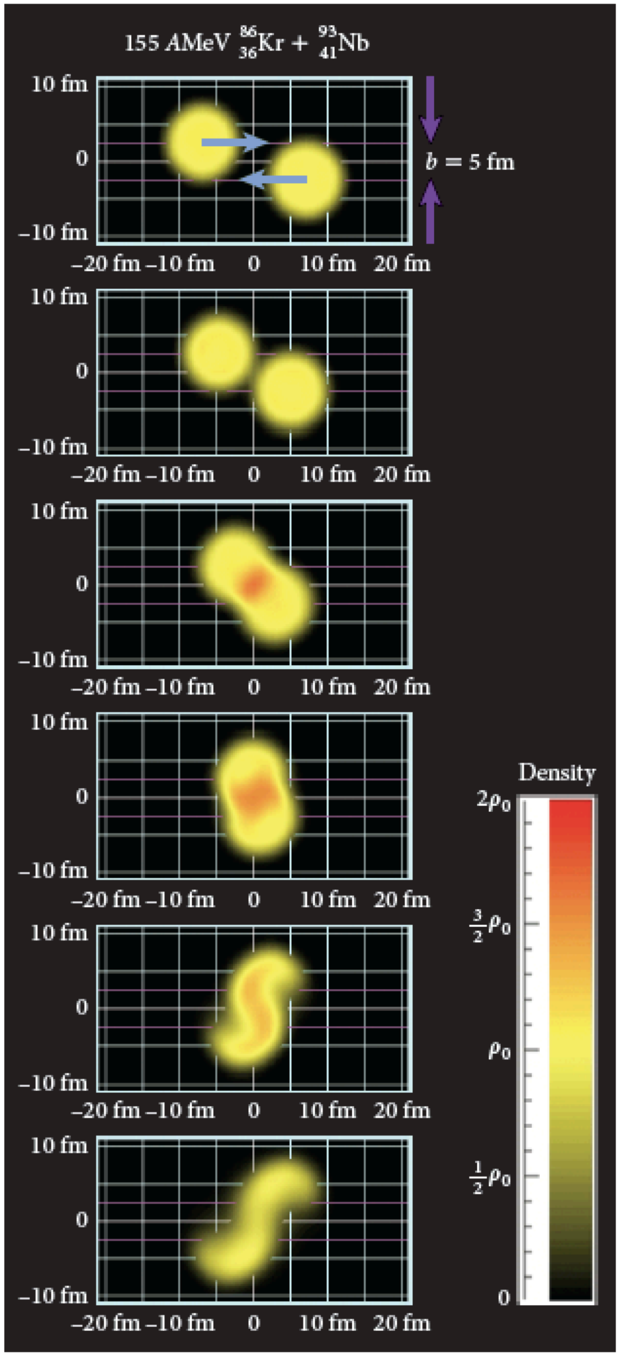


FIGURE 40.32 Time sequence of a nuclear collision, as calculated with a nuclear transport model.

EoS?

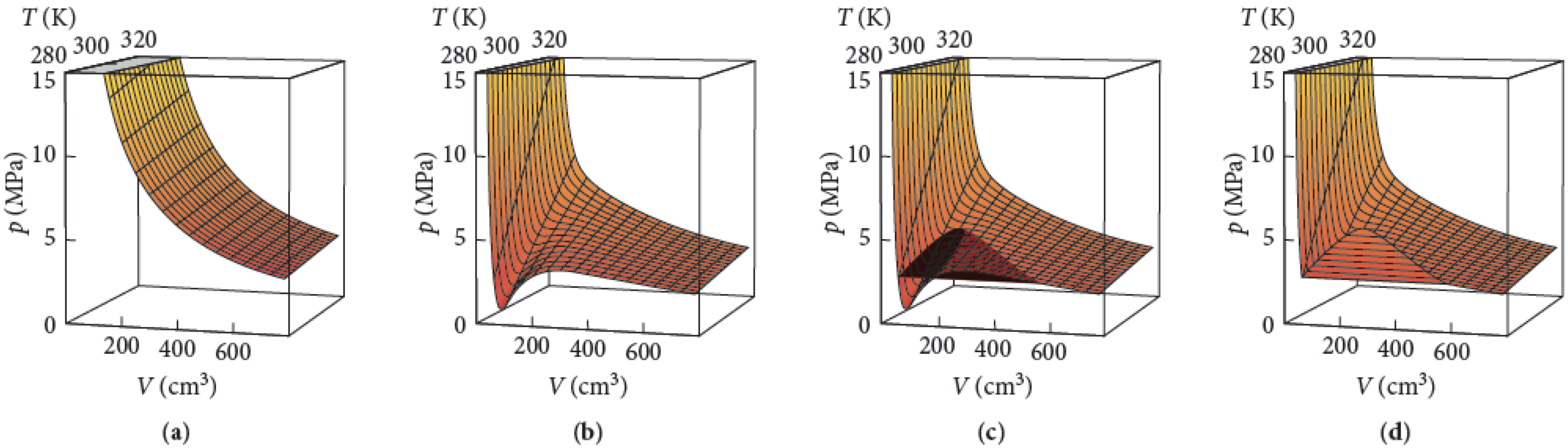
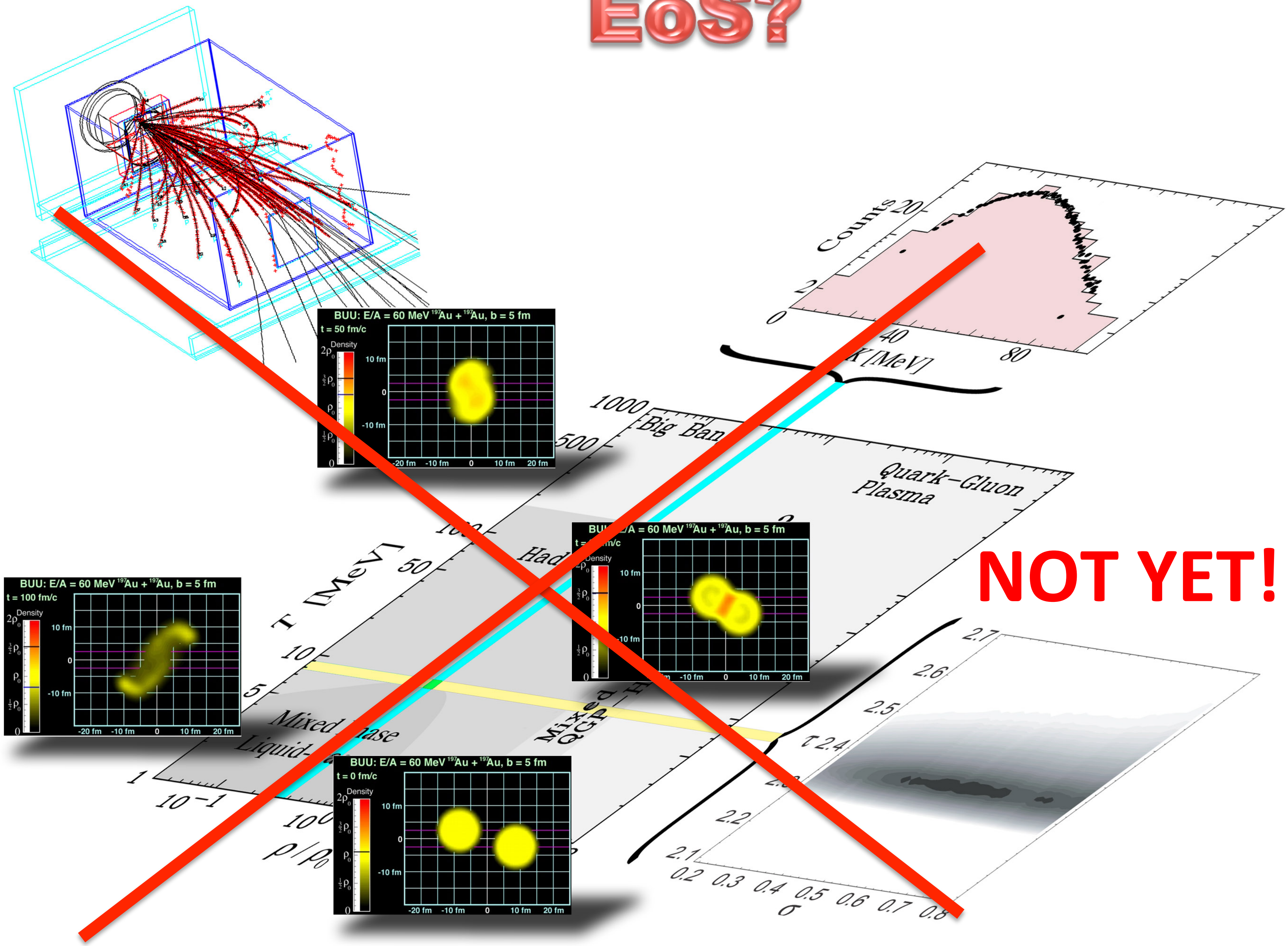


FIGURE 19.28 Pressure as a function of temperature and volume for 1 mole of ethane. (a) Ideal gas equation of state, (b) van der Waals equation, (c) Maxwell construction, (d) real equation of state for ethane.

EoS?


BUU: E/A = 60 MeV 197Au + 197Au, b = 5 fm
t = 50 fm/c
t = 100 fm/c
t = 0 fm/c
Density
Counts
K [MeV]
Big Bang
Quark–Gluon Plasma
T [MeV]
Mixed Phase
Liquid-
ρ/ρ0
σ
τ

NOT YET!


RANP 4 Advisory Board, August 1995

... just yesterday